

# Uncertainty, Reliability and Factor of Safety for Bearing Capacity of Shallow Foundations in Cohesive Soils

**Mwajuma Ibrahim Lingwanda**

*Department of Built Environment Engineering, Mbeya University of Science and Technology, Mbeya, Tanzania.  
Corresponding Author: Mwajuma Ibrahim Lingwanda*

## ABSTRACT

In geotechnical engineering practice, researchers have a task of bridging the traditional allowable stress design method with the relatively new and fast growing reliability based design method. One way of doing that is through expressing some relationship between parameters of the two methods in simple terms that can easily be understood and applied by practitioners. This study applied arbitrary chosen design parameters with simple Monte Carlo simulation procedure to determine probabilities of failure of shallow foundations on homogeneous saturated cohesive soil. The probabilities of failure were then plotted against a range of uncertainties expressed in terms of coefficient of variation and for a range of safety factors. The relationships so developed appear to be non-specific to particular type or size of shallow foundation but rather they are general. The study revealed that for undrained shear strength of soil, a factor of safety of 2.0 can accommodate uncertainties of up to 100% and result to a highly reliable design of a shallow foundation in relation to bearing capacity.

**KEYWORDS:** factor of safety, geotechnical reliability, Monte Carlo simulation, probability of failure, reliability based design, shallow foundations, uncertainty.

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## I. INTRODUCTION

Apart from structural design and adequate depth, foundations are normally designed to fulfil two main criteria; safety against bearing capacity failure and limiting settlement below an allowable value (Whitlow, 2001; Das, 2007). In relation to shallow foundations, three modes of bearing capacity failure are possible (Coduto, 2001; Whitlow, 2001; Craig, 2005); general shear failure, local shear failure and punching shear failure. In analysis of relatively incompressible and relatively strong soils, it is common to assume the general shear failure mode (Coduto, 2001) and Terzaghi's bearing capacity equation is commonly applied to shallow foundations. Traditionally, safety against shear failure is ensured by providing a lumped factor to virtually increase the applied load or reduce the ultimate resistance to an allowable value. The method is referred to as permissible or allowable stress design (ASD). Alternatively, reliability based design (RBD) may be applicable to rationally quantify the probability of failure or reliability index of the foundation for some specified uncertainties. However, due to the perceived complexity involved in RBD procedures, the method is currently not commonly applicable in routine designs.

In recent years, there has been a growing interest in application of RBD in geotechnical analysis and designs which is reflected by the amount of research related to the topic. Central to the application of RBD is the quantification of different types of uncertainties associated to soils, testing methods and transformation models. Studies by Cafaro and Cherubini (2002), Akbas and Kulhawy (2010) and Stuedlein et al. (2012a) for clay soils, Al-Naqshabandy et al. (2012) and Bergman et al. (2013) for lime cement columns, Phoon and Kulhawy (1999a) for different soil types and testing methods and Phoon and Kulhawy (1999b) for transformation uncertainties are well cited examples. Reliability concepts and methods have also found application in calibrating partial factors for load and resistance factor design method also known as the limit state method. Some examples in that area are Phoon et al. (2003), Fenton et al. (2005), Foye et al. (2006), Foye et al. (2009) and Forrest and Orr (2010). The applicability of RBD in geotechnical practice depends very much on how it is presented to practicing engineers. The current research trend in geotechnical RBD does not favour simplicity and applicability. Many existing literature involve complex mathematics which can be difficult to grasp. While the complex mathematics may be necessary for the theoretical development of RBD, there is a need to have studies which will bridge the theory to practice. Such easily understandable and user friendly studies will eventually increase the number of projects which benefit from what RBD has to offer.

One way of familiarising RBD is by being able to relate the common traditional design with some elements of RBD. This study aims at establishing relationship between uncertainties, reliability and factor of safety with respect to design of shallow foundation in saturated cohesive soils. Uncertainties are presented in terms of the coefficient of variation (COV); reliability is the probability of the design to be successful in performance and factor of safety is the traditional factor applied in deterministic design of shallow foundation that evolved through experience.

Generic values of soil properties for cohesive soils and a range of COV values were applied in a Monte Carlo simulation (MCS) process to determine the probability of exceeding the ultimate bearing capacity ( $q_f$ ) over a range of uncertainties. Relationships were then established between COV and  $p_f$  at a selected range of factors of safety. Such relationships may be useful even at early stages of planning a geotechnical investigation, for example, in predicting the consequences of selecting a certain soil investigation method to the design. As it is known, different testing methods are associated with different magnitudes of uncertainties. Lingwanda et al. (2017) obtained some significantly different magnitudes of total uncertainties between laboratory and in situ testing methods.

