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A New Method Of DOA Estimation Based On The Decomposition Of Eingenvalues Of Covariance Matrix: Analysis Under Conditions Of Mutual Coupling.

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ABSTRACT

In the context of sensor array applied to the field of Direction of Arrival (DOA) estimation there were several methods, whose development had the objective to identify the angle of incidence of electromagnetic waves for wireless communication systems. This paper proposes a new DOA estimation algorithm based on eigenvalues combination of the estimated spatial covariance matrix. The proposed method presented competitive RMSE performance when compared to MUSIC, based on orthogonal subspaces, in mutual coupling condition, specifically for large number of sources.

KEYWORDS: Sensor Array, DOA, Wireless, eigenvalues, MUSIC, Covariance Matrix, Mutual Coupling.

I. INTRODUCTION

Several DOA (Direction Of Arrival) estimation techniques have been developed over the years, however the influence of mutual coupling is little considered since it degrades the estimation. the mutual coupling causes a distortion of the phase, changing the quality of the detection of the angle of incidence [10]. In [5], was analyzed the change of Steering Vector (Directional Vector) caused by the mutual coupling and its effect in the detection of the angle of incidence. Thus, this paper deals with such an estimation problem in a Uniform Linear Array (ULA) under coupling conditions.

The most analyzed papers that involve the mutual coupling use the MUSIC(Multiple Signal Classification)[8]. The MUSIC algorithm is widely used because of its easy implementation and parametric characteristics. However, such a method suffers performance losses at low SNRs(Signal-to-Noise Ratio), especially when electromagnetic characteristics are present[10][9][4].

Looking for a robust estimation in a presence of mutual coupling and white noise, this paper proposes a new DOA technique inspired by the Differential Spectrum[2] [6]. A study of the behavior of the eigenvalues of the spatial covariance matrix allows us to determine a combination of them in order to construct a new approach for DOA detection. Thus, greater precision and robustness are sought in unfavorable circumstances, making a closer approximation to real cases.

This paper presents a brief description of the signal model with the presence of a matrix representative of the coupling between the sensors, followed by an eigenvalue analysis and the proposed new spectrum. Comparisons are made with the MUSIC method for several situations in terms of RMSE (Root Mean Square Error) versus SNR curves. Finally, conclusions and perspectives of future work are presented.

II. SIGNAL MODEL

Consider an ULA with K sensors receiving M narrowband plane waves from far-field sources, where $M < K$ and the parameter M is assumed to be known [6] [7]. A $K \times 1$ vector of sensor outputs, in presence of additive noise $\mathbf{e}(n)$ on time instant n can be modeled as [3]:

$$\mathbf{v}(n) = \mathbf{Z}_0^{-1}\mathbf{A}\mathbf{s}(n) + \mathbf{e}(n) \tag{1}$$

where $\mathbf{v}(n) \in \mathbb{C}^{K\times 1}$ is the noisy data vector (snapshot), $\mathbf{A} = [\mathbf{a}(\theta_1), ..., \mathbf{a}(\theta_M)]$ is the $K \times M$ matrix of source steering vectors, $\mathbf{s}(n)$ is the $M \times 1$ vector of random source waveforms, $\mathbf{e}(n)$ is the $K \times 1$ vector of sensor noise and $\mathbf{Z}_0$ is the $K \times K$ normalized impedance matrix.

The $K \times 1$ steering vector $\mathbf{a}(\theta)$ on direction θ, can be described as [7]:

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 & e^{\frac{j2\pi d \sin\theta}{\nu}} & \ldots & e^{(K-1)\frac{j2\pi d \sin\theta}{\nu}} \end{bmatrix} \quad (2)$$

where d is the distance between the sensors and ν is the wavelength. Additionally, the matrix $\mathbf{Z}_0$ can be written as[3]:

$$\mathbf{Z}_0 = \begin{bmatrix} 1+\frac{Z_{11}}{Z_L} & \frac{Z_{12}}{Z_L} & \cdots & \frac{Z_{1K}}{Z_L} \\ \frac{Z_{21}}{Z_L} & 1+\frac{Z_{22}}{Z_L} & \cdots & \frac{Z_{2K}}{Z_L} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{Z_{K1}}{Z_L} & \frac{Z_{N2}}{Z_L} & \cdots & 1+\frac{Z_{KK}}{Z_L} \end{bmatrix} \quad (3)$$

where Z_L is the load impedance and Z_{ij} for $j = 1, ..., K$ and $i = 1, ..., K$ represents the mutual impedance between the i-th and j-th sensors if $i \neq j$, or else the self-impedance for $i = j$. The Z_{ij} impedance is calculate according [1] .

