**B_{g**}-Closed Sets In Topological Space**

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**ABSTRACT:** In this paper we introduce and study new class of sets called \(B_{g**}\)-closed sets in topological spaces. Also we discuss some of their properties and investigate the relations between other closed sets.

**KEYWORDS:** b-closed, bcl(A), \(B_{g**}\)-closed, \(B_{g**}\)-open, g**-closed, g**-open, g*-open

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**I. INTRODUCTION**

In 1970, Levine introduced the concept of generalized closed set and discussed the properties of sets, closed and open maps, compactness, normal and separation axioms. Later in 1996 Andrjevic gave a new type of generalized closed set in topological space called b closed sets.. A.A.Omari and M.S.M. Noorani made an analytical study and gave the concepts of generalized b closed sets in topological spaces. In this paper, a new class of closed set called \(B_{g**}\)-closed set is introduced to prove that the class forms a topology. Throughout this paper \((X,\tau)\) and \((Y,\sigma)\) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let \(A\subseteq X\), the closure of \(A\) and interior of \(A\) will be denoted by \(\text{cl}(A)\) and \(\text{int}(A)\) respectively, union of all b-open sets \(X\) contained in \(A\) is called b-interior of \(A\) and it is denoted by \(\text{bint}(A)\), the intersection of all b-closed sets of \(X\) containing \(A\) is called b-closure of \(A\) and it is denoted by \(\text{bcl}(A)\).

**II. PRELIMINARIES:**

Before entering into our work we recall the following definitions which are due to Levine.

**Definition 2.1:**

(1) a pre-open set \([11]\) if \(A\subseteq \text{int}(\text{cl}(A))\) and a preclosed set if \(\text{cl}(\text{int}(A))\subseteq A\).

(2) a semi-open set \([9]\) if \(A\subseteq \text{cl}(\text{int}(A))\) and semi-closed set if \(\text{int}(\text{cl}(A))\subseteq A\).

(3) a semi-preopen set \([2]\) if \(A\subseteq \text{cl}(\text{int}(A))\) and a semi preclosed set \([1]\) if \(\text{int}(\text{cl}(A))\subseteq A\).

(4) an \(\alpha\)-open set \([15]\) if \(A\subseteq \text{cl}(\text{int}(A))\) and an \(\alpha\)-closed set \([17]\) if \(\text{cl}(\text{int}(A))\subseteq A\).

(5) a b-open set \([3]\) if \(A\subseteq \text{cl}(\text{int}(A))\cup \text{int}(\text{cl}(A))\) and a b-closed set if \(\text{cl}(\text{int}(A))\cap \text{int}(\text{cl}(A))\subseteq A\).

(6) a generalised closed set (briefly g-closed) \([8]\) if \(\text{cl}(A)\subseteq U\) whenever \(A\subseteq U\) and \(U\) is open in \((X,\tau)\).

(7) a g*-closed \([20]\) if \(\text{cl}(A)\subseteq U\) whenever \(A\subseteq U\) and \(U\) is g-open in \((X,\tau)\).

(8) a g**-closed \([19]\) if \(\text{cl}(A)\subseteq U\) whenever \(A\subseteq U\) and \(U\) is g*-open. The complement of g**-closed set is g**-open in \(X\).

(9) an generalised semi pre-closed set (briefly gsp-closed) \([6]\) if \(\text{spcl}(A)\subseteq U\) whenever \(A\subseteq U\) and \(U\) is open in \((X,\tau)\).

(10) a generalized bclosed set(briefly gb-closed) \([16]\) if \(\text{bcl}(A)\subseteq U\) whenever \(A\subseteq U\) and \(U\) is open in \(X\).

(11) a generalized \(\alpha\)-closed set (briefly \(g\alpha\)-closed) \([10]\) if \(\text{acl}(A)\subseteq U\) whenever \(A\subseteq U\) and \(U\) is \(\alpha\)-open in \(X\).

(12) a weakly closed set (briefly \(W\)-closed) \([18]\) if \(\text{cl}(A)\subseteq U\) whenever \(A\subseteq U\) and \(U\) is semi-open in \(X\).

(13) a generalized pre-closed (briefly gp-closed) \([12]\) if \(\text{pcl}(A)\subseteq U\) whenever \(A\subseteq U\) and \(U\) is open in \(X\).

(14) a semi generalized closed set (briefly sg-closed) \([5]\) if \(\text{sccl}(A)\subseteq U\) whenever \(A\subseteq U\) and \(U\) is semi open in \(X\).
A subset A of a topological space \((X,\tau)\) is called \(b^*\)-closed\([14]\) set if \(\text{int} (\text{cl}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is b-open.