## II. METHODOLOGY

### 2.1 Setup of the design problem

Three cases of shallow footings were considered for design and their reliability analysed at different levels of uncertainties and factors of safety. The footings dimensions were arbitrary chosen while observing the criteria of being shallow. Table 1 indicate dimensions of the three foundation categories; rectangular, square and strip. The footings were assumed to rest on a homogeneous saturated clay so that the shear resistance of the soil is defined only in terms of its cohesion. This assumption was intended to minimize the number of random variables in the reliability analysis hence simplify the problem.

Saturated clays are assumed to generate positive excess pore water pressures when loaded. Consequently, the most likely time for bearing capacity failure is immediately after the load is applied (Craig, 2005, p. 293) and the undrained shear strength of the soil ( $s_u, \phi_u = 0$ ) is therefore applicable in the analysis. According to the guideline by Coduto (2001), shallow foundations on undrained clays are governed by general shear mode. Therefore, Terzaghi's ultimate bearing capacity formula for saturated undrained clays is applicable as indicated by Eq. (1).

$$q_f = s_u N_c + \gamma D \tag{1}$$

In which  $q_f$  is the ultimate bearing capacity defined as the load per unit area of foundation at which the shear failure occurs in the soil,  $s_u$  is the undrained shear strength of clay,  $N_c$  bearing capacity factor,  $\gamma$  unit weight of the soil and  $D$  depth of the footing.

Satisfying the ultimate limit state, the allowable bearing pressure of a given foundation is the maximum allowable net loading intensity of the ground expressed by Eq. (2). Engineers have to decide the values of  $FS$  using their experience and judgement keeping in mind such factors as soil type, type and amount of soil characterization data, soil variability, importance of the structure and the consequences of failure if it happens. According to Coduto (2001, p. 191), typical values of  $FS$  against bearing capacity failure of shallow foundations are between 2.5 and 3.5 but can occasionally be as low as 2.0 or as high as 4.0. Factors between 2.5 to 3 were suggested by Whitlow (2001, p. 456) while Craig (2005, p. 278) and Tomlinson (2001) suggested factors between 2 to 3 with 3 being most preferred.

$$q_a = \frac{q_f - \gamma D}{FS} + \gamma D \tag{2}$$

where  $q_a$  is the allowable load and  $FS$  a factor of safety. Other parameters are as defined in Eq. (1).

**Table 1. Dimensions of footings applied in the analysis**

FOOTING	Depth (m)	Breadth (m)	Length (m)
1. SQUARE	1.5	2.5	2.5
2. RECTANGULAR	4.5	6.0	15.0
3. STRIP	2.0	3.0	>>B

### 2.2 Monte Carlo Simulation (MCS)

MCS is a robust tool that can be used to facilitate reliability analysis. The method involves repetitive calculation of a mathematical or empirical operator in which variables within the operator are random with prescribed probability distributions. The numerical result from each repetition of the numerical process is considered as a sample of the true solution of the operator, just like an observation of a physical measurement (Wang et al., 2011).

Consider the problem of determining an allowable load ( $q_a$ ) of a footing of specified dimensions: The deterministic value of ultimate load ( $q_f$ ) is obtained by application of Eq. (1). A Monte Carlo process is then performed with Eq. (2) as the mathematical operator in which ( $s_u$ ) is supplied as a random variable with a defined distribution. The probability of failure ( $p_f$ ) is defined as the probability of ( $q_a$ ) exceeding ( $q_f$ ). The random soil property has uncertainty which is defined in terms of coefficient of variation (COV). Uncertainty of the load is not considered in this study for reasons of simplification. Moreover, as noted by Wu (2013), there is already an extensive literature covering it. Finally, relationships were established between COV and  $p_f$  for distinct values of  $FS$ . The MCS method has found application in geotechnical reliability problems such as by, among others Wang and Kulhawy (2008), Akbas and Kulhawy (2009), SivakumarBabu and Srivastava (2010), Uzielli and Mayne (2011) and Stuedlein et al. (2012b).