The $K \times K$ spatial covariance matrix of the array output signal $\mathbf{v}(n)$ is given by [3]:

$$\mathbf{R} = E\left\{\mathbf{v}(n)\mathbf{v}^H(n)\right\} = \mathbf{Z}_0^{-1}\mathbf{APA}^H(\mathbf{Z}_0^{-1})^H + \sigma^2\mathbf{I} \quad (4)$$

where $\mathbf{P} = E\{\mathbf{s}(n)\mathbf{s}^H(n)\}$ is an $M \times M$ covariance matrix of signal waveform, $\mathbf{I}$ is the identity matrix, $E\{\cdot\}$ and $(\cdot)^H$ denote expectation operator and Hermitian transpose, respectively, and σ^2 is the noise power.

The sample covariance matrix can be calculated by [7]:

$$\widehat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{v}(n)\mathbf{v}^H(n)(0) \quad (5)$$

where N is the number of snapshots.

Equation 4 presents the covariance matrix modified by the coupling. The presence of the terms of the impedance matrix distorts the detection as the subspace of the signal is modified and thus the orthogonality of the signal subspace and noise subspace is changed, causing problems in the estimation of the angle

III. PROPOSED METHOD

According to [7] the eigenvalues is robust in the presence of noise and this behavior inspired the Differential Spectrum [2][6] , that perform a low estimation threshold. Looking for a robust DOA method in a presence of mutual coupling and noise this paper investigates the feature of the eigenvalues of an augmented spatial covariance matrix $\breve{R}$ to propose a new method for DOA estimation. To make $\breve{R}$ dependent on a search angle θ_S, we can write it as [2]:

$$\breve{R}(\theta_S) = \hat{R} + \Phi(\theta_S) \quad (6)$$

Where $\hat{R}$ is sample covariance matrix and $\Phi(\theta_S)$ is K x K and defined by:

$$\Phi(\theta_S) = a(\theta_S)a(\theta_S)^H \quad (7)$$

However, the eigendecomposition of $\breve{R}$ allows us to write :

$$\breve{R} = \widetilde{U_S}.\widetilde{\Sigma_S}.\widetilde{U_S^H} + \widetilde{U_\varepsilon}.\widetilde{\Sigma_\varepsilon}.\widetilde{U_\varepsilon^H} \quad (8)$$

where $\widetilde{\Sigma_S}$ is the (M+ 1)×(M+ 1) diagonal matrix of signal subspace eigenvalues, $\widetilde{\Sigma_\varepsilon}$ is the (K−M−1)×(K−M−1) diagonal matrix of noise subspace eigenvalues, $\widetilde{U_S}$ and $\widetilde{U_\varepsilon}$ contain the signal and noise-subspace eigenvectors, respectively.

The (M+ 1) largest eigenvalues of $\widetilde{R(\theta_S)}$ and their corresponding eigenvectors span the signal subspace, while the remaining eigenvalues and eigenvectors span the noise subspace. If θ_S is swept along the interval [−90◦,90◦] and coincides with the DOA angle, then the dimension of the signal subspace is reduced to M. As a consequence, more power is concentrated in the first eigenvalue $\widetilde{\lambda_1}(\theta_S)$, draining energy from the second $\widetilde{\lambda_2}(\theta_S)$, and from the noise eigenvalues, $\widetilde{\lambda_{M+1}}(\theta_S)$, $\cdots$,$\widetilde{\lambda_K}(\theta_S)$, , which attain their minima[6]. So, we exploit this behavior to introduce a new spectrum defined as:

$$S(\theta) = \frac{\widetilde{\lambda_1}(\theta)}{\widetilde{\lambda_1}(\theta).\prod_{j=M+1}^{K}\widetilde{\lambda_j}(\theta)} \quad (9)$$

whose peaks indicate the DOA angles.

Fig. 1 superimposes 1000 experiments that evaluate (9) for N= 100 snapshots taken on K= 10 mutually coupled sensors that receive M=7 completely uncorrelated incident waves under SNR = 5 dB. Additionally the function S(θ) is normalized in the range[0,1]. According to Fig. 1 all 7 signal sources were detected despite the noise and mutual coupling considered. The values of the peaks of the 1000 experiments varied little around the incidence angles, which allowed the correct detection of the angles. For a more robust analysis of the performance of the proposed method will be analyzed the behavior of the method by the variation of REQM (Root Mean Square Estimation Error) with SNR(Signal to Noise Ratio) and compared with the MUSIC method.