A subset A of a topological space \((X,\tau)\) is called \(b^*\)-open set\([17]\) if \(A \subseteq \text{int} (\text{cl} (\text{int}(A))) \cup \text{cl}(\text{int}(A))\) and \(b^*\)-closed set if \(\text{cl}(\text{int}(A)) \cap \text{cl}(\text{int}(A)) \subseteq A\).

### III. BASIC PROPERTIES OF \(B_{G^*}\)-CLOSED SETS:

**Definition 3.1:** A subset A of a topological space \((X,\tau)\) is called \(B_{G^*}\)-closed if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(G^*\)-open in \(X\). The family of all \(B_{G^*}\)-closed sets are denoted by \(B_{G^*} C(X)\).

**Definition 3.2:** The complement of \(B_{G^*}\)-closed set is called \(B_{G^*}\)-open set. The family of all \(B_{G^*}\)-open sets of \(X\) are denoted by \(B_{G^*} O(X)\).

**Example 3.3:** Let \(X=\{1,2,3\}\) with the topology \(\tau=\{\emptyset,X,\{1\},\{2\},\{1,2\}\}\) then \(\{\emptyset,X,\{2\},\{3\},\{1,3\},\{2,3\}\}\) are \(B_{G^*}\)-closed sets and \(\{\emptyset,X,\{1\},\{2\},\{1,2\},\{1,3\}\}\) are \(B_{G^*}\)-open sets in \(X\).

**Proposition 3.4:** Every closed set is \(B_{G^*}\)-closed set.

**Proof:** Let \(A\) be a closed set in \(X\) such that \(A \subseteq U\). Let \(U\) be \(G^*\)-open. Since \(A\) is closed, \(\text{cl}(A) \subseteq U\). Hence \(A\) is \(B_{G^*}\)-closed set in \(X\).

**Example 3.5:** Let \(X=\{1,2,3\}\) with the topology \(\tau=\{\emptyset,X,\{1\},\{2\},\{1,2\}\}\) then \(A=\{2\}\) is a \(B_{G^*}\)-closed set but not a closed set of \((X,\tau)\).

**Theorem 3.6:** Every \(b^*\)-closed set is \(B_{G^*}\)-closed set.

**Proof:** Let \(A\) be a \(b^*\)-closed set in \(X\) such that \(A \subseteq U\) and \(U\) is \(G^*\)-open. Since \(A\) is \(b^*\)-closed, \(\text{cl}(A) \subseteq U\). Hence \(A\) is \(B_{G^*}\)-closed set.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.7:** Let \(X=\{1,2,3\}\) with the topology \(\tau=\{\emptyset,X,\{1\},\{2\}\}\) then \(A=\{1,3\}\) is \(B_{G^*}\)-closed but not a \(b^*\)-closed set.

**Theorem 3.8:** Every \(b^*\)-closed set is \(B_{G^*}\)-closed set.

**Proof:** Follows from the definition.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.9:** Let \(X=\{1,2,3\}\) with the topology \(\tau=\{\emptyset,X,\{1\}\}\) then \(A=\{1,3\}\) is \(B_{G^*}\)-closed but not a \(b^*\)-closed set.

**Theorem 3.10:** Every \(b^*\)-closed set is \(B_{G^*}\)-closed set.

**Proof:** Follows from the definition.

**Example 3.11:** Let \(X=\{1,2,3\}\) with the topology \(\tau=\{\emptyset,X,\{1\},\{1,3\}\}\) then \(A=\{1,2\}\) is \(B_{G^*}\)-closed but not a \(b^*\)-closed set.

**Theorem 3.12:** Every \(sb^*\)-closed set is \(B_{G^*}\)-closed set.

**Proof:** Follows from the definition.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.13:** Let \(X=\{1,2,3\}\) with the topology \(\tau=\{\emptyset,X,\{2\},\{2,3\}\}\) then \(A=\{1,2\}\) is \(B_{G^*}\)-closed but not a \(sb^*\)-closed set.

**Theorem 3.14:** Every \(b_{G^*}\)-closed set is \(G^*\)-closed set.

**Proof:** Let \(A\) be a \(b_{G^*}\)-closed set in \(X\) such that \(A \subseteq U\) and \(U\) is open. Since every open set is \(G^*\)-open and \(A\) is \(B_{G^*}\)-closed, \(\text{cl}(A) \subseteq U\). Hence \(A\) is \(G^*\)-closed.

