

# Transient Heat Transfer Flow by Induced Magnetic Field along a Vertical Plate

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**ABSTRACT-** In the field of geophysical and astrophysical engineering, the thermal instability in nature, in chemical processes, in separation processes and in industrial applications the effect of thermal diffusion on the MHD heat transfer in an unsteady flow past through vertical plate is very much important. It has been investigated numerically under the action of a strong applied magnetic field taking into account the induced magnetic field. This study is performed for cooling problem with lighter and heavier particles. Numerical solutions for the velocity field, induced magnetic field and temperature distribution are obtained for associated parameters using the explicit finite difference method. The obtained results are discussed with the help of graphs using Fortran programming as well as Techplot to observe the effects of various parameters on the above mentioned quantities. Finally, the important findings of the investigation are concluded.

**Keywords-**

Transient heat, Magnetic Diffusivity, induced magnetic field, thermal diffusion, viscous dissipation

## INTRODUCTION:

Heat transfer plays an important role in fluids condensing or boiling at a solid surface. The heat transfer consideration arises due to buoyancy forces caused by thermal diffusions. Condensing and boiling are characteristic for many separation processes in chemical engineering as drying, evaporation, distillation, condensation, rectification and absorption of a fluid. [5] Pera and Gebhart (1971) was first author to investigate the combined buoyancy effects of thermal and mass diffusion on natural convection flow. [9] Lin and Wu (1997) have studied simultaneous heat and mass transfer model with the entire range of buoyancy ratio for most practical chemical species in dilute and aqueous solutions. [4] Chen (2004) have observed the MHD combined heat and mass transfer in natural convection adjacent to a vertical surface. Along with these studies, the effect of thermal diffusion on MHD free convection and mass transfer flows have also been considered by many investigators due to its important role particularly in isotope separation and immixtures between gases with very light molecular weight (incompressible viscous fluid past a continuously moving surface under only the action of a transverse magnetic field with or without thermal diffusion). But the flow under the action of a strong magnetic field that induces another magnetic field have of great interest in geophysics and astrophysics. [3] Chaudhary and Sharma (2006) have analytically analyzed the steady combined heat and mass transfer flow with induced magnetic field. There are analytical solution restrictions in their studies. Quite recently, a numerical study of steady combined heat and mass transfer by mixed convection flow past a continuously moving infinite vertical porous plate under the action of a strong magnetic field with constant suction velocity, constant heat and mass fluxes have been investigated by [11] Alam et al. (2008). For unsteady two-dimensional case, the above problem becomes more complicated. These type of problems play a special role in nature, in many separation processes as isotope separation, immixtures between gases, in many industrial applications as solidification of binary alloys as well as in astrophysical

$H_2$ ,  
 $H_e$  and medium molecular weight ( $N_2$ , air) (Eckert  
and geophysical engineering).

and Drake, 1972). Recently, [11] Alam et al. (2006) have numerically investigated the mass transfer flow past a vertical porous medium with heat generation and thermal diffusion on the combined free-forced convection under the influence of transversely applied magnetic field.

In all the papers cited earlier, the studies concentrated on MHD free convection and mass transfer flow of an

### 1 Mathematical Model

A flow model of transient MHD heat transfer of an electrically conducting incompressible viscous fluid past an electrically non-conducting continuously moving semi-infinite vertical plate with thermal diffusion is considered. Introducing the Cartesian coordinate system, the  $x$ -axis chosen along the porous plate in the direction of flow and the  $y$ -axis is normal to it. A strong uniform magnetic field is applied normal to the flow region. Because of the magnetic Reynolds number the flow is not taken to be small enough, the induced magnetic field is not negligible. The induced

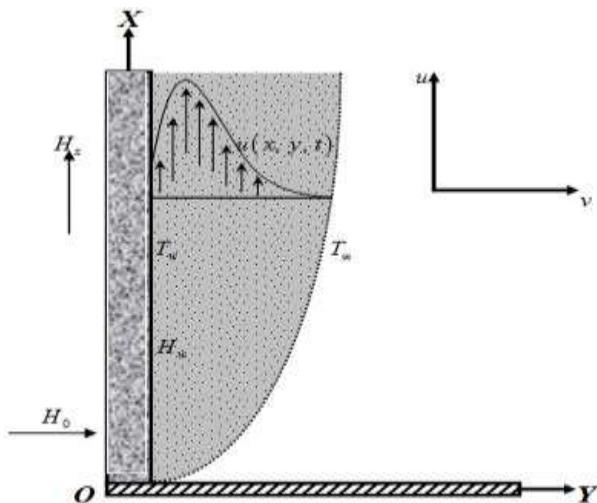
#### Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