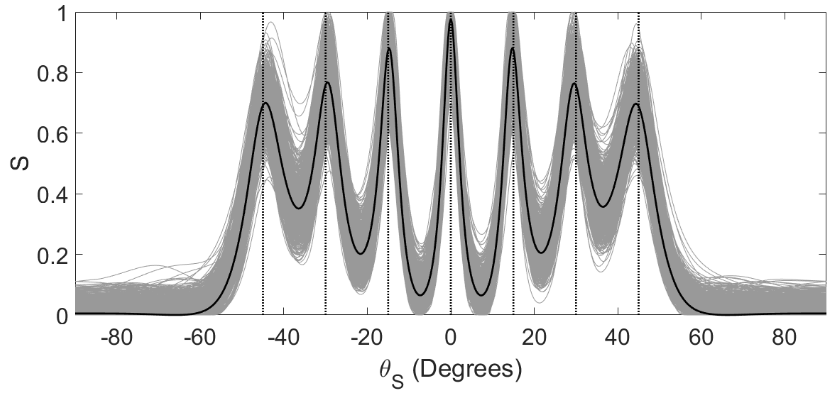

Figura 1- The gray lines represent each one of the 1000 experiments, the blackline stands for the average behavior and the vertical dotted lines indicate the true DOAs.

IV. RESULTS

In order to analyze the performance of a DOA estimation method, RMSE(Root Mean Square Error) and its variation in relation to SNR are used. The RMSE provides a reliable result as it is performed with 1000 experiments and over a wide range of SNR. We analyze the root mean square estimation error (RMSE) of proposed method and MUSIC algorithms, in order to compare the performance and robustness against noise in coupling condition. The simulation was performed using K= 10 sensors, M= 7 completely uncorrelated incident waves, N= 100 snapshots and SNR ranging from 0 to 20 dB with 1000 experiments.
Fig. 2 shows the performance of the estimation with seven DOA angles at -45°, -30°, -15°, 0°, 15°, 30° and 45°.

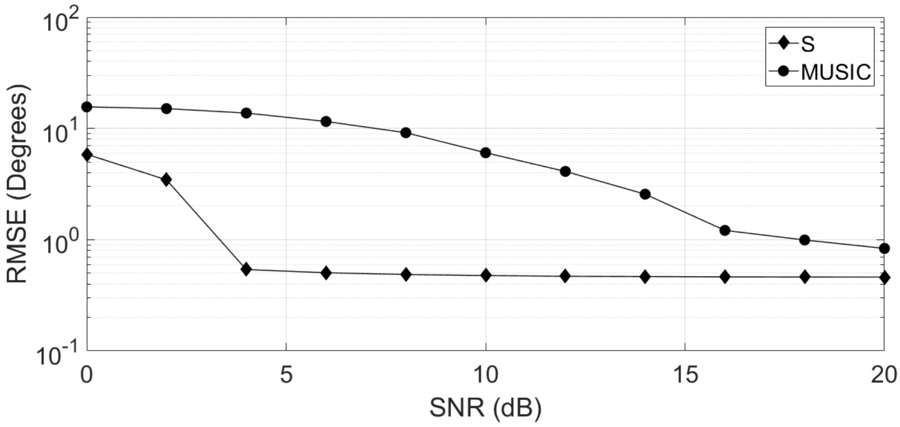

Figure 2- RMSE performance of MUSIC and the proposed method for seven sources

The MUSIC algorithm presents a RMSE below 1 degree only with SNRs above18 dBs while the proposed method with SNRs from 4 dBs, which shows a difference of 14 dBs in performance.

V. CONCLUSIONS

A new method of DOA estimation was presented considering the mutual coupling between sensors. For the analysis of its performance was used the RMSE varying with the SNR, and compared with the performance of the MUSIC algorithm. The results show that the new algorithm has a performance better than the MUSIC algorithm. The mutual coupling consideration between sensors, makes the analysis closer to reality, since the sensors are subject to interact with each other, which changes the DOA estimation. The difference of 14 dBs in performance (RMSE below 1 degree for SNR of 4 dBs for new method and SNR of 18 dB for MUSIC algorithm) allows the application of the proposed method in much more severe reception conditions (presence of higher noise considering the mutual coupling).

Future works will be able to develop coupling compensation techniques that aimto reduce SNR in relation to RMSE using the new DOA estimation method presented in this work.

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