### III. ANALYSIS

In this study, analysis of the individual footings followed the following sequence: (i) determination of the ultimate load, (ii) determination of allowable load, (iii) MCS for determination of the probability of failure and lastly (iv) development of relationships between  $p_f$  and COV. Procedure number (ii) and (iii) were actually combined and performed with software MATLAB. The following sub-sections report details of the important steps.

#### 3.1 Determination of $q_f$

First Eq. (1) was applied to determine  $q_f$  with inputs of soil properties arbitrarily assumed from the experience of undrained clays existing in literature. From an extensive study comprising of more than 500 tests, Phoon and Kulhawy (1999a) reported a range of  $s_u$  of 6 – 412 kN/m<sup>2</sup> obtained through unconfined compression test (UC). A value of  $s_u = 40$  kN/m<sup>2</sup> was adopted for this study. Phoon and Kulhawy (1999a) also reported  $\gamma$  values for fine grained soils ranging from 14 – 20 kN/m<sup>3</sup> obtained from more than 3000 records. A value of  $\gamma = 18$  kN/m<sup>3</sup> was applied in this study. The value for bearing capacity factor ( $N_c$ ) for  $\phi_u = 0$  is 5.14 as can be found in several literature including Terzaghi et al. (1996). The values of  $D$  are as expressed in Table 1 for the respective footings.

Terzaghi's bearing capacity equation was basically developed for shallow continuous foundations, i.e those with a very large length over breadth ratio. For foundations of different shape, depth (non shallow), with inclined load, inclined base or inclined ground, corrections must be applied from the general formula. In this case, shape corrections were necessary for the square and rectangular footings. The factors applied were as suggested by Tomlinson (2001) which are, for the square footing  $S_c = 1.3$  and  $S_q = 1.2$  respectively for the first and second terms of Eq. (1). For the case of the rectangular footing,  $S_c = S_q = 1.08$  was applied. The ultimate loads for the three foundation cases were found to be 241.6 kN/m<sup>2</sup>, 303.0 kN/m<sup>2</sup> and 294.3 kN/m<sup>2</sup> respectively for the strip, rectangular and square footing.

#### 3.2 Determination of $q_a$

With application of Eq. (2), the allowable loads were determined at a range of factors of safety. As previously indicated, the common range of  $FS$  for shallow foundations is between 2.0 and 4.0. However, it would be interesting to understand the relationship between COV and  $p_f$  even for cases where failure is inevitable i.e  $FS < 1$ . Therefore, initial values of  $FS$  were selected to be 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0. However, during the analysis, large intervals were observed between successive results and hence intermediate  $FS$  values were introduced for analysis as 0.8 and 1.3. Moreover, for  $FS = 2.5$  the result was  $p_f = 0$  throughout the range of COV and therefore were eliminated in further analysis. Finally, the analysis settled at  $FS$  values of 0.5, 0.8, 1.0, 1.3, 1.5 and 2.0.

#### 3.3 Monte Carlo simulation and determination of $p_f$

In the MCS process, the mathematical expression involved was the determination of  $q_a$  as described in section 3.2. The only input random variable in the MCS was  $s_u$  of which its variability and distribution was to be defined first. In real practice, values of  $s_u$  are obtained from UC tests on undisturbed samples or from field tests such as vane test, pressuremeter test, cone penetration test or standard penetration test. However, Terzaghi et al. (1996) discouraged the use of standard penetration test data to obtain  $s_u$  values for footing designs due to their crudeness.

Phoon and Kulhawy (1999a) reported a range of COV for  $s_u$  obtained by UC method to be 6 – 56%. For field obtained  $s_u$  values, COV are expected to be generally larger than the laboratory counterparts (Lingwanda et al., 2017). This is because of further uncertainties induced in the process of transforming measured data to design

data. To account for uncertainties larger than 56%, it was decided to consider a COV range of 5 – 100% at intervals of 5%.

Normal and lognormal distributions are the most commonly applied distributions for geotechnical random properties, example Baecher and Christian (2003). However, Lingwanda et al. (2015) has studied the effect of application of normal distributions in MCS problems and concluded that the amount of negative variables generated by normal distributions can be significantly large and affect the accuracy of the calculated probability of failure. To avoid such effects, lognormal distributions of  $s_u$  were applied throughout the analysis in this study.