#### Momentum equation

(1)  
magnetic field is of the form  $\langle H_x, H_y, 0 \rangle$ . The divergence

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$



**Fig. 1 Physical configuration and coordinate system.**

$$\frac{\partial u}{\partial v} = \frac{g}{\rho} T - T_0$$

$$\frac{\partial^2 u}{\partial v^2}$$

$$-\frac{\partial e_H}{\partial u}$$

$$\frac{\partial H_x}{\partial u}$$

$$(2) \quad \begin{array}{ccc} \frac{\partial t}{\partial x} & \frac{\partial x}{\partial y} \\ \frac{\partial^2 y}{\partial x^2} & 0 & k' \end{array}$$

**Magnetic induction equation**

$$\frac{\partial H_x}{\partial x} = \frac{\partial H_y}{\partial y} = \frac{\partial H_z}{\partial z}$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$1 - \frac{\partial^2 H_x}{\partial x^2}$$

$$\frac{x}{H_0} \frac{\partial u}{\partial x} - \frac{x}{H_0} \frac{\partial v}{\partial y} = \frac{x}{H_x} \frac{\partial H_x}{\partial x}$$

$$\frac{\partial H_0}{\partial x}$$

$$(3) \quad \begin{array}{ccc} \frac{\partial t}{\partial x} & \frac{\partial x}{\partial y} \\ \frac{\partial x}{\partial x} & \frac{\partial y}{\partial y} \\ \frac{\partial e}{\partial y} & 2 \end{array}$$

**Energy equation**

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial x} = \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial x^2}$$

$$1 - \frac{\partial^2 H_x}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0$$

$$\frac{x}{x} \frac{\partial x}{\partial x}$$

$$(4) \quad \frac{\partial t}{\partial x} = \frac{\partial x}{\partial y} = \frac{\partial c}{\partial y} \hat{y} = c \frac{\partial y}{\partial y}$$

p      p      p  
with the corresponding initial and boundary conditions are

$$t=0,$$

$$u=0,$$

$$v=0,$$

$$H_x=0,$$

$$T=T_0$$

everywhere      (5)

$$t=0,$$

$$u=0,$$

$$v=0,$$

$$H_x=0,$$

$$T=T_0$$

$$at x=0$$

$$u=0,$$

$$v=0,$$

$$H_x = H_w,$$

$$T = T_w$$

$$at y = 0$$

$$u = 0, \quad v = 0, \quad H_x = 0, \quad T = T_\infty \quad at y = 0 \quad (6)$$

equation  $\nabla \cdot \mathbf{H} = 0$  of Maxwell's equation for the magnetic field gives  $H_y = H_0$ .

where  $x, y$  are Cartesian coordinates system;  $u, v$  are  $x, y$  component of flow velocity respectively;  $g$  is the local acceleration due to gravity;  $\alpha$  is the thermal expansion coefficient;  $\beta$  is the kinematic viscosity;  $\mu_e$  is the magnetic permeability;  $\rho$  is the density of the fluid;  $H_0$  is the constant induced magnetic field;  $H_x$  be the  $x$ -component

Initially we consider that the plate as well as the fluid are at the same temperature  $T = T_\infty$  everywhere in the fluid is same.

Also it is assumed that the fluid and the plate is at rest after that the plate is to be moving with a constant velocity induced magnetic field;  $\kappa$  is the electrical conductivity;  $\lambda$  is the thermal conductivity;  $c_p$  is the specific heat at the constant pressure and  $H_w$  is the induced magnetic field at

$U_0$  in its own plane and instantaneously at time  $t > 0$ , the temperature of the plate is raised to  $T_w \neq T_\infty$ , which is thereafter maintained constant, where  $T_w$  is the temperature at the wall and  $T_\infty$  is the temperature of the species far away from the plate respectively.

The physical model of this study is furnished in the following figure.

