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Study of Multiple Jointed Kinematic Chains

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ABSTRACT: *In this paper, the [JJ] matrix is used to identify the distinct mechanisms of a given kinematic chain with multiple joints. The two structural invariants 'sum of the absolute values of the characteristic polynomial coefficients' [SCPC] and 'maximum absolute value of the characteristic polynomial coefficient' [MCPC] of [JJ] matrices are derived from [JJ] matrix of the mechanism kinematic chains. These structural invariants are calculated using Software MAT LAB and are same for identical or structural equivalent mechanisms but different for distinct mechanism.*

Keywords: *kinematic chain, multiple joints, distinct mechanism*

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I. INTRODUCTION

A number of researchers have discussed structural synthesis in the earlier days. Crossley [1996] proposed a collection of 10-link plane chains. During the compiling of this collection, his greatest problem was to distinguish whether two arrangements, which might appear unlike, were actually the same or different. This led to a definition of isomorphism between linkages.Mruthyunjaya [1984] made an effort to develop a fully computerized approach for structural synthesis of kinematic chains. Agrawal and Rao [1987] investigated a systematic method of analysis of the mobility properties of the kinematic chains by its loop freedom matrix and its permanent function which are used to identify it. Sethi and Agrawal [1993] proposed a classification scheme on the basis of structural properties. Madan and Jain [2002] considered the kinematic chains-isomorphism, inversions and degree of similarity using the concept of connectivity. Rao [2006] threw the light on the enumeration of distinct planar kinematic chains. They developed a very simple method based on independent loop(s) assorted and their adjacency is reported. Misti [2003] presented the position analysis in polynomial form of planar mechanisms with Assur groups of class 3 including revolute and prismatic joints. Uicker and Raicu [1975] presented a method for the identification and recognition of equivalence of kinematic chains. Later on, this method failed. Mruthyunjaya and Balasubramanian [1987] proved that the method proposed by Uicker and Raicu [1975] is not reliable. They proved that the test based on comparison of the characteristic coefficients of the adjacency matrices of the corresponding graphs for detection of isomorphism in kinematic chains failed. Shende and Rao [1994] work, which deals with the problem of detection of isomorphism which is frequently encountered in structural synthesis of kinematic chains.Chu Jin-Kui and Cao Wei-Qing [1994] proposed a method for identification of isomorphism among kinematic chains and inversions using Link's adjacent-chain-table. Yadav, et.al. [1996] Proposed a computer aided detection method of isomorphism among kinematic chains and mechanisms using the concept of modified distance. Yadav, et.al.[1996] presented a paper mechanism of a kinematic chain and the degree of structural similarity based on the concept of link path code'. Yadav, et.al.[2002] presented a paper 'computer aided detection of isomorphism among binary chains using the link-link multiplicity distance concept. Rao [2000] suggested the application of fuzzy logic for the study of isomorphism, inversions, symmetry, parallelism and mobility in kinematic chains with some necessary and sufficient conditions. Kong, et.al. [1999] Proposed a new method based on artificial neural network (ANN) to identify the isomorphism of the mechanism kinematic chain. Rao and Deshmukh [2001] proposed method does not require any separate test for isomorphism in the generation of kinematic chains. Chang, et.al. [2002] proposed method is based on the eigen vectors and eigen values to identify isomorphism of mechanism kinematic chain.. He and Jhang [2003] proposed a new method for detection of graph isomorphism based on the quadratic form. Tang and Liu [1988] established a method 'the degree code' as a new mechanism identifier. Later on this method also failed. Zhao, et.al [2004] put forward and more complete theory of degrees of freedom (DOF) for mechanisms, especially for the complex spatial mechanisms, which may not be solved correctly with traditional theories. Hasan et al. [2007] but the concept that these methods are based on seems to be unjustified as either link-link adjacency or joint-joint adjacency hardly differ in nature and are likely to fail at some stage or the other. Hasan [2007,2009] proposed a new method in which kinematic chains are represented in the form of the Joint-Joint [JJ] matrix. Two structural invariants, sum of absolute characteristic polynomial coefficients and maximum absolute value of the characteristic polynomial coefficient are derived from the characteristic polynomials of the [JJ] matrix of the kinematic chains. Dargar et al. [2009,2010] proposed Link adjacency value method to identify the isomorphism by calculating the first and second link adjacency values. Rizvi et al. [2016,2016] presented a new method for distinct inversions and isomorphism based on a link identity matrix and link signature. In [2016], the authors gave an algorithm for distinct inversions and isomorphism detection in kinematic chains using link identification number. Alam et al.[2017] presented weighted squared path technique to determine the structural similarity and dissimilarity in the kinematic chains.Still there is a need for a reliable, easy and efficient algebraic method for finding the distinct mechanisms from a given kinematic chains having multiple joints. Most of the methods available in the literature related to isomorphism and distinct mechanisms of kinematic chains are based on the graph theoretic approach. The edges and vertices of the graph represent the joints and links respectively of the kinematic chain. However, using the proposed method, such problems do not arise. In this paper, the distinct mechanism of all 45 kinematic chains having multiple joints are determined.

