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A Generalized Double Sampling Estimator of Population Mean

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ABSTRACT
A generalized double sampling estimator representing a class of estimators using information on an auxiliary variable is proposed for the estimation of population mean. Its bias and mean square error are found, and the properties of the generalized estimator are studied. Further, some classes of estimators depending on optimum and estimated optimum values in the sense of minimum mean square error are investigated. Comparison of the proposed generalized estimator with the usual double sampling linear regression estimator is also made.
KEY-WORDS: *Bias and mean square error, Efficiency, Optimum and estimated optimum estimators.*

I. INTRODUCTION

For a first phase large simple random sample of size n' from a population of size N, let the auxiliary character x be observed to find an estimate of population mean $\bar{X}$ of x, and further, let the characters y, x be observed on the second phase simple random sample of size n from the first phase sample of size n'. Let $(\bar{Y}, \bar{X})$ be the population means of the characters (y, x), $\bar{x}'$ be the sample mean of n' first phase sample values on x and $(\bar{y}, \bar{x})$ be the sample means of n second phase sample values on (y, x) respectively. Let $s_x'^2 = \frac{1}{(n'-1)}\sum_{i=1}^{n}(x_i' - \bar{x}')^2$ based on the first phase sample observations $(x_1', x_2', \ldots, x_{n'}')$, $s_x^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2$ based on the second phase sample observations $(x_1, x_2, \ldots, x_n)$ on x and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ based on the second phase sample observations $(y_1, y_2, \ldots, y_n)$ on y be the conventional estimator of the population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ of the study variable y.

For estimating $\bar{Y}$, the proposed generalized double sampling estimator is

$$\bar{y}_{gd} = g\left(\bar{y}, \bar{x}, \bar{x}', s_x^2, s_x'^2\right) \tag{1.1}$$

$$= g(t) \tag{1.2}$$

where $t = \left(\bar{y}, \bar{x}, \bar{x}', s_x^2, s_x'^2\right)$and $g(t)$ satisfying the validity conditions of Taylor's series expansion is a bounded function of t such that at the point

$T = \left(\bar{Y}, \bar{X}, \bar{X}, S_x^2, S_x^2\right)$,

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(i) $g(t=T)=\bar{Y}$ (1.3)

and, for first order partial derivatives $g_0=\frac{\partial g}{\partial \bar{y}}\Big]_T$, $g_1=\frac{\partial g}{\partial \bar{x}}\Big]_T$, $g_2=\frac{\partial g}{\partial \bar{x}'}\Big]_T$, $g_3=\frac{\partial g}{\partial s_{x_2}^2}\Big]_T$, $g_4=\frac{\partial g}{\partial s_x'^2}\Big]_T$ of $g(t)$ with respect to $\bar{y},\ \bar{x},\ \bar{x}',\ s_x^2,\ s_x'^2$ respectively at the point $t=T$ and second order partial derivatives $g_{00}=\frac{\partial^2 g}{\partial \bar{y}^2}\Big]_T$, $g_{01}=\frac{\partial^2 g}{\partial \bar{y}\partial \bar{x}}\Big]_T$, $g_{02}=\frac{\partial^2 g}{\partial \bar{y}\partial \bar{x}'}\Big]_T$, $g_{03}=\frac{\partial^2 g}{\partial \bar{y}\partial s_x^2}\Big]_T$, $g_{04}=\frac{\partial^2 g}{\partial \bar{y}\partial s_x'^2}\Big]_T$ of $g(t)$ with respect to $\bar{y}$, $(\bar{y},\bar{x})$, $(\bar{y},\bar{x}')$, $(\bar{y},s_x^2)$, $(\bar{y},s_x'^2)$ respectively at the point $t=T$,

(ii) $g_0=1$ (1.4)

(iii) $g_1=-g_2$ (1.5)

(iv) $g_3=-g_4$ (1.6)

(v) $g_{00}=0$ (1.7)

(vi) $g_{01}=-g_{02}$ (1.8)

(vii) $g_{03}=-g_{04}$. (1.9)