Also the analysis is based on the following assumptions:

- All the physical properties of the fluid are considered to be constant except the influence of variations of density with the wall.

## 1 Mathematical Formulation

Since the solutions of the governing equations (1) to (4) under the initial (5) and boundary (6) conditions will be based on the finite difference method it is required to make the said equations dimensionless. For this purpose we now introduce the following dimensionless quantities; temperature are considered only on the body force term, in accordance with the Boussinesq's approximation.

ii. There is no chemical reaction taking place between the foreign mass and the fluid.

iii. The equation of conservation of electric charge,  $\nabla \cdot \mathbf{J} = 0$

$$X = \frac{x}{U_0}, Y = \frac{y}{U_0}$$

$$H_x = \frac{H_x}{U_0}$$

$$\frac{U}{U_0} = \frac{v}{U_0}, \quad \frac{V}{U_0} = \frac{u}{U_0}, \quad \frac{T}{T_\infty} = \frac{T - T_\infty}{T_w - T_\infty}$$

$$T = T_\infty + \frac{T_w - T_\infty}{U_0} X$$

$$\frac{v}{\sqrt{\frac{\mu_e}{\rho}}} = \frac{u}{U_0} = \frac{v}{U_0} = t U$$

—

gives  $J_y$

$\square$  constant where the current density

$H_x \square$ , And  $T \square$

$0$

$T_w \square T \square$

$U$

$\mathbf{J} \square (J_x, J_y, J_z)$ , because the direction of propagation is considered only along the  $y$ -axis and  $\mathbf{J}$  does not have any variation along the  $y$ -axis. Since the plate is electrically non-conducting, the constant is zero i.e.  $J_y \square 0$  at the plate and everywhere.

From the above dimensionless variable we have

$$u \square U_0 U, v \square U_0 V, H_x \square U_0$$

$$\square H_x \text{ and } e \quad \sqrt{\frac{U}{\mu}} -$$

$$\square \frac{U^2}{U^2}$$

$$T \square T_0 \square T_w \square \bar{T} \square T.$$

Also the associated initial (5) and boundary (6) conditions become  
Using these relations we have the following derivatives

$$\square 0,$$

$$U \square 0,$$

$$V \square 0,$$

$$\bar{H}_x \square 0,$$

$$\bar{T} \square 0$$

everywhere (11)

$$\square u \square U^3 \square U \quad \square u \square U^2 \square U \quad \square u \square U^2 \square U$$

$$\frac{\square}{\square} \frac{\square}{\square} \frac{\square}{\square} \frac{\square}{\square} \frac{\square}{\square}, \quad \frac{0}{0} \frac{\square}{\square},$$

$$\frac{0}{0} \frac{\square}{\square},$$

$$U \square 0,$$

$$U \square 0,$$

$$V \square 0,$$

$$H \square 0,$$

$$T \square 0$$

at  $X \square 0$

$$\square t \quad \square \square \square$$

$$\square x \quad \square \square X \quad \square y$$

$$\square \square Y \quad x$$

$$\frac{\square}{\square} \frac{\square}{\square} \frac{\square}{\square} \frac{\square}{\square} \frac{\square}{\square} \frac{\square}{\square}, \quad \frac{2}{2} \frac{\square}{\square}$$

$$\frac{U^3}{U^3} \frac{\square}{\square} \frac{\square}{\square} \frac{\square}{\square} \frac{\square}{\square} \frac{\square}{\square}$$

$$= \frac{\frac{\partial}{\partial x} v}{\frac{\partial^2}{\partial x^2} v} - \frac{U^2 \frac{\partial}{\partial x} v}{\frac{\partial^2}{\partial x^2} H + U^3}, \quad x=0$$

$$\frac{\partial}{\partial x} H_x = \sqrt{\frac{\rho}{\mu_e}}, \quad x=0, \\ U=0, \\ U_x=0, \\ V=0, \\ V_x=0, \\ H_x=1,$$

$$\frac{\partial}{\partial x} H_x = \sqrt{\frac{\rho}{\mu_e}}, \quad x=0, \\ T=1 at Y=0 \\ T=0 as Y \rightarrow \infty \quad (12) \quad - \\ \frac{\partial^2}{\partial x^2} Y = \frac{\partial^2}{\partial t^2} Y = 0, \quad x=0, \\ \frac{\partial^2}{\partial x^2} H + U^2 = 0, \quad x=0$$