II. KINEMATIC CHAINS HAVING MULTIPLE JOINTS

A kinematic chain may consist of simple joints or a combination of simple and multiple joints. The types of joint in a kinematic chain are given in Table-1.

Table-1: Types of joints

No of links joined at a point(n)	n-1	Type of joint
2	1	simple joint
3	2	double joint
4	3	ternary joint
5	4	quaternary joint
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n	m	m-nary joint

2.1 The Joint-Joint [Jj] Matrix

This matrix is based upon the connectivity of the joints through the links and defined, as a square symmetric matrix of size n x n, where n is the number of joints in a kinematic chain.this matrix is given by equation (1).

$$[JJ] = \left\{ L_{ij} \right\}_{n \times n} \quad \text{--------} \quad (1)$$

Where

$$L_{ij} \left\{ \begin{array}{l} = \text{Degree of link between } i^{th} \text{ and } j^{th} \text{ joints} \\ \text{those are directly connected} \end{array} \right\}$$

Off course all the diagonal elements $L_{ii} = 0$

Thus the form of [JJ] matrix will be

$$[JJ] = \begin{pmatrix} 0 & L_{12} & L_{13} & - & - & - & L_{1n} \\ L_{21} & 0 & L_{23} & - & - & - & L_{2n} \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ L_{n1} & L_{n2} & L_{n3} & - & - & - & 0 \end{pmatrix}$$

III. METHODOLOGY

A kinematic chain is represented by the [JJ] matrix. When any link of a kinematic chain is fixed, a mechanism results. It means that the corresponding joints of the fixed link work as pivots. If in the [JJ] matrix, the diagonal elements of the corresponding joints of the fixed link-'a' are changed from 0 to 1 (zero to one), it will be the representation of the first mechanism with fixed link 'a'. Then this new [JJ] matrix is represented by [JJ-a] matrix. The structural invariants of this [JJ-a] matrix are then calculated using software MAT LAB. These invariants 'SCPC-a' and 'MCPC-a' are the characteristic numbers of the first mechanism. This process is repeated for the second link and so on. In this way, a set of invariants equal to the number of the links are obtained. Some of them may be same and others are different. The same structural invariants represent the corresponding structurally equivalent links that constitute one distinct mechanism.

3.1 Illustrative Example

Consider an example of a 6-link, 7-joints (5 simple and 1 double joint), 1-DOF kinematic chain shown in Fig.1.The [JJ] matrix representing the kinematic chain [Fig.1] using equation (1) is given by [M1]. The set of structural invariants derived from the [M1] matrix using software MAT LAB for the kinematic chain shown in Fig.1 are; SCPC = 1.2500e+003, MCPC = 528.0000. Now by Fixing link 'a', the first mechanism developed. The link 'a' is a ternary link having joints 1, 2, and 7. Hence, changing the diagonal element L_{11}, L_{22} and L_{77} from 0 to 1 of [JJ] matrix. The [JJ-a] matrix is obtained and represented by matrix [M1-a]. The structural invariants of the first mechanism are derived from the kinematic chain shown in Fig.1 are [SCPC-a] = 1.1680e+003, [MCPC-a] = 464.0000. Fixing link 'b', the second mechanism developed. The link 'b' is a binary link having joints 2, and 3. Hence, changing the diagonal element L_{22} and L_{33} from zero to 1 of [M1] matrix. The [M1-b] matrix is obtained. The structural invariants of the second mechanism are; [SCPC-b] = 1.5680e+003, [MCPC-b] = 648.0000. By fixing link 'c', the third mechanism developed. Link 'c' makes double joint with link 'f' and link 'd'. This double joint is equivalent to two simple joints 4 and 5 as shown in Fig.1. For third mechanism, the diagonal element L_{33} ,L_{44} and L_{55} from 0 to 1 of [M1] matrix. The [M1-c] matrix is obtained. Structural invariants of the third mechanism are; [SCPC-c] = 2.5680e+003, [MCPC-c] = 911.0000.Similarly, the links –d, e and f are fixed in turn and the diagonal elements of all the corresponding joints of the fixed link are changed from 0 to 1 in the [M1] matrix. The structural invariants of other mechanisms obtained and given as: [SCPC-d] = 2.5680e+003, [MCPC-d] = 911.0000, [SCPC-e] = 1.5680e+003, [MCPC-e] = 648.0000, [SCPC-f] = 1.8560e+003, [MCPC-f] = 781.0000.Observing the structural invariants for the above six mechanisms, it is found that the structural invariants of link – b and e are same and form only one distinct mechanism. Similarly, the structural invariants of link 'c' and'd' are same and formed second distinct mechanism. Link 'e' and 'f' both have distinct invariants and forms the third and fourth distinct mechanism. Therefore, 4 distinct mechanisms obtained from kinematic chain shown in Fig 6.1 Note that by using other method available in the literature, the same conclusion is obtained.