II. BIAS AND MEAN SQUARE ERROR

Let

$$\mu_{rs}=\frac{1}{N}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)^r\left(X_i-\bar{X}\right)^s, \qquad \text{for } r,s=0,1,2,3,4;$$

$$e_0=\frac{\bar{y}-\bar{Y}}{\bar{Y}},\ e_1=\frac{\bar{x}-\bar{X}}{\bar{X}},\qquad e_1'=\frac{\bar{x}'-\bar{X}}{\bar{X}},\ e_2=s_x^2-S_x^2,\ e_2'=s_x'^2-S_x^2$$

so that ignoring fpc (finite population correction) for simplicity,

$$E(e_0)=E(e_1)=E(e_1')=E(e_2)=E(e_2')=0,$$

$$E(e_0^2)=\frac{\mu_{20}}{n\bar{Y}^2},\qquad E(e_1^2)=\frac{\mu_{02}}{n\bar{X}^2},\qquad E(e_1'^2)=\frac{\mu_{02}}{n'\bar{X}^2},$$

$$E(e_2^2)=\frac{\mu_{02}^2(\beta_{2x}-1)}{n},\qquad E(e_2'^2)=\frac{\mu_{02}^2(\beta_{2x}-1)}{n'},$$

$$E(e_0e_1)=\frac{\mu_{11}}{n\overline{YX}},\qquad E(e_0e_1')=\frac{\mu_{11}}{n'\overline{YX}},$$

$$E(e_1e_1')=\frac{\mu_{02}}{n'\bar{X}^2},\qquad E(e_1'e_2)=\frac{\mu_{03}}{n'\bar{X}},$$

$$E(e_0e_2)=\frac{\mu_{12}}{n\bar{Y}},\qquad E(e_1'e_2')=\frac{\mu_{03}}{n'\bar{X}},$$

$$E(e_0e_2') = \frac{\mu_{12}}{n'\bar{Y}} \quad , \quad E(e_1e_2) = \frac{\mu_{03}}{n\bar{X}} \quad ,$$

$$E(e_1e_2') = \frac{\mu_{03}}{n'\bar{X}} \quad \text{and} \quad E(e_2e_2') = \frac{\mu_{02}^2(\beta_{2x}-1)}{n'} \; ,$$

where $\beta_{2x} = \frac{\mu_{04}}{\mu_{02}^2}$ is the coefficient of kurtosis of x.

Further, it is assumed that the sample is large enough to ignore terms involving e_0 , e_1 , e_1' , e_2 , e_2' of degree greater than two, to justify the first degree approximation [see Murthy (1967) for more details].

Expanding $\bar{y}_{gd} = g(t)$ about the point $T = \left(\bar{Y}, \bar{X}, \bar{X}, S_x^2, S_x^2\right)$ in third order Taylor's series, we have

$$\begin{aligned}
\bar{y}_{gd} = {} & g(T) + (\bar{y}-\bar{Y})g_0 + (\bar{x}-\bar{X})g_1 + (\bar{x}'-\bar{X})g_2 + \left(s_x^2 - S_x^2\right)g_3 \\
& + \left(s_x'^2 - S_x^2\right)g_4 + \frac{1}{2!}\Big\{ (\bar{y}-\bar{Y})^2 g_{00} + (\bar{x}-\bar{X})^2 g_{11} \\
& + (\bar{x}'-\bar{X})^2 g_{22} + \left(s_x^2 - S_x^2\right)^2 g_{33} + \left(s_x'^2 - S_x^2\right)^2 g_{44} \\
& + 2(\bar{y}-\bar{Y})(\bar{x}-\bar{X})g_{01} + 2(\bar{y}-\bar{Y})(\bar{x}'-\bar{X})g_{02} \\
& + 2(\bar{y}-\bar{Y})\left(s_x^2 - S_x^2\right)g_{03} + 2(\bar{y}-\bar{Y})\left(s_x'^2 - S_x^2\right)g_{04} \\
& + 2(\bar{x}-\bar{X})(\bar{x}'-\bar{X})g_{12} + 2(\bar{x}-\bar{X})\left(s_x^2 - S_x^2\right)g_{13} \\
& + 2(\bar{x}-\bar{X})\left(s_x'^2 - S_x^2\right)g_{14} + 2(\bar{x}'-\bar{X})\left(s_x^2 - S_x^2\right)g_{23} \\
& + 2(\bar{x}'-\bar{X})\left(s_x'^2 - S_x^2\right)g_{24} + 2\left(s_x^2 - S_x^2\right)\left(s_x'^2 - S_x^2\right)g_{34} \Big\} \\
& + \frac{1}{3!}\Big\{ (\bar{y}-\bar{Y})\frac{\partial}{\partial \bar{y}} + (\bar{x}-\bar{X})\frac{\partial}{\partial \bar{x}} \\
& + (\bar{x}'-\bar{X})\frac{\partial}{\partial \bar{x}'} + \left(s_x^2 - S_x^2\right)\frac{\partial}{\partial s_x^2} \\
& + \left(s_x'^2 - S_x^2\right)\frac{\partial}{\partial s_x'^2} \Big\}^3 g\left(\bar{y}_*, \bar{x}_*, \bar{x}_*', s_{x*}^2, s_{x*}'^2\right)
\end{aligned} \tag{2.1}$$