$$\frac{\partial}{\partial x} H_x = \sqrt{\frac{\rho}{\mu_e}}, \quad x=0$$

$$\frac{\partial}{\partial x} H_x = \sqrt{\frac{\rho}{\mu_e}}, \quad x=0$$

$$\boxed{1} \quad \textbf{Numerical Solutions}$$

$$\frac{\partial}{\partial x} x = 0 \\ \frac{\partial^2}{\partial x^2} H = U^3 \\ \frac{\partial}{\partial x} X = \frac{\partial}{\partial y} Y = 0, \quad x=0 \\ \frac{\partial^2}{\partial x^2} H = \frac{\sqrt{\frac{\rho}{\mu_e}} T}{U^2} - \frac{U^2 \frac{\partial}{\partial x} T}{\frac{\partial^2}{\partial x^2} H} = 0, \quad x=0$$

The explicit finite difference method has been used to solve equations (7) to (10) subject to the conditions given by (11) and

$$x=0$$

$$\frac{x}{w}, \quad 0 \quad \square$$

$$, \quad \frac{y^2}{Y^2} \quad \frac{t^2}{T}$$

$$\bar{t} \quad \square \quad \square \quad \square$$

(12). To obtain the difference equations the region of the flow is divided into a grid or mesh of lines parallel to X and Y axes

$$\frac{\frac{U_0 T_w}{U_0 T_w} \frac{T_w - T}{T}}{\frac{U_0 T_w}{U_0 T_w} \frac{T_w - T}{T}},$$

where X-axis is taken along the plate and Y-axis is normal to

$$\frac{x}{x^2 T} \quad \frac{y}{T^2} \quad - \quad \frac{Y}{Y^2}$$

the plate.

Here we consider that the plate of height

$$X_{\max} = 100 \text{ i.e. } X$$

$$= \frac{0}{\frac{y^2}{Y^2}} \quad \frac{w}{\frac{t^2}{T}}$$

$$\frac{y^2}{Y^2} \quad \text{varies from 0 to 100 and regard } Y_{\max} = 25 \text{ as corresponding}$$

$$\frac{x}{X} \quad \frac{y}{Y} \quad \frac{t}{T} \quad \frac{Y^2}{Y^2} \quad \frac{Y}{Y}$$

$$\underline{\underline{H}} \quad \underline{\underline{H}} \quad \underline{\underline{H}} \quad \underline{\underline{U}} \quad \underline{\underline{U}} \quad \underline{\underline{H}}^2$$

$$\underline{\underline{X}} \quad \underline{\underline{U}} \quad \underline{\underline{V}} \quad \underline{\underline{H}_x} \quad \underline{\underline{M}}$$

$$\underline{\underline{x}}$$

(9)

$$\frac{x}{x} \quad \frac{X}{X} \quad \frac{Y}{Y}$$

$$\frac{X}{X} \quad \frac{Y}{Y} \quad P_m$$

$$\frac{Y^2}{Y^2}$$

$$\bar{T} \quad \bar{T}$$

$$\frac{T}{T} \quad \frac{1}{T} \quad \frac{E}{E} \quad \frac{H^2}{H^2}$$

$$\frac{U^2}{U^2} \quad - \quad - \quad -$$

$$\frac{U}{U} \quad \frac{V}{V}$$

$$\begin{array}{c}
 \square \quad \square c \quad \square x \\
 \square \quad \square \quad \square \\
 + \quad E_c \quad \square \\
 \hline
 (10) \\
 \square \quad \square X \\
 \square Y \quad P \square Y^2 \quad P \square \square Y \square \\
 \square \square Y \square \\
 r \quad m
 \end{array}$$

$$\square g \square T_w \square T \square$$

where \_\_\_\_\_

$$G_r \square \quad 3 \quad U$$

(GrashofNumber),

$$\begin{array}{c}
 H_0 \quad \square e \\
 M \quad \square \quad (\text{MagneticForceNumber}), \\
 U_0 \quad \square
 \end{array}$$

$$P_m \square \square \square e \quad (\text{MagneticdiffusivityNumber}),$$

$$\square \square c_p$$

shown in Fig.2.

