

Definite Integral Relation Involving H-Function of Multivariable Function and General Class of Polynomial

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ABSTRACT

In this paper, we obtain an integral relation containing to a product of Fox's H-function [2], , general class of multivariable polynomials srivastava and Garg [11],generalized polynomials srivastava [10] and H-function of several complex variables given by srivastava and panda [14] with general argument of quadratic nature. This paper is capable of yielding numerous result involving classical orthogonal polynomials.

Key Words: Fox's H-function, general polynomials, general class of polynomials, generalized lauricella function, G-function, multivariables H-function

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I. INTRODUCTION

The H-function of several complex variable is defined by Srivastava and Panda [14] as:

$$H[x_1, \dots, x_r] = H_{A,C;[B',D'];\dots;[B^{(r)},D^{(r)}]}^{0,\lambda:(u',v');\dots;(u^{(r)},v^{(r)})} [(p):\theta'; \dots;\theta^{(r)}] : [(q):\Delta] ; \dots; [(q^{(r)}):\Delta^{(r)}] ; x_1, \dots, x_r \quad \dots(1.1)$$

The Fox's H-function [2]:

$$H_{P,Q}^{L,R} \left[x \left| \begin{matrix} (m_P, M_P) \\ (n_Q, N_Q) \end{matrix} \right. \right] = \sum_{G=0}^{\infty} \sum_{g=1}^L \frac{(-1)^G}{G! N_g} \phi_{(\eta_G)} x^{\eta_G}, \quad \dots(1.2)$$

where

$$\phi_{(\eta_G)} = \frac{\prod_{j=1}^L \Gamma(n_j - N_j \eta_G) \prod_{j=1}^R \Gamma(1 - m_j + M_j \eta_G)}{\prod_{j=L+1}^P \Gamma(1 - n_j + N_j \eta_G) \prod_{j=R+1}^Q \Gamma(m_j - M_j \eta_G)}$$

and

$$\eta_G = \frac{(\eta_g + G)}{\eta_g}$$

The H-function of multivariable in (1.1) converges absolutely if

$$\left| \arg(x_i) \right| < \frac{1}{2} \pi T_i, \quad \dots(1.3)$$

where

$$T_i = - \sum_{j=1+\lambda}^A \theta_j^{(i)} + \sum_{j=1}^{v^{(i)}} \Delta_j^{(i)} - \sum_{j=1+v^{(i)}}^{B^{(i)}} \Delta_j^{(i)} - \sum_{j=1}^C \Psi_j^{(i)} + \sum_{j=1}^{u^{(i)}} \delta_j^{(i)} - \sum_{j=1+u^{(i)}}^{D^{(i)}} \delta_j^{(i)} > 0, \quad \forall i \in (1, \dots, r) \quad \dots(1.4)$$

general class of polynomials introduced by srivastava [10] is as follows

$$S_N^M(Z) = \sum_{\ell=0}^{(N/M)} \frac{(-N)_{M\ell}}{\ell!} B_{N,k} Z^\ell \quad \dots(1.5)$$

Where M,N are arbitrary positive integer and the coefficients B_{N,k} are arbitrary constant ,real or complex. Srivastava has defined and introduced the general polynomials ([10], p.185, eq.(7))

$$S_{N_1, \dots, N_s}^{M_1, \dots, M_s} [z_1, \dots, z_r] = \sum_{\beta_1=0}^{[N_1/M_1]} \dots \sum_{\beta_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1\beta_1}}{\beta_1!} \dots \frac{(-N_s)_{M_s\beta_s}}{\beta_s!} \cdot B [N_1, \beta_1; \dots; N_s, \beta_s] z_1^{\beta_1} \dots z_s^{\beta_s}, \quad \dots(1.6)$$

where N_i = 0,1,2,..., $\forall i = (1, \dots, s)$; M₁, ..., M_s are arbitrary positive integers and the coefficients B [N₁,β₁ ; ...; N_s,β_s] are arbitrary constants, real or complex.