The number of simulations ( $N$ ) of a Monte Carlo process is known to affect the accuracy of results such that to achieve a high accuracy,  $N$  should be sufficiently high. An approach adopted by Uzielli and Mayne (2011) to determine  $N$  seems objective and clear, and therefore was applied in this study. In their approach, a confidence level ( $\alpha$ ) is specified together with a target probability of failure ( $\hat{p}_f$ ) and  $N$  is determined through Eq. (3). A value of  $\alpha = 0.99$  and  $\hat{p}_f = 10^{-6}$  were adopted, giving  $N = 5,000,000$  to the nearest millions. It is important to note that the adopted  $\hat{p}_f$  value considered to be a high reliability according to U.S Army Corps of Engineers (1997).

$$N = \frac{-\ln(1 - \alpha)}{\hat{p}_f} \quad (3)$$

After performing a simulation, the number events in which the simulated allowable load exceeds the ultimate load,  $n(q_a > q_f)$  were counted, recorded and used to calculate  $p_f$  values with application of Eq. (4). Lastly, the relationships between  $p_f$  and COV were determined using the ordinary least squares method adopting correlation equations with highest coefficient of determination.

$$p_f = \frac{n(q_a > q_f)}{N} \quad (4)$$

#### IV. RESULTS AND DISCUSSION

Results from the analysis were tabulated for each footing (and a factor of safety) in terms of  $p_f$  against corresponding COV. At first, plots were established for COV against  $p_f$  for each individual footing and  $FS$  value. The curves looked very much alike for all the three footings. It was then decided to combine the data and consider plots for all three footings together. This way eliminated the need to identify individual footings and hence results of this study represents a general case of shallow footings regardless of their configuration.

It can be observed from Figure 1 that within the range of COV from 5% to 100%, the probability of failure ranges from 0 (high reliability) to 1.0 (hazardous). When  $F = 0.5$ , failure seems to be inevitable as  $p_f = 1$  for all cases of COV. Similarly for  $F = 0.8$ ,  $p_f = 1$  for COV up to 30% but slightly decreases with increase in COV. The decrease in  $p_f$  may be attributed to the non-symmetrical appearance of lognormal distributions which in this case may be resulting lower  $q_a$  values at higher COVs hence a lower  $p_f$ . The curve for  $F = 1.0$  is almost constant at  $p_f$  slightly below 0.5 with a slight decrease for increased COVs.

Figure 1 was then divided into six distinct zones ranging from high reliability to hazardous. The zones were suggested by U.S Army Corps of Engineers (1997), also previously applied by Phoon (2008). It can be seen from Figure 1 that the curves for  $FS = 0.5$ ,  $FS = 0.8$  and  $FS = 1$  fall in the zone for hazardous reliability. However, the zones described as high reliability to poor could not be clearly presented in Figure 1 due to scale effects. Therefore, Figure 2 was created to magnify and clarify the poor, below average and above average zones of reliability.

It is indicated in Figure 2 that for  $FS = 1.3$ , one can obtain high, good, above average, below average or poor reliability depending on the magnitude of uncertainties. The relationship between  $p_f$  and COV can also be expressed by the strong polynomial function indicated in the figure. It can be observed that Figure 2 could not clearly show the zones of high to above average reliability due to scale effects. Therefore, Figure 3 was constructed to magnify these two reliability zones.