It is assumed that  $\square X, \square Y$  are constant mesh sizes along  $X$

$$P_r \square \quad (\text{PrandtlNumber}),$$

$$U^2$$

and  $Y$  directions respectively and taken as follows,

$$\square X \square 0.8 \square 0 \quad x \square 100 \square$$

$$\frac{E_c}{c_p T_w} = \frac{0}{T_w - T_0}$$

(Eckert Number) and

$$Y = 0.2, \quad y = 25$$

with the small time-step,  $\Delta t = 0.005$

Let  $U_i, V_i, H_i$ ,

$H_i$  and  $T_i$  denote the values of  $U, V, H$

$$U_{i,j} = 1$$

$$\frac{U_{i,j}}{H_{x_{i,j}}} = \frac{-}{-} = \frac{-}{-}$$

$$= 2H$$

$$x_{i,j}$$

$H_{x_{i,j}} = 1$   
and  $T_i$  at the end of a time-step respectively. Using the explicit finite difference approximation we have,

$$\begin{aligned} Y &= P_m \\ Y^2 &= \\ (15) \end{aligned}$$

$$\frac{U_{i,j}}{U_{i,j}} = \frac{U_{i,j}}{U_{i,j}}$$

$$\frac{U_{i,j}}{U_{i,j}} = \frac{U_{i,j}}{U_{i,j}}$$

$$\begin{aligned} \bar{T}_{i,j} &= \frac{\bar{T}_{i,j} - \bar{T}_{i,j} + \bar{T}_{i,j+1} - \bar{T}_{i,j-1}}{4\Delta x} \\ \bar{T}_{i,j+1} &= \frac{\bar{T}_{i,j+1} - \bar{T}_{i,j} + \bar{T}_{i,j+2} - \bar{T}_{i,j+1}}{4\Delta x} \\ \frac{\bar{T}_{i,j+1} - \bar{T}_{i,j}}{\Delta x} &= \frac{\bar{T}_{i,j+2} - \bar{T}_{i,j+1}}{\Delta x} \\ \frac{\bar{T}_{i,j+1} - \bar{T}_{i,j}}{\Delta x} &= \frac{\bar{T}_{i,j+2} - \bar{T}_{i,j+1}}{\Delta x} \end{aligned}$$

$$\begin{aligned}
 & i,j \\
 & \frac{\partial Y}{\partial r} = P - \frac{\partial^2 Y}{\partial x^2} \\
 & \frac{\partial^2 U}{\partial x^2} = \frac{\rho c_p}{k} \frac{\partial T}{\partial x} + \frac{\rho g}{k} \frac{\partial H_x}{\partial x} \\
 & \frac{\partial^2 H_x}{\partial x^2} = \frac{\rho c_p}{k} \frac{\partial^2 T}{\partial x^2} + \frac{\rho g}{k} \frac{\partial^2 H_x}{\partial x^2} \\
 & \frac{\partial^2 V}{\partial x^2} = \frac{\rho c_p}{k} \frac{\partial^2 T}{\partial x^2} - \frac{\rho g}{k} \frac{\partial^2 H_x}{\partial x^2}
 \end{aligned}$$

$$\begin{aligned}
 & (16) \\
 & P_m = \frac{Y''(0)}{Y'(0)} = \frac{c}{|c|} \\
 & \frac{V_{i,j}}{H_x''(i,j)} = \frac{V_{i,j}}{H_x'(i,j)} = \frac{1}{1 - \frac{E}{E_c}}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{V_{i,j}}{H_x''(i,j)} = \frac{V_{i,j}}{H_x'(i,j)} = \frac{1}{1 - \frac{E}{E_c}} \\
 & \frac{\partial V_{i,j}}{\partial \tau} = \frac{\partial H_x}{\partial x}(i,j)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\partial^2 V_{i,j}}{\partial x^2} = \frac{\partial^2 H_x}{\partial x^2}(i,j) \\
 & \frac{\partial^2 H_x}{\partial x^2}(i,j) = \frac{\partial^2 V_{i,j}}{\partial x^2} - \frac{\partial^2 H_x}{\partial x^2}(i,j) \\
 & \text{and the initial and boundary conditions with the finite differences scheme are} \\
 & H_x(0) = H_x(L) = 0
 \end{aligned}$$

