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Cost Analysis of Two-Phase M/M/1 Queueing system in the Transient state with N-Policy and Server Breakdowns

Dr V.N.Rama Devi[1], K.Satish Kumar[2], G.Sridhar[3], Dr K.Chandan[4]
[1]Associate Professor, GokarajuRangaraju Instsitute of Engineering and Technology, Hyderabad, Telangana,India
[2]Research Scholar, Acharya Nagarjuna University, Guntur, Andhrapradesh, India
[3]Research Scholar, Acharya Nagarjuna University, Guntur, Andhrapradesh, India
[4]Professor, Acharya Nagarjuna University, Guntur, Andhrapradesh, India

ABSTRACT
This paper deals with the cost function analysis in Transient state of Two-Phase M/M/1 Queueing system with N-Policy, Server Breakdowns and Repairs. Customers arrive in the system in Poisson fashion and served in exponential service time distribution. They receive the first phase service as a batch followed by second essential phase of individual service. Lack of service occurs when the server is on vacation, busy during the service mode or due to the sudden breakdowns of the server. MATLAB is used to obtain transient solution for the system size probabilities. Further, Expected length of the system, waiting time in the system and cost of the model are analyzed. Finally Numerical illustrations are provided to see the effect of parameters on system performance measures.

Key words: Time dependent probabilities, Server Breakdowns and Repairs, Startup.

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I. INTRODUCTION

During the last three decades, Queues with vacations or simply called *vacation models* attracted great attention of queueing researchers and became an active research area due to their wide applications in many areas like computer systems, communication networks, etc.. Many studies on vacation models were published from the 1970's to the mid 1980' and were summarized in two survey papers by Doshi and Teghem, respectively, in 1986[2,3]. Stochastic decomposition theorems were established as the core of vacation queueing theory. In the early 1990‘Takagi published a set of three volume books entitled Queueing Analysis. One of Takagi's books was devoted to vacation models of both continuous and discrete time types and focus mainly on M/G/1 type and Geo/G/l type queues with vacations. Takagi's book certainly advanced further research and wide applications of vacation models.

Many authors have studied the utilization of server's idle time in queueing systems. These queueing systems have got wide applications in computer, communication, production and other stochastic systems. Miller (1964) [10] was the first to study an M/G/1 queueing system where the server is unavailable during some random length of time (referred to as vacation).Levy and Yechiali (1975) [8, 9] have found server's idle time utilization in the M/G/1 queue based on the assumption that as the queue becomes empty, the server takes vacation of exponential distribution during which he does some secondary work.

Yadin and Naor (1963) [14] were first to study the concept of N-policy. They studied an M/G/1 queueing system and obtained the optimal value of the queue size at which to start on a single server, assuming that the form of the policy is to turn on the server when the queue size reaches a certain number N and to turn him off when the system size is empty. Baker (1973) [1] studied the M/M/1 queue with exponential startups. He derived the optimum number of customers present that minimizes the mean time cost when startup times are zero and non-zero respectively.

Two-phase queueing system with two essential phases of service was first introduced by Krishna and Lee (1990) [6]. They considered the exhaustive service with and without gating for the M/M/1 queueing system and derived the sojourn time distribution and its mean for an arbitrary customer. Kim and Chae (1998) [7] analyzed a single server two-phase queueing system with N-policy where the First phase of service is batch service and the second phase of service is individual. Vasanta Kumar.V, Chandan.K et. al (2010) [13] Studied Two-phase M/M/1 queueing system with N-policy for exhaustive batch service with gating, server start-ups and breakdowns.

Explicit expressions for the steady state distribution of the number of customers in the system are obtained and also derived the expected system length. The total expected cost function is developed to determine the optimal threshold of N at a minimum cost.

In many applications one has to find transient solutions in Queueing systems. Transient analysis is dependent on time, it uses different analysis algorithms, control options with different convergence related issues and different initialization parameters. A time-dependent solution for the number in a single-server queueing system with Poisson arrivals and exponential service times is derived in a direct way by P. R. Parthasarathy[11]. Jacob.M.J. and Madhusoodanan.T.P. (1988) [5] examined the transient behaviour of the infinite capacity M/G/1 model with batch arrivals and server vacations. Dong-Yuh Yang and Ying-Yi Wu (2014) [4] presented Transient Behavior Analysis of a Finite Capacity Queue with Working Breakdowns and Server Vacations. The transient analysis of a finite capacity Markovian queueing system with discouraged arrivals and retention of reneging customers is well presented by Rakesh Kumar and Sapana Sharma(2017)[12].