where g_0 , g_1 , g_2 , g_3 , g_4 , g_{00} , g_{01} , g_{02} , g_{03} , g_{04} are already defined, g_{11} , g_{22} , g_{33} , g_{44} , g_{12} , g_{13} , g_{14} , g_{23} , g_{24} , g_{34} are the second order partial derivatives with respect to $\bar{x}, \bar{x}'$, $s_x^2, s_x'^2$, $(\bar{x}, \bar{x}')$, $\left(\bar{x}, s_x^2\right)$, $\left(\bar{x}, s_x'^2\right)$, $\left(\bar{x}', s_x^2\right)$, $\left(\bar{x}', s_x'^2\right)$, $\left(s_x^2, s_x'^2\right)$ respectively,

and $\bar{y}_* = \bar{Y} + \theta(\bar{y}-\bar{Y})$, $\bar{x}_* = \bar{X} + \theta(\bar{x}-\bar{X})$,

$\bar{x}_*' = \bar{X} + \theta(\bar{x}'-\bar{X})$, $s_{x*}^2 = S_x^2 + \theta\left(s_x^2 - S_x^2\right)$,

$s_{x*}'^2 = S_x^2 + \theta\left(s_x'^2 - S_x^2\right)$ for $0 < \theta < 1$.

From regularity conditions (1.3) to (1.9), substituting $g(T)=\bar{Y}$, $g_0=1$, $g_1=-g_2$, $g_3=-g_4$, $g_{00}=0$, $g_{01}=-g_{02}$, and $g_{03}=-g_{04}$ in (2.1), we have

$$\begin{aligned}
\bar{y}_{gd}-\bar{Y} &= (\bar{y}-\bar{Y})+(\bar{x}-\bar{X})g_1-(\bar{x}'-\bar{X})g_1+\left(s_x^2-S_x^2\right)g_3 \\
&\quad -\left(s_x'^2-S_x^2\right)g_3+\frac{1}{2}\Big\{(\bar{x}-\bar{X})^2 g_{11}+(\bar{x}'-\bar{X})^2 g_{22} \\
&\quad +\left(s_x^2-S_x^2\right)^2 g_{33}+\left(s_x'^2-S_x^2\right)^2 g_{44}+2(\bar{y}-\bar{Y})(\bar{x}-\bar{X})g_{01} \\
&\quad -2(\bar{y}-\bar{Y})(\bar{x}'-\bar{X})g_{01}+2(\bar{y}-\bar{Y})\left(s_x^2-S_x^2\right)g_{03} \\
&\quad -2(\bar{y}-\bar{Y})\left(s_x'^2-S_x^2\right)g_{03}+2(\bar{x}-\bar{X})(\bar{x}'-\bar{X})g_{12} \\
&\quad +2(\bar{x}-\bar{X})\left(s_x^2-S_x^2\right)g_{13}+2(\bar{x}-\bar{X})\left(s_x'^2-S_x^2\right)g_{14} \\
&\quad +2(\bar{x}'-\bar{X})\left(s_x^2-S_x^2\right)g_{23}+2(\bar{x}'-\bar{X})\left(s_x'^2-S_x^2\right)g_{24} \\
&\quad +2\left(s_x^2-S_x^2\right)\left(s_x'^2-S_x^2\right)g_{34}\Big\}+\frac{1}{3!}\Big\{(\bar{y}-\bar{Y})\frac{\partial}{\partial\bar{y}} \\
&\quad +(\bar{x}-\bar{X})\frac{\partial}{\partial\bar{x}}+(\bar{x}'-\bar{X})\frac{\partial}{\partial\bar{x}'}+\left(s_x^2-S_x^2\right)\frac{\partial}{\partial s_x^2} \\
&\quad +\left(s_x'^2-S_x^2\right)\frac{\partial}{\partial s_x'^2}\Big\}^3 g\left(\bar{y}_*,\bar{x}_*,\bar{x}'_*,s_{x*}^2,s_{x*}'^2\right). \qquad (2.2)
\end{aligned}$$