$$\begin{aligned}
 & (17) \\
 & \bar{U}^0 = 0, \quad \bar{V}^0 = 0, \quad \bar{H}^0 = 0, \quad \bar{T}^0 = 0 \\
 & \frac{x_{i,j}}{x_i} = \frac{x_{i,j}}{x_i} = 1
 \end{aligned}$$

i,j      i,j      xi,j      i,j

— —

$$\square \square X^{\square}$$

$$\square X^{\square}, \\ U^n=0, \quad V^n=0, \quad H^n=0, \\ T^0=0$$

$$\square \square i,j \\ 0,j \\ 0,j \\ 0,j \\ H \quad \underline{H}$$

$$— \quad \square U^n$$

$$\overline{=}0, \\ V^n=0,$$

$$\overline{H}^n=1,$$

$$\overline{T}^n=1 \\ \square \square H_x \square$$

$$\overline{\square x_{i,j}} \square 1 x_{i,j} \\ \square \square T \square$$

$$\overline{T_{i,j}} \square T_{i,j}$$

$$\overline{i,0} \\ \overline{i,0} \\ \overline{x,0}$$

$$\overline{i,0}$$

—

$$\square \square Y^{\square}$$

$$\square Y^{\square},$$

$$\square \square \square \square \square \square, \\ U^n=0, \quad V^n=0, \quad H^n=0, \quad T^n=0 \\ \square \square i,j \\ \square \square i,j \\ i,L \quad i,L \quad x,L \quad i,L$$

$$\overline{\square \square T \square}$$

$$\square \square X^{\square}$$

$$( \quad )_{i,j}$$

$$\overline{\square \square}$$

$$T_{i,j} \square T_{i,j}$$

$$(\partial^2 U)$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{Y^2}$$

$\square T$

$$, \frac{\partial^2 T}{\partial y^2}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{Y^2}$$

$$\text{where } L = . \quad (18)$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{Y^2}$$

are treated as constants. Then at the end of any time-step

$\square$ , the new temperature  $T$ , the new velocity  $U$ , the new induced magnetic field  $H$  and  $V$  at all interior nodal points may be obtained by successive applications of equations (16), (14), (15) and (13) respectively. This process is repeated in time and provided the time-step is sufficiently small,  $U, V$ ,

$\bar{H}_x$  and  $\bar{T}$  should eventually converge to

$$\frac{U_{i,j+1} - U_{i,j-1}}{2Y} = V \quad \frac{\partial V}{\partial x}$$

values which approximate the steady-state solution of

$$\frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{Y^2} = 0$$

$\square Y$

$$\frac{U_{i,j+1} - U_{i,j-1}}{2Y} = V$$

(13)

$$\frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{Y^2} = 0$$

$\square GT$

equations (7) to (10). These converged solutions are shown graphically in Figs. 3 to 38.

For the purpose of discussing the results of the problem, the approximate solutions are obtained for various parameters

Here the subscripts  $i$  and  $j$  designate the grid points with  $x$  and  $y$  coordinates respectively and the superscript  $n$

$$\frac{\partial^2 U_{i,j}}{\partial X^2} + \frac{\partial^2 U_{i,j}}{\partial Y^2} = 0$$

$$\frac{\bar{H}_{x,i,j}}{r_{i,j}} = \frac{1}{2} \left( \frac{\partial U_{i,j}}{\partial X} \right)_Y$$

with small values of Eckert number. In order to analyze the physical situation of the model, we have computed the steady state numerical values of the non-dimensional velocity  $U$ ,

$$\frac{\partial^2 U_{i,j}}{\partial Y^2} = M \frac{\partial U_{i,j}}{\partial Y}$$

(14)

induced magnetic field

$\bar{H}_x$  and temperature  $T$  within the

$$-\bar{H}_x \leq H_x \leq \bar{H}_x$$

$$\bar{H}_x = \frac{\partial T}{\partial Y}$$

$$U = U_0 \sin(\frac{\pi Y}{L})$$

boundary layer for different values of magnetic parameter

$$\frac{\partial^2 U_{i,j}}{\partial Y^2} = \frac{i-j}{Pr} \frac{\partial U_{i,j}}{\partial X} - \frac{G_r}{Pr} \frac{\partial^2 U_{i,j}}{\partial X^2}$$

$M$ , magnetic diffusivity  $Pr$ , Grashof number  $G_r$ ,

$$\frac{\partial U_{i,j}}{\partial X} = X_{i,j} \frac{\partial U}{\partial X}$$

$$X_{i,j} = \frac{1}{L} i \cos(\frac{\pi Y}{L})$$

Prandtl number  $Pr$  and Eckert number  $E_c$ .