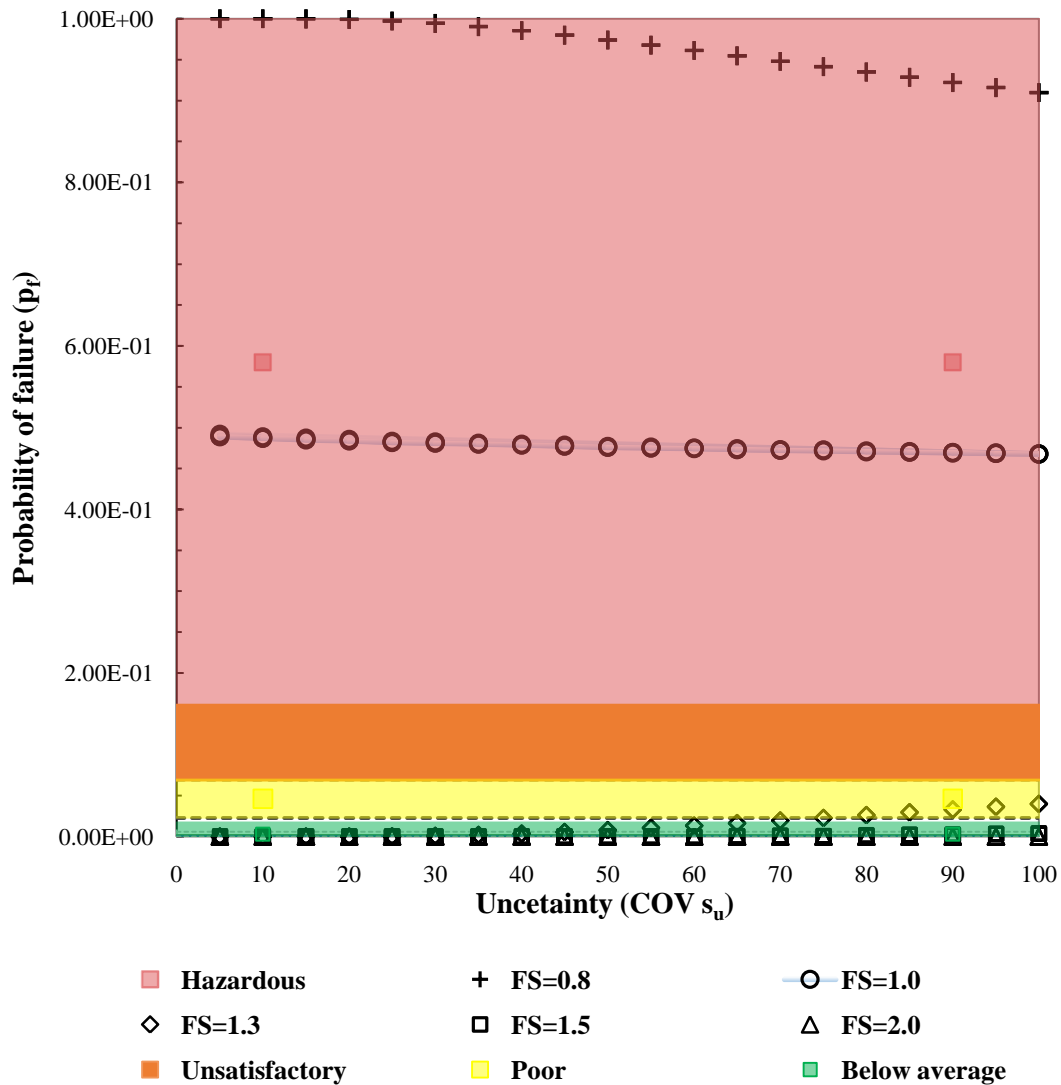


Figure 1. Relationship between uncertainties (COV) and the probability of exceeding ultimate bearing capacity ( $p_f$ ) for a range of  $p_f$  from 0 to 1 (high reliability to hazardous)

As indicated in Figure 3, the relationship between  $p_f$  and COV for  $FS = 1.5$  can be represented by the polynomial function indicated. The polynomial has a very strong coefficient of determination which is also shown in the figure. The curve spreads within zones of high, good and above average reliability. For COV less than and up to 45%, the value of  $p_f$  is almost zero for a factor of safety of 1.5. Recall that Phoon and Kulhawy (1999a) suggested a range of COV from 6 to 56% for  $s_u$  obtained through laboratory UC tests. From results of this study it can be concluded that, by applying a factor of safety  $FS = 1.5$ , the designer is likely to produce foundations of high to good reliability in saturated clays under undrained conditions when  $s_u$  data are obtained from UC tests provided that the only uncertainties come from the undrained shear strength.

When the factor of safety is increased to 2.0, Figure 3 indicates that the values of  $p_f$  will almost be zero throughout the studied range of COV and the reliability can be rated as high according to the definition by U.S Army Corps of Engineers (1997). By assuming that uncertainties due to other factors than the soil properties are kept minimum, such that the total design uncertainties are not greater than 100%, a factor of safety of 2.0 will be sufficiently high enough for deterministic design of shallow foundations under undrained condition.