Further, from (2.2), we have

$$\begin{aligned}
\bar{y}_{gd}-\bar{Y} &= \bar{Y}e_0+\bar{X}(e_1-e_1')g_1+(e_2-e_2')g_3+\frac{1}{2!}\Big\{\bar{X}^2 e_1^2 g_{11} \\
&\quad +\bar{X}^2 e_1'^2 g_{22}+e_2^2 g_{33}+e_2'^2 g_{44}+2\overline{YX}(e_0e_1-e_0e_1')g_{01} \\
&\quad +2\bar{Y}(e_0e_2-e_0e_2')g_{03}+2\bar{X}^2 e_1e_1' g_{12}+2\bar{X}e_1e_2 g_{13} \\
&\quad +2\bar{X}e_1e_2' g_{14}+2\bar{X}e_1'e_2 g_{23}+2\bar{X}e_1'e_2' g_{24}+2e_2e_2' g_{34}\Big\} \\
&\quad +\frac{1}{3!}\Big\{(\bar{y}-\bar{Y})\frac{\partial}{\partial\bar{y}}+(\bar{x}-\bar{X})\frac{\partial}{\partial\bar{x}}+(\bar{x}'-\bar{X})\frac{\partial}{\partial\bar{x}'} \\
&\quad +\left(s_x^2-S_x^2\right)\frac{\partial}{\partial s_x^2}+\left(s_x'^2-S_x^2\right)\frac{\partial}{\partial s_x'^2}\Big\}^3 g\left(\bar{y}_*,\bar{x}_*,\bar{x}'_*,s_{x*}^2,s_{x*}'^2\right). \qquad (2.3)
\end{aligned}$$

Taking expectation on both sides of (2.3), to the first degree of approximation

$$\begin{aligned}
E(\bar{y}_{gd})-\bar{Y} &= \Big[\frac{1}{2}\Big\{\bar{X}^2 \mathrm{E}\left(e_1^2\right)g_{11}+\bar{X}^2 E\left(e_1'^2\right)g_{22} \\
&\quad +E\left(e_2^2\right)g_{33}+E\left(e_2'^2\right)g_{44}+2\overline{YX}E(e_0e_1-e_0e_1')g_{01} \\
&\quad +2\bar{Y}\,E(e_0e_2-e_0e_2')g_{03}+2\bar{X}^2 E(e_1e_1')g_{12} \\
&\quad +2\bar{X}E(e_1e_2)g_{13}+2\bar{X}E(e_1e_2')g_{14}
\end{aligned}$$

$$+2\bar{X}E(e_1'e_2)g_{23}+2\bar{X}E(e_1'e_2')g_{24}+2E(e_2e_2')g_{34}\}]$$

or $$Bias(\bar{y}_{gd})=\frac{1}{2}\left\{\frac{\mu_{02}}{n}g_{11}+\frac{\mu_{02}}{n'}(g_{22}+2g_{12})+\frac{\mu_{02}^2}{n}(\beta_{2x}-1)g_{33}\right.$$

$$+\frac{\mu_{02}^2}{n'}(\beta_{2x}-1)g_{44}+2\left(\frac{1}{n}-\frac{1}{n'}\right)\overline{YX}\left(\frac{\mu_{11}}{\overline{YX}}\right)g_{01}$$

$$+2\left(\frac{1}{n}-\frac{1}{n'}\right)\bar{Y}\left(\frac{\mu_{12}}{\bar{Y}}\right)g_{03}+2\frac{\bar{X}}{n}\left(\frac{\mu_{03}}{\bar{X}}\right)g_{13}$$

$$+2\frac{\bar{X}}{n'}\left(\frac{\mu_{03}}{\bar{X}}\right)(g_{14}+g_{23}+g_{24})$$

$$\left.+2\frac{\mu_{02}^2}{n'}(\beta_{2x}-1)g_{34}\right\}$$

or $$Bias(\bar{y}_{gd})=\frac{1}{2}\left\{\frac{\mu_{02}}{n}g_{11}+\frac{\mu_{02}}{n'}(g_{22}+2g_{12})\right.$$

$$\left.+\frac{\mu_{02}^2}{n}(\beta_{2x}-1)g_{33}+\frac{\mu_{02}^2}{n'}(\beta_{2x}-1)g_{44}\right\}$$

$$+\left(\frac{1}{n}-\frac{1}{n'}\right)(\mu_{11}g_{01}+\mu_{12}g_{03})+\frac{\mu_{03}}{n}g_{13}$$

$$+\frac{1}{n'}\{\mu_{03}(g_{14}+g_{23}+g_{24})+\mu_{02}^2(\beta_{2x}-1)g_{34}\}. \tag{2.4}$$