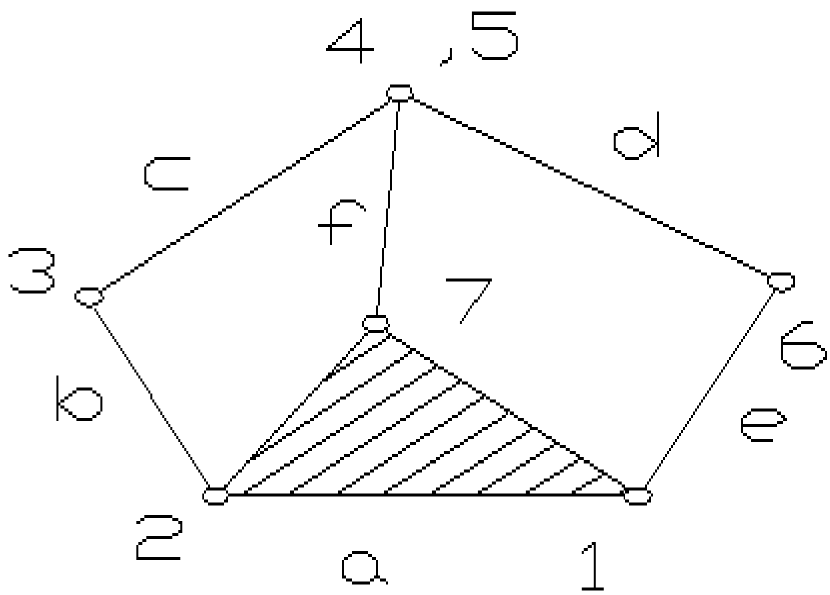

Fig 1: 6-Link, 1-dof Kinematic Chain with One Double Joint

$$[M1]=\begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 2 & 3 \\ 3 & 0 & 2 & 0 & 0 & 0 & 3 \\ 0 & 2 & 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 0 & 0 & 2 & 2 \\ 2 & 0 & 0 & 2 & 2 & 0 & 0 \\ 3 & 3 & 0 & 2 & 2 & 0 & 0 \end{pmatrix} \quad [M1\text{-}a]=\begin{pmatrix} 1 & 3 & 0 & 0 & 0 & 2 & 3 \\ 3 & 1 & 2 & 0 & 0 & 0 & 3 \\ 0 & 2 & 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 0 & 0 & 2 & 2 \\ 2 & 0 & 0 & 2 & 2 & 0 & 0 \\ 3 & 3 & 0 & 2 & 2 & 0 & 1 \end{pmatrix}$$

$$[M1\text{-}b]=\begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 2 & 3 \\ 3 & 1 & 2 & 0 & 0 & 0 & 3 \\ 0 & 2 & 1 & 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 0 & 0 & 2 & 2 \\ 2 & 0 & 0 & 2 & 2 & 0 & 0 \\ 3 & 3 & 0 & 2 & 2 & 0 & 0 \end{pmatrix} \quad [M1\text{-}c]=\begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 2 & 3 \\ 3 & 0 & 2 & 0 & 0 & 0 & 3 \\ 0 & 2 & 1 & 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 2 & 2 \\ 0 & 0 & 2 & 0 & 1 & 2 & 2 \\ 2 & 0 & 0 & 2 & 2 & 0 & 0 \\ 3 & 3 & 0 & 2 & 2 & 0 & 0 \end{pmatrix}$$

IV. RESULTS

The number of distinct mechanisms calculated from 1-DOF, 6, 8 and 10 links kinematic chains having simple joints are 5, 71 and 1856 respectively. The number of distinct mechanism derived from 1-DOF, 6 and 8 links are in complete agreement as reported those reported already in the literature. The distinct mechanisms derived from 1-F, 10-link and 13 joined kinematic chains are reported as 1821 to 1844 by other researchers.