To get the steady state solutions, the computations have been carried out up to  $80$

. We observed that the results of the computations, however, show no visible changes after

$40$ . Thus the solution for  $40$  are essentially steady state solutions.

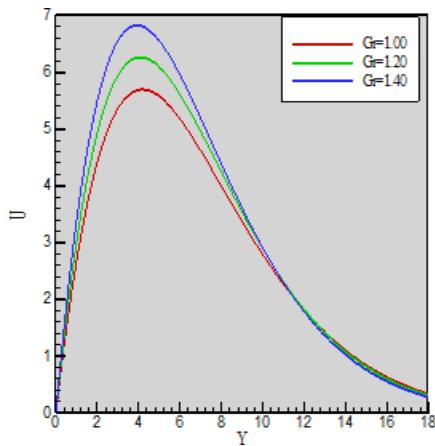
Because of the special importance of cooling problems in nuclear engineering in connection with the cooling of reactors, the value of the Grashof number for heat transfer is

taken to be positive  $G_r > 0$  and the present study has

considered  $G_r = 1.0, 1.2$  and  $1.4$ . Since the most important fluids are atmospheric air, saltwater and water so the

results are limited to  $Pr = 0.71$  (Prandtl number for air at

$20^{\circ}\text{C}$ ,  $P = 1.0$  (Prandtl number for saltwater at  $20^{\circ}\text{C}$ )



**Fig.4** Velocity profiles for different values of  $G_r$

r

and  $P = 7.0$  (Prandtl number for water at  $20^{\circ}\text{C}$ ). However  
where  $M = 0.10$ ,  $P_m = 1.00$ ,  $P_r = 0.71$  and

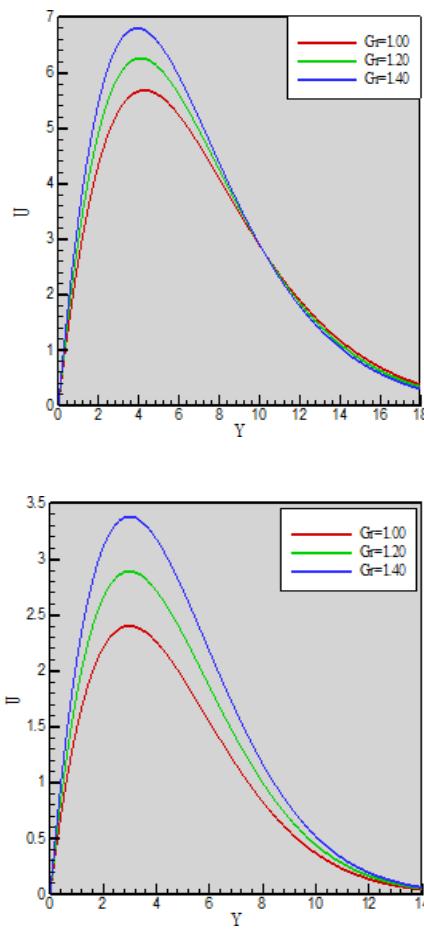
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the values of other parameters  $M$ ,  $P_m$  and  $E_c$  are  
 $E_c = 0.01$  at time  $t = 30$ .

chosen arbitrarily as  $M=0.10, 0.20$  and  $0.30$ ;  
and  $2.0$ ;  $E_c=0.01, 0.02$  and  $0.03$ .

$P_m=1.0, 1.5$

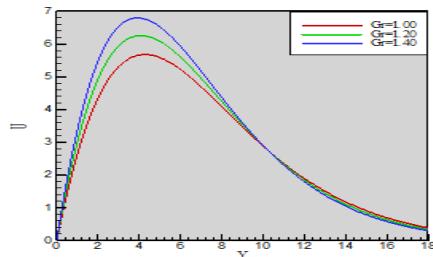
Along with the obtained steady state solutions, the flow behaviors in case of cooling problem are discussed graphically. The profiles of the transient velocity, induced magnetic field and temperature versus Y are illustrated in Figs.3 to 38.