II. THE MAIN INTEGRAL RESULT

Here we obtain following integral :

$$\int_0^\infty w^{1-\beta} (a + bw + cw^2)^{\beta-3/2} H_{P,Q}^{L,R} \left[\left(\frac{w}{a + bw + cw^2} \right)^\sigma \middle| \begin{matrix} (m_p, M_p) \\ (n_q, N_q) \end{matrix} \right] \cdot S_{N_1, \dots, N_s}^{M_1, \dots, M_s} \left[z_1 \left(\frac{w}{a + bw + cw^2} \right)^{n_1}, \dots, z_s \left(\frac{w}{a + bw + cw^2} \right)^{n_s} \right] \cdot S_N^M \left[z \left(\frac{w}{a + bw + cw^2} \right)^n \right] \cdot H \left[x_1 \left(\frac{w}{a + bw + cw^2} \right)^{\sigma_1}, \dots, x_r \left(\frac{w}{a + bw + cw^2} \right)^{\sigma_r} \right] dw = \sqrt{\frac{\pi}{c}} \sum_{G=0}^\infty \sum_{g=1}^L \sum_{\beta_1=0}^{[N_1/M_1]} \dots \sum_{\beta_s=0}^{[N_s/M_s]} \sum_{\ell=0}^{(N/M)} \frac{(-N)_{M\ell}}{\ell!} B_{N,k} \frac{(-1)^G}{G! F_g} \frac{(-N_1)_{M_1\beta_1}}{\beta_1!} \dots \frac{(-N_s)_{M_s\beta_s}}{\beta_s!} \Phi(\eta_G)$$

$$B [N_1, \beta_1; \dots; N_s, \beta_s] z_1^{\beta_1} \dots (z_s)^{\beta_s} z^\ell (b + 2\sqrt{ca})^{\beta - \sigma\eta_G - \sum_{i=1}^s n_i\beta_i - 1} \cdot H_{A+1, C+H[B;D]; \dots; [B^{(r)}, D^{(r)}]}^{0, \lambda+1; (u, v); \dots; (u^{(r)}, v^{(r)})} \left[\begin{matrix} x_1 (b + 2\sqrt{ca})^{-\sigma_1} \\ \vdots \\ x_r (b + 2\sqrt{ca})^{-\sigma_r} \end{matrix} \middle| \begin{matrix} [\beta - \sigma\eta_G - \sum_{i=1}^s n_i\beta_i : \sigma_1; \dots; \sigma_r], \\ [(s) : \Psi'; \dots; \Psi^{(r)}], \end{matrix} \right]$$

$$\left[\begin{array}{l} [(p):\theta'; \dots; \theta^{(r)}] : [(q'): \Delta']; \dots; [(q^{(r)}): \Delta^{(r)}] \\ [\beta - \sigma - \sum_{i=1}^s n_i \beta_i - \frac{1}{2} : \sigma_1; \dots; \sigma_r] : [(t'): \delta']; \dots; [(t^{(r)}): \delta^{(r)}] \end{array} \right] \dots(2.1)$$

provided that $\text{Re}(a) > 0, \text{Re}(b) > 0, c > 0$ and

$$\sigma \min \left[\text{Re} \left(\frac{n_j}{N_j} \right) \right] + \sum_{i=1}^r \sigma'_i \min \left[\text{Re} \left(\frac{t_{j'}^{(i)}}{\delta_{j'}^{(i)}} \right) \right] > \beta - 2, j = 1, \dots, M \text{ and } j' = 1, \dots, u^{(i)}.$$

Proof:

For getting the result (2.1) first we express the Fox H-function and a general polynomials in form of series and the H-function of multivariable in terms of Mellin-Barnes contour integrals. Now interchanging the order of summations and integration which is permissible under the stated condition, we get

$$\begin{aligned} & \sum_{G=0}^{\infty} \sum_{g=1}^L \sum_{\beta_1=0}^{[N_1/M_1]} \dots \sum_{\beta_s=0}^{[N_s/M_s]} \sum_{\ell=0}^{(NM)} \frac{(-N)_{Mk}}{\ell!} B_{N,k} \frac{(-1)^G (-N_1)_{M_1 \beta_1}}{\beta_1!} \dots \frac{(-N_s)_{M_s \beta_s}}{\beta_s!} \phi(\eta_G) \\ & \cdot B[N_1, \beta_1; \dots; N_s, \beta_s](z_1)^{\beta_1} \dots (z_s)^{\beta_s} z^\ell \\ & \cdot \frac{1}{(2\pi i)^r} \int_{I_1} \dots \int_{I_r} \Psi(\xi \gamma_1, \dots, \gamma_r) \Delta_1(v_1) \dots \Delta_r(\gamma_r) x_1^{\gamma_1} \dots x_r^{\gamma_r} \\ & \cdot \left\{ \int_0^\infty w^{1 - [\beta - \sigma \eta_G - \sum_{i=1}^s n_i \beta_i - n\ell - \sigma_1 \gamma_1 - \dots - \sigma_r \gamma_r]} \right. \\ & \left. \cdot (a + bw + cw^2)^{\left(\beta - \sigma \eta_G - \sum_{i=1}^s n_i \beta_i - n\ell - \sigma_1 \gamma_1 - \dots - \sigma_r \gamma_r \right) - \frac{3}{2}} dw \right\} d\gamma_1 \dots d\gamma_r, \dots(2.2) \end{aligned}$$