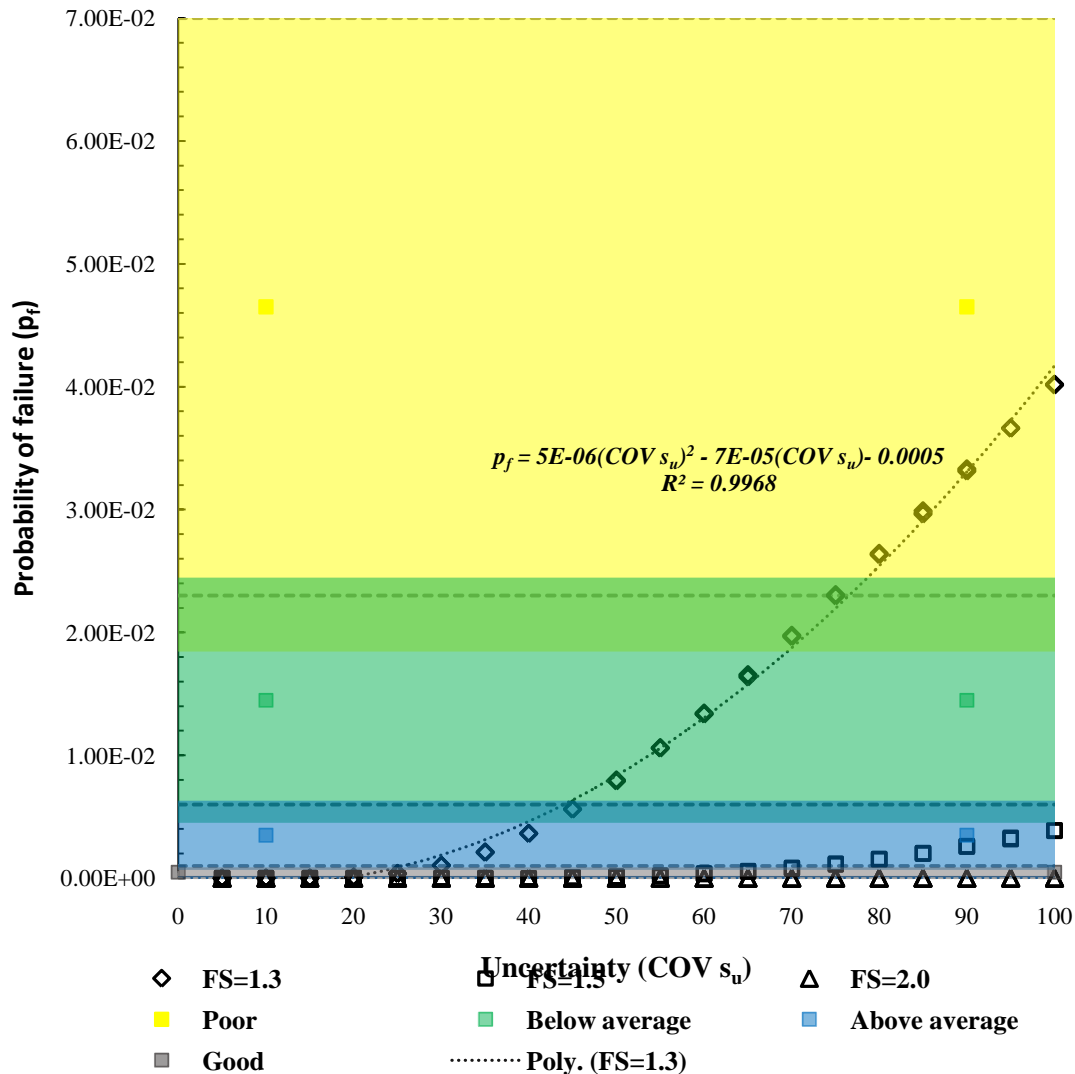


Figure 2. Relationship between uncertainties (COV) and the probability of exceeding ultimate bearing capacity ( $p_f$ ) for a range of  $p_f$  from 0 to 0.07 (high reliability to poor)

### V. CONCLUSIONS

A study has been conducted to establish relationship between uncertainties in terms of coefficient of variation, reliability in terms of probability of failure and factor of safety for shallow foundations in saturated cohesive soils. Monte Carlo simulation process has been employed with arbitrary chosen parameters for determination of bearing capacity. The procedure used can easily be followed and replicated in similar problems. Besides bringing understanding of the consequences of different magnitudes of uncertainties in design of shallow foundations, results of this study can be applied in deciding the type of geotechnical testing method during planning since individual soil testing methods are accompanied with different magnitudes of uncertainties. The following conclusions can be drawn from this study;

- (i) Relationships between uncertainties in terms of COV and reliability of a bearing capacity design at a given factor of safety are independent of design problems. That is, the relationships suggested in this study represent a general case for shallow footings under undrained conditions regardless their shape or size.
- (ii) For shallow foundations constructed over saturated clay soils under undrained conditions, a factor of safety against bearing capacity failure of 2.0 is sufficiently high to cater for uncertainties up to 100%. In other words, providing factors of safety larger than that may be considered uneconomical and unnecessary provided that the 100% represent total uncertainties of a particular foundation design.