Squaring both sides of (2.3) and taking expectation, $MSE(\bar{y}_{gd})=E(\bar{y}_{gd}-\bar{Y})^2$ to the first degree of approximation, is

$$MSE(\bar{y}_{gd})=E\left[\bar{Y}^2e_0^2+\bar{X}^2(e_1-e_1')^2g_1^2+(e_2-e_2')^2g_3^2\right.$$
$$+2\overline{YX}(e_1-e_1')e_0g_1+2\bar{Y}(e_2-e_2')e_0g_3$$
$$\left.+2\bar{X}(e_1-e_1')(e_2-e_2')g_1g_3\right]$$

or $$MSE(\bar{y}_{gd})=\bar{Y}^2E(e_0^2)+\bar{X}^2\{E(e_1^2)+E(e_1'^2)-2E(e_1e_1')\}g_1^2$$

$$+\{E(e_2^2)+E(e_2'^2)-2E(e_2e_2')\}g_3^2$$

$$+2\overline{YX}\{E(e_0e_1)-E(e_0e_1')\}g_1$$

$$+2\bar{Y}\{E(e_0e_2)-E(e_0e_2')\}g_3$$

$$+2\bar{X}\{E(e_1e_2)-E(e_1e_2')-E(e_1'e_2)+E(e_1'e_2')\}g_1g_3$$

$$=\frac{\mu_{20}}{n}+\left(\frac{1}{n}-\frac{1}{n'}\right)\mu_{02}g_1^2+\left(\frac{1}{n}-\frac{1}{n'}\right)\mu_{02}^2(\beta_{2x}-1)g_3^2$$

$$+2\left(\frac{1}{n}-\frac{1}{n'}\right)\mu_{11}g_1+2\left(\frac{1}{n}-\frac{1}{n'}\right)\mu_{12}g_3$$

$$+2\left(\frac{1}{n}-\frac{1}{n'}\right)\mu_{03}g_1g_3$$

or $$MSE(\bar{y}_{gd})=\frac{\mu_{20}}{n}+\left(\frac{1}{n}-\frac{1}{n'}\right)\Big\{\mu_{02}g_1^{\,2}+\mu_{02}^2(\beta_{2x}-1)g_3^{\,2}$$

$$+2\mu_{11}g_1+2\mu_{12}g_3+2\mu_{03}g_1g_3\Big\}\ . \tag{2.5}$$

III. OPTIMUM AND ESTIMATED OPTIMUM VALUES

From (2.5), we see that values of g_1 and g_3 for which $MSE(\bar{y}_{gd})$ is minimized, are given by

$$g_{1*}=\frac{\mu_{11}}{\mu_{02}}\left\{\frac{(\beta_{2x}-1)(1-\Delta_1)}{(\beta_{2x}-\beta_{1x}-1)}\right\} \tag{3.1}$$

$$=\frac{\mu_{11}}{\mu_{02}}\omega$$

and $$g_{3*}=\frac{\mu_{11}}{\mu_{02}^2}\left\{\frac{1-\Delta_2}{(\beta_{2x}-\beta_{1x}-1)}\right\} \tag{3.2}$$

$$=\frac{\mu_{11}}{\mu_{02}^2}\xi$$

where $\omega=\left\{\frac{(\beta_{2x}-1)(1-\Delta_1)}{(\beta_{2x}-\beta_{1x}-1)}\right\}$, $\xi=\left\{\frac{1-\Delta_2}{(\beta_{2x}-\beta_{1x}-1)}\right\}$, $\Delta_1=\frac{\mu_{12}\mu_{03}}{\mu_{11}\mu_{02}^2}$ and $\Delta_2=\frac{\mu_{11}\mu_{03}}{\mu_{12}\mu_{02}}$, and the minimum mean square error is