On solving above w-integral by help of known theorem (Saxena [8]) and reinterpreting the result obtained in terms of H-function of r variable, we get the desired result.

III. PARTICULAR CASES

(a) When we put $\lambda = A = C = 0$ in (2.1), we get the following integral result

$$\begin{aligned} & \int_0^\infty w^{1-\beta} (a + bw + cw^2)^{\beta-3/2} H_{P,Q}^{L,R} \left[\left(\frac{w}{a + bw + cw^2} \right)^\sigma \middle| \begin{array}{l} (m_P, M_P) \\ (n_Q, N_Q) \end{array} \right] \\ & \cdot S_{N_1 \dots N_s}^{M_1 \dots M_s} \left[z_1 \left(\frac{w}{a + bw + cw^2} \right)^{n_1}, \dots, z_s \left(\frac{w}{a + bw + cw^2} \right)^{n_s} \right] \\ & \cdot S_N^M \left[z \left(\frac{w}{a + bw + cw^2} \right)^n \right] \\ & \cdot \prod_{i=1}^r H_{B^{(i)}, D^{(i)}}^{u^{(i)}, v^{(i)}} \left[x_i \left(\frac{w}{a + bw + cw^2} \right)^{\sigma_i} \middle| \begin{array}{l} [(b^{(i)}): \phi^{(i)}] \\ [d^{(i)} : \delta^{(i)}] \end{array} \right] dw \\ & = \sqrt{\frac{\pi}{c}} \sum_{G=0}^{\infty} \sum_{g=1}^L \sum_{\beta_1=0}^{[N_1/M_1]} \dots \sum_{\beta_s=0}^{[N_s/M_s]} \sum_{\ell=0}^{(NM)} \frac{(-N)_{Mk}}{\ell!} B_{N,k} \frac{(-1)^G (-N_1)_{M_1 \beta_1}}{G! F_g} \frac{(-N_s)_{M_s \beta_s}}{\beta_s!} \phi(\eta_G) \end{aligned}$$

$$\cdot (B[N_1, \beta_1; \dots; N_s, \beta_s] z_1^{\beta_1} \dots (z_s)^{\beta_s} z^l$$

$$\begin{aligned} & (b + 2\sqrt{ca})^{\beta - \sigma\eta_G - \sum_{i=1}^s n_i \beta_i - 1} H_{1,1;\{B;D\};\dots;\{B^{(r)},D^{(r)}\}}^{0,1;(u',v');\dots;(u^{(r)},v^{(r)})} \\ & \left[\begin{array}{l} x_1 (b + 2\sqrt{ca})^{-\sigma_1} \\ \vdots \\ x_r (b + 2\sqrt{ca})^{-\sigma_r} \end{array} \right] \left[\begin{array}{l} [\beta - \sigma\eta_G - \sum_{i=1}^s n_i \beta_i : \sigma_1; \dots; \sigma_r] : [(q') : \Delta']; \dots; [(q^{(r)}) : \Delta^{(r)}] \\ [\beta - \sigma\eta_G - \sum_{i=1}^s n_i \beta_i - \frac{1}{2} : \sigma_1; \dots; \sigma_r] : [(t') : \delta']; \dots; [(t^{(r)}) : \delta^{(r)}] \end{array} \right] \end{aligned} \dots(3.1)$$

valid under the same condition which is obtained from (2.1).