However, to the best of our knowledge, for two –phase queueing systems with N-Policy, server breakdowns, there is no literature which takes time dependent probabilities into consideration. This motivates us to study a two-phase queueing system with N-policy, server start-up, breakdowns in Transient State. Thus, in this present paper, we consider Cost analysis of two-phase M/M/1 queueing system with server Start-up, N-Policy and unreliable server in Transient mode.

This paper is organized in V sections. Section II describes the mathematical model and includes the set of governing differential equations of the model. In Section III, some performance measures are provided using the solution of the system of differential equations. Numerical results are given in Section IV. Section V concludes the paper.

The main objectives of the analysis carried out in this paper for the Cost control policy are:

i. To establish the Transient state equations and obtain the Transient state probability distribution of the number of customers in the system in each state.
ii. To derive values for the expected number of customers in the system when the server is in vacation, in startup, in batch service (working and broken conditions) and in individual service (working and broken conditions) respectively.
iii. To formulate the total expected cost functions for the system and determine its value.
iv. To carry out sensitivity analysis on the System performance measures and the expected cost for various system parameters through numerical experiments.

II. THE SYSTEM AND ASSUMPTIONS

We consider the M/M/1 queueing system with N-policy, two phases of service and server Breakdowns in Transient state with the following assumptions:

1. Customers are assumed to arrive according to Poisson process with mean arrival rate λ and join the batch queue. Customers will get the service in the order in which they arrive.
2. The service is in two phases. The first phase of service is batch service to all customers waiting in the queue. On completion of batch service, the server immediately proceeds to the second phase to serve all customers in the batch individually. Batch service time is assumed to follow exponential distribution with mean $1/\beta$ which is independent of batch size. Individual service times are assumed to be exponentially distributed with mean $1/\mu$. On completion of individual service, the server returns to the batch queue to serve the customers who have arrived. If at-least one customer is waiting, the server starts the batch service followed by individual service to each customer in the batch. If no customer is waiting the server takes a vacation.
3. Whenever the system becomes empty, the server is turned off. As soon as the total number of arrivals in the queue reaches or exceeds the pre-determined threshold N, the server is turned on and is temporarily unavailable for the waiting customers. The server needs a startup time which follows an exponential distribution with mean $1/\theta$. As soon as the server finishes startup, it starts serving the first phase of waiting customers.
4. The customers who arrive during the batch service are also allowed to join the batch queue which is in service.
5. The breakdowns are generated by Poisson process with rates ξ_1 for the first phase of service and α_1 for the second phase of service. When the server fails it is immediately repaired at a repair rate ξ_2 in first phase and α_2 in second phase, where the repair times are exponentially distributed. After repair the server immediately resumes the concerned service.

Notations:

$p_{0,i,0}(t)$ =The probability that there are i customers in the batch queue when the server is on vacation by the time t, where i = 0,1,2,3,…,N-1

$p_{1,i,0}(t)$ = The probability that there are i customers in the batch queue when the server is doing pre-service (startup work) by the time t,, where i = N,N+1,N+2,…,S
$p_{2,i,0}(t)$= The probability that there are i customers in the batch queue when the server is in batch service by the time t, where i = 1,2,3,…,S
$p_{3,i,0}(t)$= The probability that there are i customers in batch queue when the server is working but found to be broken down by the time t, where i = 1,2,3,…,S
$p_{4,i,j}(t)$= The probability that there are i customers in the batch queue and j customers in individual queue when the server is in individual service by the time t,, where i=0,1,2…S-1 and j=1,2,3,…,S
$p_{5,i,j}(t)$=The probability that there are i customers in the batch queue and j customers in individual queue when the server is working but found to be broken down by the time t,, where i = 0,1,2,…,S-1 and j = 1,2,3, …,S
The transient-state equations governing the system size probabilities at an arbitrary time t, are given by the following set of Differential equations:

$$\frac{dp_{0,0,0}(t)}{dt} = -\lambda p_{0,0,0}(t) + \mu p_{4,0,1}(t). \quad (1)$$
$$\frac{dp_{0,i,0}(t)}{dt} = -\lambda p_{0,0,0}(t) + \lambda p_{0,i-1,0}(t), 1 \le i \le N-1. \quad (2)$$
$$\frac{dp_{1,N,0}(t)}{dt} = -(\lambda+\theta) p_{1,N,0}(t) + \lambda p_{0,N-1,0}(t). \quad (3)$$
$$\frac{dp_{1,i,0}(t)}{dt} = -(\lambda+\theta) p_{1,i,0}(t) + \lambda p_{1,i-1,0}(t), N+1 \le i \le S-1. \quad (4)$$
$$\frac{dp_{1,S,0}(t)}{dt} = -(\theta) p_{1,S,0}(t) + \lambda p_{1,S-1,0}(t) \quad (5)$$
$$\frac{dp_{2,i,0}(t)}{dt} = -(\lambda+\beta+\xi_1) p_{2,i,0}(t) + \lambda p_{2,i-1,0}(t) + \mu\, p_{4,i,1}(t) + \xi_2 p_{3,i,0}(t), 1 \le i \le N-1. \quad (6)$$
$$\frac{dp_{2,i,0}(t)}{dt} = -(\lambda+\beta+\xi_1) p_{2,i,0}(t) + \lambda p_{2,i-1,0}(t) + \mu p_{4,i,1}(t) + \xi_2 p_{3,i,0}(t) + \theta p_{1i,0}(t), N \le i \le S-1. \quad (7)$$
$$\frac{dp_{2,S,0}(t)}{dt} = -(\beta+\xi_1) p_{2,i,0}(t) + \lambda p_{2,i-1,0}(t) + \xi_2 p_{3,i,0}(t) + \theta p_{1,S,0}(t). \quad (8)$$
$$\frac{dp_{3,i,0}(t)}{dt} = -(\lambda+\xi_2) p_{3,i,0}(t) + \lambda p_{3,i-1,0}(t) + \xi_1 p_{2,i,0}(t), 1 \le i \le S-1. \quad (9)$$
$$\frac{dp_{3,S,0}(t)}{dt} = -(\xi_2) p_{3,i,0}(t) + \lambda p_{3,S-1,0}(t) + \xi_1 p_{2,S,0}(t). \quad (10)$$
$$\frac{dp_{4,0,j}(t)}{dt} = -(\lambda+\alpha_1+\mu) p_{4,0,j}(t) + \mu\, p_{4,0,j+1}(t) + \beta p_{2,j,0}(t) + \alpha_2 p_{5,0,j}(t), 1 \le j \le S-1. \quad (11)$$
$$\frac{dp_{4,0,S}(t)}{dt} = -(\alpha_1+\mu) p_{4,0,S}(t) + \beta p_{2,S,0}(t) + \alpha_2 p_{5,0,S}(t). \quad (12)$$
$$\frac{dp_{4,i,j}(t)}{dt} = -(\lambda+\alpha_1+\mu)\, p_{4,i,j}(t) + \mu p_{4,i,j+1}(t) + \lambda p_{4,i-1,j}(t) + \alpha_2 p_{5,i,j}(t), 1 \le i \le S-1, 1 \le i \le S-1, i+j \le S-1. \quad (13)$$
$$\frac{dp_{4,i,j}(t)}{dt} = -(\alpha_1+\mu)\, p_{4,i,j}(t) + \lambda p_{4,i-1,j}(t) + \alpha_2 p_{5,i,j}(t), 1 \le i \le S-1, 1 \le i \le S-1, i+j = S. \quad (14)$$
$$\frac{dp_{5,0,j}(t)}{dt} = -(\lambda+\alpha_2) p_{5,0,j}(t) + \alpha_1 p_{4,0,j}(t), 1 \le j \le S-1. \quad (15)$$
$$\frac{dp_{5,0,S}(t)}{dt} = -(\alpha_2) p_{5,0,S}(t) + \alpha_1 p_{4,0,S}(t). \quad (16)$$
$$\frac{dp_{5,i,j}(t)}{dt} = -(\lambda+\alpha_2)\, p_{5,i,j}(t) + \alpha_1 p_{4,i,j}(t) + \lambda\, p_{5,i-1,j}(t), 1 \le i \le S-1,\ i+j \le S-1. \quad (17)$$
$$\frac{dp_{5,i,j}(t)}{dt} = -(\alpha_2)\, p_{5,i,j}(t) + \alpha_1 p_{4,i,j}(t) + \lambda\, p_{5,i-1,j}(t), 1 \le i \le S-1, i+j = S. \quad (18)$$

III. PERFORMANCE MEASURES

Some performance measures are calculated to predict the system's behavior using the probabilities obtained in previous section with the help of Runge-Kutta method.