The converse implication need not be true as seen from the following example.

**Example 3.15:** Let \(X=\{1,2,3\}\) with the topology \(\tau=\{\emptyset,X,\{1\}\}\) then \(A=\{1,3\}\) be the subset of \((X,\tau)\). Here \(A\) is \(G^*\)-closed but not a \(b_{G^*}\)-closed set of \((X,\tau)\).

**Theorem 3.16:** Every \(\alpha\)-closed set is \(B_{G^*}\)-closed set.

**Proof:** Let \(A\) be a \(\alpha\)-closed set in \(X\) such that \(A \subseteq U\) and \(U\) be \(G^*\)-open. Since \(A\) is \(\alpha\)-closed \(\text{cl}(A) \subseteq \text{cl}(A) \subseteq U\). Therefore \(\text{cl}(A) \subseteq U\). Hence \(A\) is \(B_{G^*}\)-closed.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.17:** Let \(X=\{1,2,3\}\) with the topology \(\tau=\{\emptyset,X,\{2\}\}\) then \(A=\{3\}\) be the subset of \((X,\tau)\). Here \(A\) is \(B_{G^*}\)-closed but not a \(\alpha\)-closed set of \((X,\tau)\).

**Theorem 3.18:** Every semi-closed set is \(B_{G^*}\)-closed.

**Proof:** Let \(A\) be a semiclosed set in \(X\) such that \(A \subseteq U\) where \(U\) is \(G^*\)-open. Since \(A\) is semiclosed \(\text{cl}(A) \subseteq \text{cl}(A) \subseteq U\). Therefore \(\text{cl}(A) \subseteq U\). Hence \(A\) is \(B_{G^*}\)-closed set.

The converse implication need not be true as seen from the following example.

**Example 3.19:** Let \(X=\{1,2,3\}\) with the topology \(\tau=\{\emptyset,X,\{3\}\}\) then \(A=\{2,3\}\) be the subset of \((X,\tau)\). Here \(A\) is \(B_{G^*}\)-closed but not semi-closed set of \((X,\tau)\).

**Theorem 3.20:** Every pre-closed set is \(B_{G^*}\)-closed.

**Proof:** Let \(A\) be a preclosed set in \(X\) such that \(A \subseteq U\) where \(U\) is \(G^*\)-open. Since \(A\) is preclosed,
bcl(A)⊆ pcl(A)⊆ U. Therefore bcl(A)⊆ U. Hence A is B\textsuperscript{g*} -closed set in X.
The reverse implication need not be true as seen from the following example.

**Example 3.32:** Let X={1,2,3} with the topology τ={∅,X,{1,2,3}} then A={1,2} is B\textsuperscript{g*} -closed but not a pre-closed set of (X,τ).

**Theorem 3.31:** Every g\textsuperscript{*}-closed set is B\textsuperscript{g*} -closed.

**Proof:** Let A be a g\textsuperscript{*}-closed set in X such that A⊆ U where U is g\textsuperscript{**}-open. Since A is g\textsuperscript{*}-closed, bcl(A)⊆ cl(A)⊆ U. Therefore bcl(A)⊆ U. Hence A is B\textsuperscript{g*} -closed set in X.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.32:** Let X={1,2,3} with the topology τ={∅,X,{1,2,3}} then A={1,2,3} is B\textsuperscript{g*} -closed but not a g\textsuperscript{*}-closed set of (X,τ).

**Theorem 3.32:** Every ga-closed set is B\textsuperscript{g*} -closed.

**Proof:** Let A be a ga-closed set in X such that A⊆ U where U is g\textsuperscript{**}-open. Since A is ga-closed, bcl(A)⊆ acl(A)⊆ U. Therefore bcl(A)⊆ U. Hence A is B\textsuperscript{g*} -closed set in X.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.34:** In example 3.11, A={1,2} is B\textsuperscript{g*} -closed but not a-closed.

**Theorem 3.35:** Every B\textsuperscript{g*} -closed set is gsp-closed set.

**Proof:** Let A be a B\textsuperscript{g*} -closed set in X such that A⊆ U where U is open in X. Since every open set is g\textsuperscript{**}-open and A is B\textsuperscript{g*} -closed, bcl(A)⊆ acl(A)⊆ U. Hence A is gsp-closed.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.36:** Let X={1,2,3} with the topology τ={∅,X,{1}} then A={1} is gsp-closed but not B\textsuperscript{g*} -closed set of (X,τ).