(b) If $\lambda = A$, $u^{(i)} = 1$, $v^{(i)} = B^{(i)}$ and $D^{(i)} = D^{(i)} + 1, \forall i \in (1, \dots, r)$ the result in (2.1) reduces to the following integral transformation:

$$\begin{aligned} & \int_0^\infty w^{1-\beta} (a + bw + cw^2)^{\beta-3/2} H_{P,Q}^{L,R} \left[\left(\frac{w}{a + bw + cw^2} \right)^\sigma \middle| \begin{array}{l} (m_p, M_p) \\ (n_q, N_q) \end{array} \right] \\ & \cdot S_{N_1, \dots, N_s}^{M_1, \dots, M_s} \left[z_1 \left(\frac{w}{a + bw + cw^2} \right)^{n_1}, \dots, z_s \left(\frac{w}{a + bw + cw^2} \right)^{n_s} \right] \cdot S_N^M \left[z \left(\frac{w}{a + bw + cw^2} \right)^n \right] \\ & \cdot E_{C:D;\dots;D^{(r)}}^{A:B;\dots;B^{(r)}} \left[-x_1 \left(\frac{w}{a + bw + cw^2} \right)^{\sigma_1}, \dots, -x_r \left(\frac{w}{a + bw + cw^2} \right)^{\sigma_r} \middle| \begin{array}{l} [1-(p); \theta'; \dots; \theta^{(r)}] \\ [1-(s); \Psi'; \dots; \Psi^{(r)}] \\ [1-(q); \Delta']; \dots; [1-(b^{(r)}); \Delta^{(r)}] \\ [1-(t); \delta']; \dots; [1-(d^{(r)}); \delta^{(r)}] \end{array} \right] dw \\ & = \sqrt{\frac{\pi}{c}} \sum_{G=0}^\infty \sum_{g=1}^L \sum_{\beta_1=0}^{[N_1/M_1]} \dots \sum_{\beta_s=0}^{[N_s/M_s]} \sum_{\ell=0}^{(N/M)} \frac{(-N)_{Mk}}{\ell!} B_{N,k} \frac{(-1)^G}{G! F_g} \frac{(-N_1)_{M_1 \beta_1}}{\beta_1!} \dots \frac{(-N_s)_{M_s \beta_s}}{\beta_s!} \phi(\eta_G) \\ & \cdot B(N_1, \beta_1; \dots; N_s, \beta_s) \cdot z_1^{\beta_1} \dots (z_s)^{\beta_s} z^\ell (b + 2\sqrt{ca})^{\beta - \sigma\eta_G - \sum_{i=1}^s n_i \beta_i - 1} \\ & \frac{\Gamma(1 - \beta + \sigma\eta_G + \sum_{i=1}^s n_i (\beta_i + k_i))}{\Gamma(\frac{3}{2} - \beta + \sigma\eta_G + \sum_{i=1}^s n_i (\beta_i + k_i))} E_{C+1:D;\dots;D^{(r)}}^{A+1:B;\dots;B^{(r)}} \\ & \left[\begin{array}{l} [-x_1 (b + 2\sqrt{ca})^{-\sigma_1}, \dots, -x_r (b + 2\sqrt{ca})^{-\sigma_r}] \left[\begin{array}{l} [1 - \beta + \sigma\eta_G + \sum_{i=1}^s n_i \beta_i : \sigma_1; \dots; \sigma_r], \\ [1 - (s) : \Psi'; \dots; \Psi^{(r)}], \\ [1 - (p) : \theta'; \dots; \theta^{(r)}] : [1 - (q') : \Delta']; \dots; [1 - (q^{(r)}) : \Delta^{(r)}] \end{array} \right] \\ [\frac{3}{2} - \beta + \sigma\eta_G + \sum_{i=1}^s n_i \beta_i : \sigma_1; \dots; \sigma_r] : [1 - (t') : \delta']; \dots; [1 - (t^{(r)}) : \delta^{(r)}] \end{array} \right] \end{aligned} \dots(3.2)$$

provided that $Re(a) > 0, Re(b) > 0, c > 0$, the series on the right side exists.