The total number of distinct mechanisms obtained from the family of 1-DOF, 8-link, 45 kinematic chains with multiple joints redrawn in Table-2 is 286 (detail is given in Table-3).

V. CONCLUSIONS

In this paper, a new method to identify the distinct mechanisms from a given kinematic chain having a combination of simple and multiple joints is presented. The method based upon characteristic polynomial of [JJ] matrix. The two structural invariants [SCPC] and [MCPC] are calculated from the characteristic polynomial of the [JJ] matrix. These invariants are able to detect the isomorphism between the mechanism kinematic chains with multiple joints and even the kinematic chain with co-spectral graphs. Using this method, the distinct mechanism form kinematic chain with simple joints, single, or multi-degree of freedom can be identified.

Table-2: 8-Link, 1-Dof Kinematic Chains with Multiple Joints.

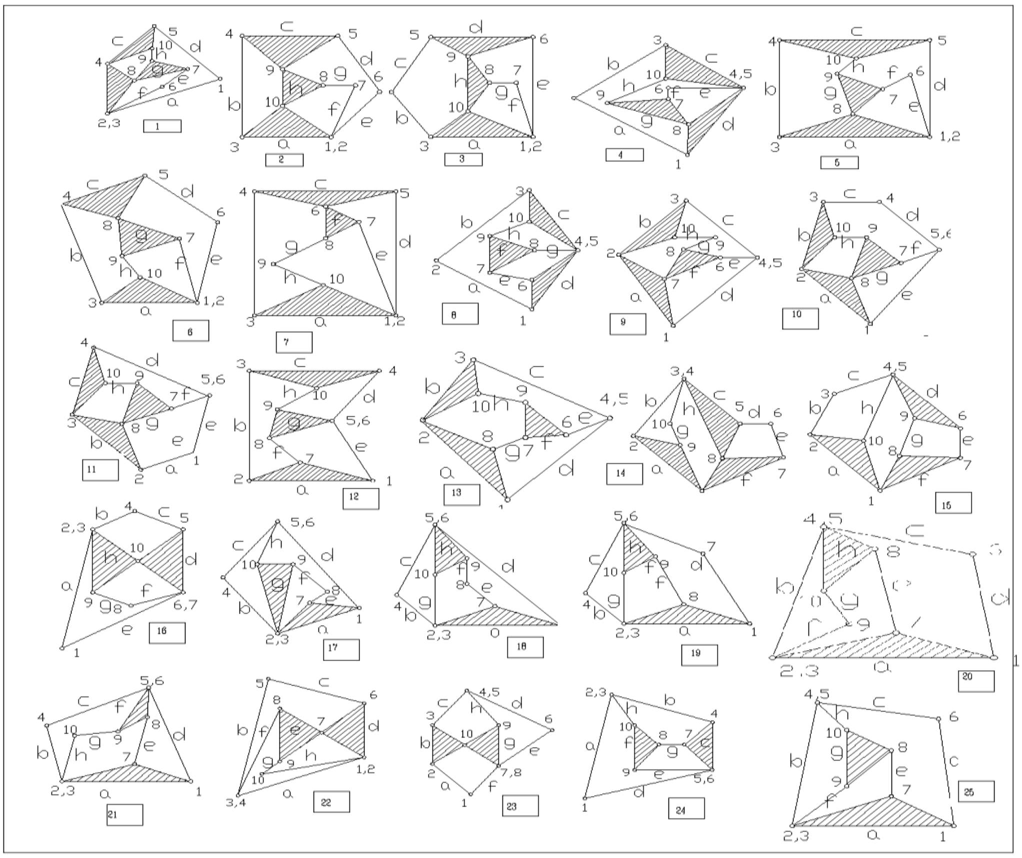

Table-2: 8-Link, 1-Dof Kinematic Chains with Multiple Joints. (Contd--)