To expand the knowledge gained in this study, it is recommended to perform similar studies incorporating other design models and more than one probabilistic parameter.

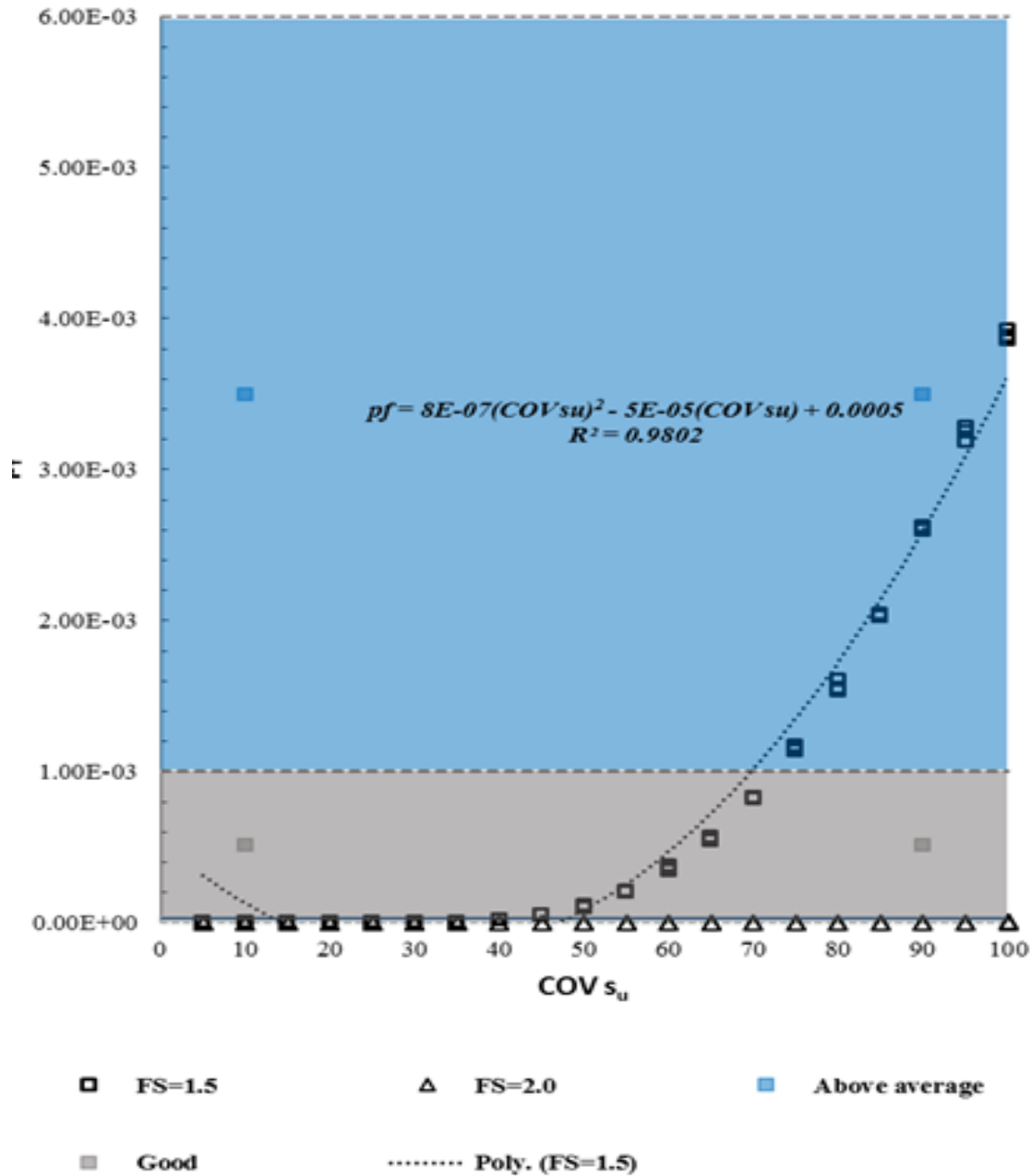


Figure 3. Relationship between uncertainties (COV) and the probability of exceeding ultimate bearing capacity ( $p_f$ ) for a range of  $p_f$  from 0 to 0.006 (high reliability to above average)

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