$$MSE(\bar{y}_{gd})_{min}=MSE(\bar{y}_{ld})-\left(\frac{1}{n}-\frac{1}{n'}\right)\left[\frac{(1-\Delta_2)^2}{\mu_{02}^4(\beta_{2x}-\beta_{1x}-1)}\right] \tag{3.3}$$

where $\beta_{2x}=\frac{\mu_{04}}{\mu_{02}^2}$, $\beta_{1x}=\frac{\mu_{03}^2}{\mu_{02}^3}$, $\bar{y}_{ld}$ is the linear regression estimate of population mean in double sampling and $MSE(\bar{y}_{ld})=\frac{\mu_{20}}{n}-\left(\frac{1}{n}-\frac{1}{n'}\right)\frac{\mu_{11}^2}{\mu_{02}}$ [see Sukhatme et. al. (1984) for more details] .

Practically, the optimum values g_{1*} and g_{3*} in (3.1) and (3.2) may not be available always, hence the alternative is to replace the parameters involved therein by their unbiased or consistent estimators and thus get the estimated optimum values. Defining $m_{rs}=\frac{1}{(n-1)}\sum_{i=1}^{n}(y_i-\bar{y})^r(x_i-\bar{x})^s$, replacing

μ_{11}, Δ_1, Δ_2, β_{2x} and β_{1x} by m_{11}, $\hat{\Delta}_1 = \frac{m_{12} m_{03}}{m_{11} m_{02}^2}$, $\hat{\Delta}_2 = \frac{m_{11} m_{03}}{m_{12} m_{02}}$, $\hat{\beta}_{2x} = \frac{m_{04}}{m_{02}^2}$ and $\hat{\beta}_{1x} = \frac{m_{03}^2}{m_{02}^3}$, we get the estimated optimum values $\hat{g}_1$ and $\hat{g}_3$ to be

$$\hat{g}_1 = \frac{m_{11}}{m_{02}} \hat{\omega} \tag{3.4}$$

and $$\hat{g}_3 = \frac{m_{11}}{m_{02}^2} \hat{\xi} \tag{3.5}$$

where $\hat{\omega} = \left\{ \frac{(\hat{\beta}_{2x} - 1)(1 - \hat{\Delta}_1)}{(\hat{\beta}_{2x} - \hat{\beta}_{1x} - 1)} \right\}$, $\hat{\xi} = \left\{ \frac{1 - \hat{\Delta}_2}{(\hat{\beta}_{2x} - \hat{\beta}_{1x} - 1)} \right\}$.

The generalized double sampling estimator $\bar{y}_{gd}$ attains the minimum mean square error in (3.3) if the conditions from (1.3) to (1.9), (3.1) and (3.2) are satisfied for the estimator $\bar{y}_{gd}$.

This means that the function $\bar{y}_{gd} = g(\bar{y}, \bar{x}, \bar{x}', s_x^2, s_x'^2)$ as an estimator of $\bar{Y}$ should not involve only $(\bar{y}, \bar{x}, \bar{x}', s_x^2, s_x'^2)$ but also g_{1*} and g_{3*} for the conditions (3.1) and (3.2) to be satisfied. Thus, we get the resulting estimator as a function $g(\bar{y}, \bar{x}, \bar{x}', s_x^2, s_x'^2, g_{1*}, g_{3*})$ satisfying the conditions (1.3) to (1.9) along with the conditions (3.1) and (3.2) to attain the minimum mean square error in (3.3). Replacing unknowns g_{1*} and g_{3*} in $g(\bar{y}, \bar{x}, \bar{x}', s_x^2, s_x'^2, g_{1*}, g_{3*})$, we get the estimator as a function $\bar{y}_{ge} = g(\bar{y}, \bar{x}, \bar{x}', s_x^2, s_x'^2, \hat{g}_1, \hat{g}_3)$ or equivalently the function $\bar{y}_{ge} = g^*(\bar{y}, \bar{x}, \bar{x}', s_x^2, s_x'^2, \hat{\omega}, \hat{\xi})$ as an estimator depending upon estimated optimum values. Now expanding $g^*(\bar{y}, \bar{x}, \bar{x}', s_x^2, s_x'^2, \hat{\omega}, \hat{\xi})$ about the point $T^* = (\bar{Y}, \bar{X}, \bar{X}', S_x^2, S_x'^2, \omega, \xi)$ in Taylor's series, we have