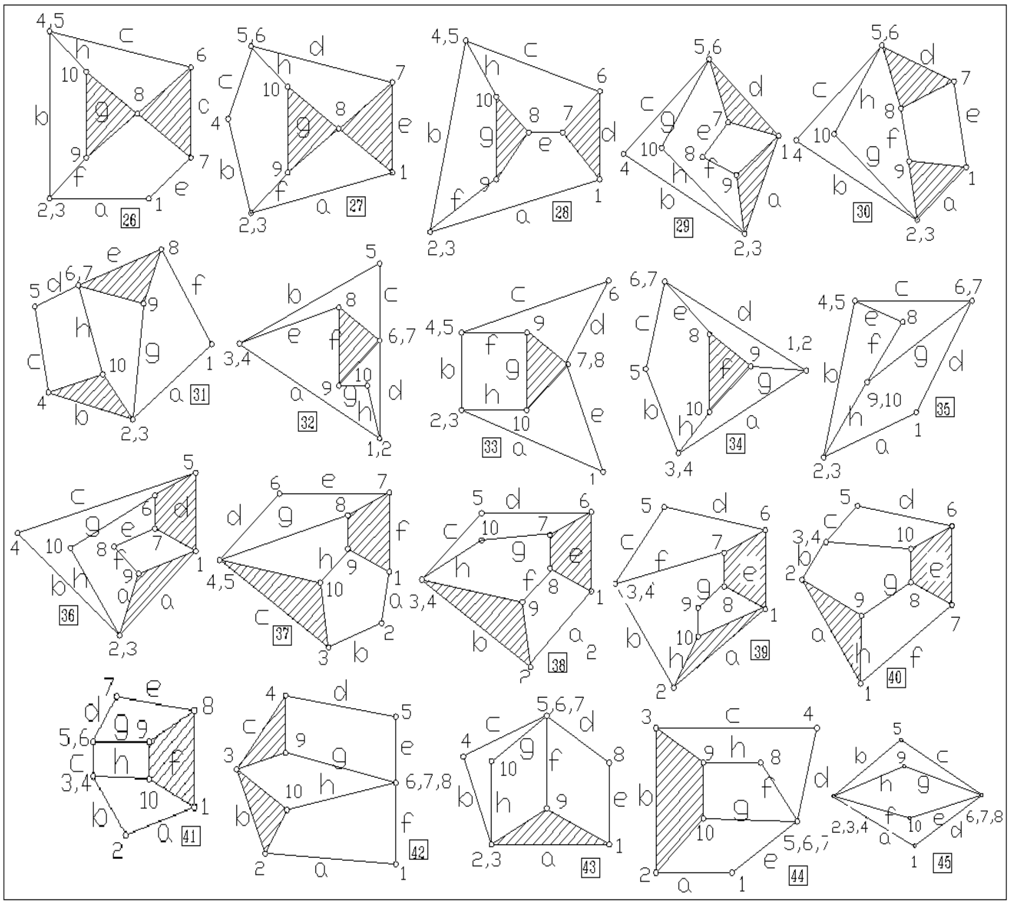

Table-3: Distinct Mechanisms calculated from 8-Link, 1-DOF Kinematic Chains with Multiple Joints.

K.C. No.	Number of distinct mechanisms	Equivalent kinematic links
1.	8	nil
2.	8	nil
3.	8	nil
4.	8	nil
5.	8	nil
6.	8	nil
7.	8	nil
8.	5	a=b, c=d, g=h
9.	5	b=f, c=e, g=h
10.	8	nil
11.	6	c=g, d=f
12.	6	d=e, f=h
13.	5	a=b, c=d, g=h
14	8	Nil
15	7	e=g
16	4	a=e, b=f, c=g, d=h
17	8	Nil
18	5	b=c, d=g, e=f
19	8	Nil
20	8	Nil
21	8	Nil
22	8	Nil
23	8	Nil
24	7	b=d
25	8	Nil
26	8	Nil
27	3	a=f=h=d, b=c, e=g
28	4	a=c=f=h, b=g
29	7	g=h
30	3	a=d, b=c=g=h=f
31	4	a=d, b=e, c=f, g=h
32	8	Nil
33	5	a=f, c=h, d=g
34	5	a=d, b=c, e=h
35	3	a=d=e=f, c=g
36	8	Nil
37	8	Nil
38	5	a=f, c=h, d=g
39	8	Nil
40	8	Nil
41	5	a=e, b=d, g=h
42	4	a=d, b=c, e=f, g=h
43	6	b=h, c=g
44	5	a=h, d=g, e=f
45	1	a=b=c=d=e=f=g=h
Total distinct mechanisms = 286		

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