(c) If $\theta', \dots, \theta^{(r)} = \Delta', \dots, \Delta^{(r)} = \Psi', \dots, \Psi^{(r)} = \delta', \dots, \delta^{(r)} = \sigma_1, \dots, \sigma_r = \beta', \dots, \beta^{(r)}$ in (2.1), we obtain the following integral result:

$$\begin{aligned}
 & \int_0^\infty w^{1-\beta} (a + bw + cw^2)^{\beta-3/2} H_{P,Q}^{L,R} \left[\left(\frac{w}{a + bw + cw^2} \right)^\sigma \middle| \begin{matrix} (m_P, M_P) \\ (n_Q, N_Q) \end{matrix} \right] \\
 & \cdot S_{N_1, \dots, N_s}^{M_1, \dots, M_s} \left[Z_1 \left(\frac{w}{a + bw + cw^2} \right)^{n_1}, \dots, Z_s \left(\frac{w}{a + bw + cw^2} \right)^{n_s} \right] \cdot S_N^M \left[Z \left(\frac{w}{a + bw + cw^2} \right)^n \right] \\
 & \cdot F_{A,C:[B',D']; \dots; [B^{(r)}, D^{(r)}]}^{0,\lambda:(u',v'); \dots; (u^{(r)}, v^{(r)})} \left[X_1^{1/\beta'} \left(\frac{w}{a + bw + cw^2} \right), \dots, X_r^{1/\beta^r} \left(\frac{w}{a + bw + cw^2} \right) \middle| \begin{matrix} (p):(q); \dots; (q^{(r)}) \\ (s):(r); \dots; (r^{(r)}) \end{matrix} \right] dw \\
 & = \sqrt{\frac{\pi}{c}} \sum_{G=0}^\infty \sum_{g=1}^L \sum_{\beta_1=0}^{[N_1/M_1]} \dots \sum_{\beta_s=0}^{[N_s/M_s]} \sum_{\ell=0}^{(N/M)} \frac{(-N)_{Mk}}{\ell!} B_{N,k} \frac{(-1)^G}{G! F_g} \frac{(-N_1)_{M_1 \beta_1}}{\beta_1!} \dots \frac{(-N_s)_{M_s \beta_s}}{\beta_s!} \phi(\eta_G) \\
 & \cdot B(N_1, \beta_1; \dots; N_s, \beta_s) \cdot z_1^{\beta_1} \dots (z_s)^{\beta_s} z^\ell (b + 2\sqrt{ca})^{\beta - \sigma \eta_G - \sum_{i=1}^s n_i \beta_i - 1} \\
 & \frac{\Gamma(1 - \beta + \sigma \eta_G + \sum_{i=1}^s n_i \alpha_i \beta_i)}{\Gamma\left(\frac{3}{2} - \beta + \sigma \eta_G + \sum_{i=1}^s n_i \beta_i\right)} E_{\substack{A+1:B'; \dots; B^{(r)} \\ C+1:D'; \dots; D^{(r)}}} \\
 & \cdot F_{A+1, C+1:[B', D']; \dots; [B^{(r)}, D^{(r)}]}^{0, \lambda+1:(u', v'); \dots; (u^{(r)}, v^{(r)})} \left[X_1^{1/\beta'} (b + 2\sqrt{ca})^{-1}, \dots, X_r^{1/\beta^r} (b + 2\sqrt{ca})^{-1} \right] \\
 & \left[\beta - \sigma \eta_G - \sum_{i=1}^s n_i \beta_i \right], (p):(q); \dots; (q^{(r)}) \\
 & (s), \left[\beta - \sigma \eta_G - \sum_{i=1}^s n_i \beta_i - \frac{1}{2} \right]; (t); \dots; (t^{(r)}) \dots (3.3)
 \end{aligned}$$

provided that $\text{Re}(a) > 0, \text{Re}(b) > 0, c > 0; \beta^{(i)} > 0 (i = 1, \dots, r), 2(u^{(i)} + v^{(i)}) > (A + C + B^{(i)} + D^{(i)})$

$$|\arg(z_i)| < \left[u^{(i)} + v^{(i)} - \frac{A}{2} - \frac{C}{2} - \frac{B^{(i)}}{2} - \frac{D^{(i)}}{2} \right] \pi \text{ and}$$

$$\sigma \left\{ \min_{1 \leq j \leq M} [\text{Re}(n_j / N_j)] \right\} + \sum_{i=1}^r \left\{ \min_{1 \leq j \leq u^{(i)}} [\text{Re}(t_j^{(i)})] \right\} > \beta - 2.$$

IV. CONCLUSION

The result so obtained may be found useful in several interesting situation appearing in the literature on Mathematical analysis applied Mathematics and Mathematical physics. These results are basic in nature and likely to find useful application in the study of simple and multivariable hypergeometric series.

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