1. Probability of server being idle at time t: $I(t) = \sum_{i=0}^{N-1} p_{0,i,0}(t) + \sum_{i=N}^{S} p_{1,i,0}(t)$
2. Probability of server being busy at time t: $B(t) = \sum_{i=1}^{S} p_{2,i,0}(t) + \sum_{i=1}^{S} p_{4,i,0}(t) + \sum_{i=1}^{S-1}\sum_{j=1}^{S-1} p_{4,i,j}$
3. Probability of server under repair at time t: $R(t) = \sum_{i=1}^{S} p_{3,i,0}(t) + \sum_{j=1}^{S} p_{5,0,j}(t) + \sum_{i=1}^{S-1}\sum_{j=1}^{S-1} p_{5,i,j}$
4. Expected length of system at time t: $L(t) = \sum n * p_n(t)$,'n' is number of customers in the system
5. Waiting time in the system: $W(t) = L(t)/\lambda(1 - p_S(t))$

Cost Function: Let C (t) be the average cost per unit of time, then

$$C(t) = C_h L(t) + C_o(B(t)) + C_m(P_{1,i,0}(t)) + C_b(R(t)) + C_s(\lambda p_{0,0,0}(t)) - C_r\left(\frac{N}{p_{0,0,0}(t)}\right).$$

$C(t) = C_h * Expected\ length(t) + C_o * (p(server\ is\ on\ and\ in\ operation)) + C_m * (p(Start - up)) + C_b * (p(Breakdown))$+ $C_s(\lambda p_{0,0,0}(t))$-$C_r\left(\frac{N}{p_{0,0,0}(t)}\right)$)

Where

C_h= Holding cost per unit time for each customer present in the system,
C_o = Cost per unit time for keeping the server on and in operation,
C_m = Startup cost per unit time,
C_s = Setup cost per cycle,
C_b = Break down cost per unit time for the unavailable server in batch as well as individual service,
C_r = Reward per unit time as the server is doing secondary work in vacation.

IV. NUMERICAL RESULTS

The obtained results are numerically evaluated based on specific parameters. The effect of various parameters on the system performance measures such as expected number of customers in the system and mean waiting time in the system are studied and also the corresponding cost function is also analyzed. MATLAB software is used to develop the computational program. The effect of different parameters on the sys in the system performance measures is summarized in Tables I-VIII.

In all numerical computations, the model parameters are taken as

N = 4, S=10, □=0.4, μ =0.6, β=0.5, $\theta = .02$, ξ_1=.01, ξ_2=.02, α_1 =.01, α_2=.02, T=2 and h=0.5 .

Costs are assumed as $C_h = 5, C_o = 50, C_m = 200, C_s = 1000, C_b = 70, C_r$ =15

From Tables 1-8, it leads to the following observations:

- With regard to the measure $L(t)$ for a particular value of t , we observe
- an increasing trend as λ increases, θ increases, ξ_1 increases and α_1 increases
- a decreasing trend with the increase of $\mu(initially\ increasing\ as\ it\ is\ the\ second\ phase\ swrvice\ but\ finally\ decreases)$, β, ξ_2 and α_2
- With regard to the measure $W(t)$ for a particular value of t, *we observe*
- an increasing trend as λ increases, θ increases, ξ_1 increases and α_1 increases
- a decreasing trend with the increase of μ , β, ξ_2 and α_2
- With regard to the measure $C(t)$ for a particular value of t , we observe
- an increasing trend as λ increases, θ increases, ξ_1 increases and α_1 increases
- a decreasing trend with the increase of μ , β, ξ_2 and α_2

V. Conclusions and further Scope of study

Cost analysis of Two-phase M/M/1 queueing system with Start-up and Breakdowns in transient state is presented. Sensitivity analysis is also performed to know the influence of non monetary parameters on system performance measures. In the above study Gating and Customer impatient behavior can also be included.