$$\begin{aligned} g^*(\bar{y}, \bar{x}, \bar{x}', s_x^2, s_x'^2, \hat{\omega}, \hat{\xi}) = g^*(T^*) + (\bar{y} - \bar{Y}) \left.\frac{\partial g^*}{\partial \bar{y}}\right]_{T^*} &+ (\bar{x} - \bar{X}) g_1^* \\ &+ (\bar{x}' - \bar{X}) g_2^* + (s_x^2 - S_x^2) g_3^* \\ &+ (s_x'^2 - S_x^2) g_4^* + (\hat{\omega} - \omega) g_5^* \\ &+ (\hat{\xi} - \xi) g_6^* + \ldots \end{aligned} \tag{3.6}$$

where $g^*(T^*) = \bar{Y}$, $g_1^* = \left.\frac{\partial g^*}{\partial \bar{x}}\right]_{T^*} = 1$, $g_2^* = \left.\frac{\partial g^*}{\partial \bar{x}'}\right]_{T^*}$, $g_3^* = \left.\frac{\partial g^*}{\partial s_x^2}\right]_{T^*}$,

$g_4^* = \left.\frac{\partial g^*}{\partial s_x'^2}\right]_{T^*}$, $g_5^* = \left.\frac{\partial g^*}{\partial \hat{\omega}}\right]_{T^*}$ and $g_6^* = \left.\frac{\partial g^*}{\partial \hat{\xi}}\right]_{T^*}$

or $$\begin{aligned} g^*(\bar{y}, \bar{x}, \bar{x}', s_x^2, s_x'^2, \hat{\omega}, \hat{\xi}) - \bar{Y} = (\bar{y} - \bar{Y}) + (\bar{x} - \bar{X}) g_1^* + (\bar{x}' - \bar{X}) g_2^* + (s_x^2 - S_x^2) g_3^* \\ + (s_x'^2 - S_x^2) g_4^* + (\hat{\omega} - \omega) g_5^* + (\hat{\xi} - \xi) g_6^* + \ldots \end{aligned} \tag{3.7}$$

Squaring both the sides of (3.7) and taking expectation, we see that the mean square error $E\left[g^*\left(\bar{y},\bar{x},\bar{x}',s_x^2,s_x'^2,\hat{\omega},\hat{\xi}\right)-\bar{Y}\right]^2$ to the first degree of approximation, becomes equal to $MSE\left(\bar{y}_{gd}\right)_{min}$ given by (3.3) if $g_5^* = g_6^* = 0$, and thus the estimator taken as a function $\bar{y}_{ge} = g^*\left(\bar{y},\bar{x},\bar{x}',s_x^2,s_x'^2,\hat{\omega},\hat{\xi}\right)$ depending upon estimated optimum values attains the minimum mean square error given by (3.3) if

$$\left.\begin{aligned}
&g^*\left(\bar{y},\bar{x},\bar{x}',s_x^2,s_x'^2,\hat{\omega},\hat{\xi}\right)\Big]_{T^*} = \bar{Y},\ \frac{\partial g^*}{\partial \bar{y}}\bigg]_{T^*} = 1,\\
&g_1^* = \frac{\partial g^*}{\partial \bar{x}}\bigg]_{T^*} = -\frac{\partial g^*}{\partial \bar{x}'}\bigg]_{T^*} = -g_2^*,\\
&g_3^* = \frac{\partial g^*}{\partial s_x^2}\bigg]_{T^*} = -\frac{\partial g^*}{\partial s_x'^2}\bigg]_{T^*} = -g_4^*,\ \frac{\partial^2 g^*}{\partial(\bar{y})^2}\bigg]_{T^*} = 0,\\
&g_{01}^* = \frac{\partial^2 g^*}{\partial\bar{y}\partial\bar{x}}\bigg]_{T^*} = -\frac{\partial^2 g^*}{\partial\bar{y}\partial\bar{x}'}\bigg]_{T^*} = -g_{02}^*,\\
&g_{03}^* = \frac{\partial^2 g^*}{\partial\bar{y}\partial s_x^2}\bigg]_{T^*} = -\frac{\partial^2 g^*}{\partial\bar{y}\partial s_x'^2}\bigg]_{T^*} = -g_{04}^*,\\
&\frac{\partial g^*}{\partial \bar{x}}\bigg]_{T^*} = \omega,\ \frac{\partial g^*}{\partial s_x^2}\bigg]_{T^*} = \xi,\\
&g_5^* = 0 \quad and \quad g_6^* = 0 .
\end{aligned}\right\} \qquad (3.8)$$

Satisfying the conditions in (3.8), some particular estimators depending on estimated optimum values $\hat{\omega},\hat{\xi}$ and attaining the minimum mean square error in (3.3), are given in Concluding Remarks.