**Theorem 3.37:** A set A is B\textsuperscript{g*} -closed iff bcl(A)-A contains no nonempty g\textsuperscript{**}-closed set.

**Proof:** Necessity: Let F be a g\textsuperscript{**}-closed set of (x,τ) such that F⊆ bcl(A)-A. Then A⊆ F. Since A is B\textsuperscript{g*} -closed and F is g\textsuperscript{**}-open, then bcl(A)⊆ X-F. This implies F⊆ X-bcl(A) so F⊆(X-bcl(A))∩(X-bcl(A))∩(bcl(A))=∅. Therefore F=∅.

Sufficiency: Assume that bcl(A)-A contains no nonempty g\textsuperscript{**}-closed set. Let A⊆ U and U is g\textsuperscript{**}-open. Suppose that bcl(A) is not contained in U, then bcl(A)∩ U is a nonempty g\textsuperscript{**}-closed set of Bcl(A)-A which is a contradiction. Therefore bcl(A)⊆ U and hence A is B\textsuperscript{g*} -closed.

**Theorem 3.38:** A B\textsuperscript{g*} -closed set A is b-closed iff bcl(A)-A is b-closed.

**Proof:** If A is b-closed then bcl(A)-A =∅ Conversely suppose bcl(A)-A is b-closed in X. Since A is B\textsuperscript{g*} -closed by theorem 3.27, bcl(A)-A contains no nonempty g\textsuperscript{**}-closed set in X. Then bcl(A)-A =∅.

Hence A is b-closed.

**Theorem 3.39:** If A and B are B\textsuperscript{g*} -closed then A∪B is also B\textsuperscript{g*} -closed.

**Proof:** Given that A and B are two B\textsuperscript{g*} -closed sets in X. Let A∪B⊆ U, U is g\textsuperscript{**}-open set in X. Since A is B\textsuperscript{g*} -closed, bcl(A)⊆ U whenever A⊆ U, U is g\textsuperscript{**}-open in X. Since B is B\textsuperscript{g*} -closed, bcl(A)⊆ U whenever B⊆ U, U is g\textsuperscript{**}-open in X.

**Corollary 3.30:** The intersection of a B\textsuperscript{g*} -closed set and a closed set is a B\textsuperscript{g*} -closed set.

The above corollary can be proved by the following example.

**Example 3.31:** Let X={1,2,3,4} with the topology τ={∅,X,{1,2,4}}. {3} is the only closed set of (X,τ). B\textsuperscript{g*} -closed sets of (X,τ) are {1} , {2} , {3} , {4} , {1,2} , {1,3} , {1,4} , {2,3} , {2,4} , {3,4} , {1,2,3} , {1,3,4} , {2,3,4}.

The intersection of a B\textsuperscript{g*} -closed set and {3} is again a B\textsuperscript{g*} -closed set.

**Remark 3.32:** If A and B are B\textsuperscript{g*} -closed then their union need not be B\textsuperscript{g*} -closed.

The above remark can be proved by the following Example.

**Example 3.33:** Let X={1,2,3} with the topology τ={∅,X,{1,2}}. Here A={1} and B={2} are B\textsuperscript{g*} -closed. But A∪B={1,2} is not B\textsuperscript{g*} -closed set.

**Theorem 3.44:** If A is both g\textsuperscript{**}-open and B\textsuperscript{g*} -closed then A is b-closed.

**Proof:** Since A is g\textsuperscript{**}-open and B\textsuperscript{g*} -closed in X, bcl(A)⊆ A. But always A⊆ bcl(A). Then A=bcl(A). Hence A is b-closed.

**Theorem 3.35:** For x∈ X, the set X-{x} is B\textsuperscript{g*} -closed or g\textsuperscript{**}-open.

**Proof:** Suppose X-{x} is not g\textsuperscript{**}-open, then X is the only g\textsuperscript{**}-open set containing X-{x}. This implies bcl(X-{x})⊆ X. Then X-{x} is B\textsuperscript{g*} -closed in X.

**Theorem 3.36:** If A is B\textsuperscript{g*} -closed and A⊆ B⊆ bcl(A) then B is B\textsuperscript{g*} -closed.

**Proof:** Let U be a g\textsuperscript{**}-open set of X such that B⊆ U. Then A⊆ U. Since A is B\textsuperscript{g*} -closed then bcl(A)⊆ U. Now bcl(B)⊆ bcl(bcl(A))=bcl(A)⊆ U. Therefore B is B\textsuperscript{g*} -closed in X.