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Table 1: Effect of λ on system performance measures under different values of't'

	Time	0.5	1	1.5	2
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.37E-07	6.92E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
λ=.4	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.999991	0.999941	0.999783
	B(t)	0	8.71E-06	5.88E-05	0.000215
	R(t)	0	1.28E-08	1.50E-07	7.55E-07
	L(t)	0.205	0.41	0.61499	0.819985
	W(t)	0.500571	1.007575	1.531223	2.079828
λ=.41	C(t)	286.1673	234.5005	193.0272	160.1954
	I(t)	1	0.99999	0.999936	0.999764
	B(t)	0	9.54E-06	6.41E-05	0.000234
	R(t)	0	1.41E-08	1.63E-07	8.22E-07
	L(t)	0.21	0.42	0.629998	0.839983
	W(t)	0.500609	1.008059	1.533051	2.084036
λ=.42	C(t)	292.8777	238.8259	195.6753	161.7238

Table 2: Effect of μon system performance measures under different values of't'

	Time	0.5	1	1.5	2
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.37E-07	6.92E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
μ=.6	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.37E-07	6.92E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
μ=.61	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.37E-07	6.92E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529043	2.075692
μ=.62	C(t)	279.3827	230.0666	190.2631	158.5609

Table 3: Effect of β on system performance measures under different values of'ť'

	Time	0.5	1	1.5	2
	I(t)	1.00E+00	1.00E+00	1.00E+00	0.999801
	B(t)	0.00E+00	7.94E-06	5.38E-05	0.000197
	R(t)	0.00E+00	1.17E-08	1.37E-07	6.92E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
β=.5	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.37E-07	6.91E-07
	L(t)	0.2	0.4	0.599999	0.799986
	W(t)	0.500533	1.007107	1.529442	2.075688
β=.51	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.36E-07	6.91E-07
	L(t)	0.2	0.4	0.599999	0.799986
	W(t)	0.500533	1.007107	1.529441	2.075684
β=.52	C(t)	279.3827	230.0666	190.2631	158.5609

Table 4: Effect of θ on system performance measures under different values of'ť'

	Time	0.5	1	1.5	2
	I(t)	1	0.999992	0.999946	1.00E+00
	B(t)	0	7.94E-06	5.38E-05	1.97E-04
	R(t)	0	1.17E-08	1.37E-07	6.92E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
θ=.05	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.99999	0.999935	0.999762
	B(t)	0	9.51E-06	6.44E-05	0.000236
	R(t)	0	1.40E-08	1.64E-07	8.27E-07
	L(t)	0.2	0.4	0.599998	0.799984
	W(t)	0.500533	1.007106	1.529435	2.07565
θ=.06	C(t)	279.3827	230.0664	190.2615	158.555
	I(t)	1	0.999989	0.999925	0.999723
	B(t)	0	1.11E-05	7.49E-05	0.000274
	R(t)	0	1.63E-08	1.90E-07	9.62E-07
	L(t)	0.2	0.4	0.599998	0.799981
	W(t)	0.500533	1.007106	1.529427	2.075608
θ=.07	C(t)	279.3827	230.0661	190.2599	158.5492

Table 5: Effect of ξ_1 on system performance measures under different values of't'

	Time	0.5	1	1.5	2
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.37E-07	6.92E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
ξ_1=.01	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.93E-06	5.38E-05	0.000197
	R(t)	0	1.39E-08	1.61E-07	8.14E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
ξ_1=.012	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.93E-06	5.37E-05	0.000197
	R(t)	0	1.61E-08	1.86E-07	9.36E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
ξ_1=.014	C(t)	279.3827	230.0666	190.2631	158.5609

Table 6: Effect of ξ_2 on system performance measures under different values of't'

	Time	0.5	1	1.5	2
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.37E-07	6.92E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
ξ_2=.02	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.36E-07	6.91E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
ξ_2=.022	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.36E-07	6.91E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
ξ_2=.024	C(t)	279.3827	230.0666	190.2631	158.5609

Table 7: Effect of α_1 on system performance measures under different values of't'

	Time	0.5	1	1.5	2
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.37E-07	6.92E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
α_1=.01	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.18E-08	1.39E-07	7.07E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
α_1=.012	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.19E-08	1.41E-07	7.23E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
α_1=.014	C(t)	279.3827	230.0666	190.2631	158.5609

Table 8: Effect of α_2 on system performance measures under different values of't'

	Time	0.5	1	1.5	2
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.37E-07	6.92E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
α_2=.02	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.37E-07	6.91E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
α_2=.022	C(t)	279.3827	230.0666	190.2631	158.5609
	I(t)	1	0.999992	0.999946	0.999801
	B(t)	0	7.94E-06	5.38E-05	0.000197
	R(t)	0	1.17E-08	1.37E-07	6.91E-07
	L(t)	0.2	0.4	0.599999	0.799987
	W(t)	0.500533	1.007107	1.529443	2.075692
α_2=.024	C(t)	279.3827	230.0666	190.2631	158.5609