IV. CONCLUDING REMARKS

(a) From Sukhatme et. al. (1984, page 245) and (3.3), we have

$$MSE\left(\bar{y}_{ld}\right) = \frac{\mu_{20}}{n} - \left(\frac{1}{n} - \frac{1}{n'}\right)\frac{\mu_{11}^2}{\mu_{02}} \qquad (4.1)$$

and $$MSE\left(\bar{y}_{ge}\right) = MSE\left(\bar{y}_{gd}\right)_{min}$$

$$= MSE\left(\bar{y}_{ld}\right) - \left(\frac{1}{n} - \frac{1}{n'}\right)\left\{\frac{(1-\Delta)^2}{\mu_{02}^4\left(\beta_{2x} - \beta_{1x} - 1\right)}\right\} \qquad (4.2)$$

which shows that the estimator $\bar{y}_{ge}$ depending upon estimated optimum values is more efficient than the usual double sampling regression estimator $\bar{y}_{ld}$ in the sense of having smaller mean square error.

(b) The estimators $\bar{y}_{d_1} = \bar{y}\left(\frac{\bar{x}}{\bar{x}'}\right)^{k_1}\left(\frac{s_x^2}{s_x'^2}\right)^{k_2}$ and $\bar{y}_{d_2} = \bar{y} + k_1(\bar{x} - \bar{x}') + k_2(s_x^2 - s_x'^2)$, belonging to the class $\bar{y}_{gd}$ of estimators and having the values of (g_1^*, g_2^*) to be

$$\left(k_1\frac{\bar{Y}}{\bar{X}}, k_2\frac{\bar{Y}}{S_x^2}\right) \text{ and } (k_1, k_2) \tag{4.3}$$

respectively, will attain the minimum mean square error given in (4.3.3) or (4.2) for the optimum values of (k_1, k_2) equal to $\left(\frac{\omega\bar{X}}{\mu_{02}\bar{Y}}, \frac{\xi}{\mu_{02}\bar{Y}}\right)$ obtained by equating each of (4.3) to the optimum values $\left(\frac{\omega}{\mu_{02}}, \frac{\xi}{\mu_{02}^2}\right)$, that is, the mean square error of the estimators $\bar{y}_{d_1} = \bar{y}\left(\frac{\bar{x}}{\bar{x}'}\right)^{\frac{\omega\bar{X}}{\mu_{02}\bar{Y}}}\left(\frac{s_x^2}{s_x'^2}\right)^{\frac{\xi}{\mu_{02}\bar{Y}}}$ and $\bar{y}_{d_2} = \bar{y} + \frac{\omega}{\mu_{02}}(\bar{x} - \bar{x}') + \frac{\xi}{\mu_{02}^2}(s_x^2 - s_x'^2)$ to the first degree of approximation will be equal to that of (4.2). But $\frac{\omega}{\mu_{02}}$ or $\frac{\xi}{\mu_{02}^2}$ may be rarely known, hence replacing $\frac{\omega}{\mu_{02}}$ or $\frac{\xi}{\mu_{02}^2}$ by consistent estimates from sample values, we get the estimators depending upon estimated optimum values to be

$$\bar{y}_{e_1} = \bar{y}\left(\frac{\bar{x}}{\bar{x}'}\right)^{\frac{\hat{\omega}\ \bar{x}}{s_x^2\bar{y}}}\left(\frac{s_x^2}{s_x'^2}\right)^{\frac{\hat{\xi}}{s_x^2\bar{y}}} \text{ and } \bar{y}_{e_2} = \bar{y} + \frac{\hat{\omega}}{s_x^2}(\bar{x} - \bar{x}') + \frac{\hat{\xi}}{s_x^4}(s_x^2 - s_x'^2),$$

which may belong to the class $\bar{y}_{ge}$ and satisfy the conditions in (3.8), and also attain the minimum mean square error given by (3.3) or (4.2) to the first degree of approximation. The general result regarding $\bar{y}_{ge}$ which attains the minimum mean square error (to the first degree of approximation) given in (3.3) or (4.2).

(c) Single sampling results may be easily found as the special cases of this study for $n' = N$.

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