**Theorem 3.37:** Let A ⊆ Y ⊆ X and suppose that A is B\textsuperscript{g*} -closed in X, then A is B\textsuperscript{g*} -closed relative to Y.


**Proof:** Given that $A \subseteq Y \subseteq X$ and $A$ is $B_{g^{**}}$-closed in $X$. To show that $A$ is $B_{g^{**}}$-closed relative to $Y$. Let $A \subseteq Y \cap U$, where $U$ is $g^{**}$-open in $X$. Since $A$ is $B_{g^{**}}$-closed, $A \subseteq U$, implies $bcl(A) \subseteq U$. It follows that $Y \cap bcl(A) \subseteq Y \cap U$. Thus $A$ is $B_{g^{**}}$-closed relative to $Y$.

**Theorem 3.31** Suppose that $B \subseteq A \subseteq X$, $B$ is $B_{g^{**}}$-closed set relative to $A$ and that $A$ is both $g^{**}$-open and $B_{g^{**}}$-closed subset of $X$, then $B$ is $B_{g^{**}}$-closed set relative to $X$.

**Proof:** Let $B \subseteq G$ and $G$ be an open set in $X$. But given that $B \subseteq A \subseteq X$, therefore $B \subseteq A$ and $B \subseteq G$. This implies $B \subseteq A \subseteq G$. Since $B$ is $B_{g^{**}}$-closed relative to $A$, $A \subseteq bcl(B) \subseteq A \cap G$. Implies $(A \cap bcl(B)) \subseteq A \cap G$. Thus $(A \cap bcl(B)) \subseteq (G \cap bcl(B))^c$. Therefore $bcl(B)$ is $g^{**}$-closed in $X$, we have $bcl(A) \subseteq G \cup bcl(B))^c$. Also $B \subseteq A$ implies $bcl(B) \subseteq bcl(A)$. Thus $bcl(B) \subseteq bcl(A) \subseteq G \cup (bcl(B))^c$. Therefore $bcl(B) \subseteq G$ since $bcl(B)$ is not contained in $bcl(B)^c$. Thus $B$ is $B_{g^{**}}$-closed set relative to $X$.

**IV. B_{G^{**}}CLOSED SET IS INDEPENDENT OF OTHER CLOSED SETS**

**Remark 4.1:** The following example shows that the concept of $W$-closed and $B_{g^{**}}$-closed sets are independent.

**Example 4.2:** Let $X=\{1,2,3\}$ with the topology $\tau=\{\emptyset, X, \{2\}\}$. In this topological space the subset $A=\{2\}$ is $W$-closed but not $B_{g^{**}}$-closed set. Also the subset $B=\{1\}$ is $B_{g^{**}}$-closed but not $W$-closed.

**Remark 4.3:** The following example shows that the concept of $sg$-closed and $B_{g^{**}}$-closed sets are independent.

**Example 4.4:** Let $X=\{1,2,3\}$ with the topology $\tau=\{\emptyset, X, \{1,2\}\}$. In this topological space the subset $A=\{1,2\}$ is $sg$-closed but not $B_{g^{**}}$-closed set. For the topology $\tau_2=\{\emptyset, X, \{1,2\}\}$ in this topological space the subset $B=\{1,3\}$ is $B_{g^{**}}$-closed but not $sg$-closed set.

**Remark 4.5:** The following example shows that the concept of $gp$-closed and $B_{g^{**}}$-closed sets are independent.

**Example 4.6:** Let $X=\{1,2,3\}$ with the topology $\tau_1=\{\emptyset, X, \{1\}\}$. In this topological space the subset $A=\{1,3\}$ is $gp$-closed but not $B_{g^{**}}$-closed set. For the topology $\tau_2=\{\emptyset, X, \{1\}\}$ in this topological space the subset $B=\{1\}$ is $B_{g^{**}}$-closed but not $gp$-closed set.

**Remark 4.7:** The following example shows that the concept of $g$-closed and $B_{g^{**}}$-closed sets are independent.

**Example 4.8:** Let $X=\{1,2,3\}$ with the topology $\tau=\{\emptyset, X, \{1,3\}\}$. In this topological space the subset $A=\{1\}$ is $B_{g^{**}}$-closed but not $g$-closed set. For the topology $\tau_2=\{\emptyset, X, \{1\}\}$ in this topological space the subset $B=\{2,1\}$ is $g$-closed but not $B_{g^{**}}$-closed set.

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