**Fig.5** Velocity profiles for different values of  $Gr$

where  
 $M = 0.10$ ,  
 $P_m = 1.00$ ,  
 $P_r = 0.71$   
and

$E_c = 0.01$  at time  $t = 40$ .



**Fig.3** Velocity profiles for different values of  $Gr$

where

$$M = 0.10,$$

$$\alpha = 10.$$

$$P_m = 1.00,$$

$$P_r = 0.71$$

and

$$E_c = 0.01$$

at time

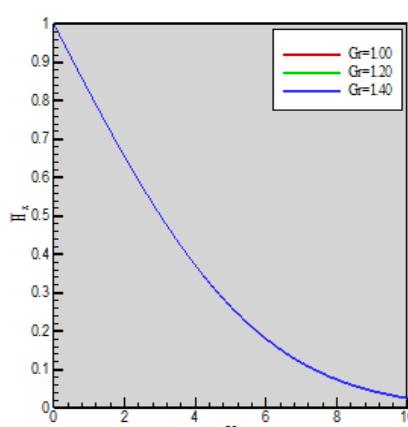
**Fig. 6** Velocity profiles for different values of  $G_f$

where  $M = 0.10, P_m = 1.00, P_r = 0.71$

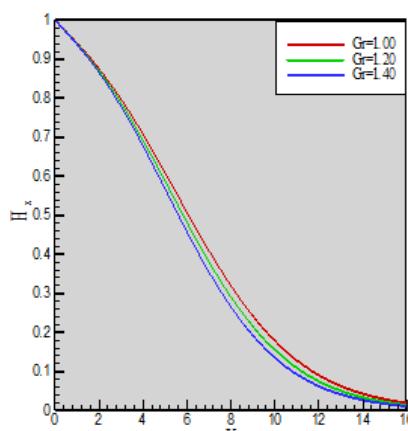
and

where  $M = 0.10, P_m = 1.00, P_r = 0.71$  and  $E_c = 0.01$

$E_c = 0.01$  at time  $t = 50$ .



**Fig. 7** Induced magnetic fields for different values of  $G_f$  where  $M = 0.10, P_m = 1.00, P_r = 0.71$  and  $E_c = 0.01$  at time  $t = 10$ . attime  $t = 40$ .



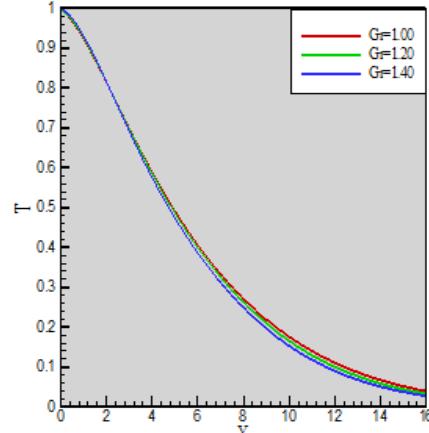
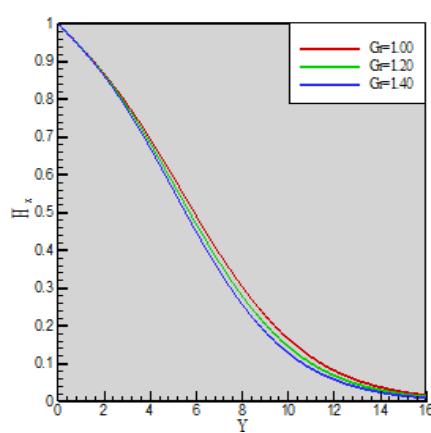
**Fig. 10** Induced magnetic fields for different values of  $G_f$  where  $M = 0.10, P_m = 1.00, P_r = 0.71$  and  $E_c = 0.01$  at time  $t = 50$ .

**Fig. 8** Induced magnetic fields for different values of  $G_r$  where  $M = 0.10, P_m = 1.00, P_r = 0.71$  and  $E_c = 0.01$  at time  $t = 30$ .

**Fig.11** Temperature profiles for different values of  $G_r$  where

$M = 0.10, P_m = 1.00, P_r = 0.71$  and  $E_c = 0.01$  at time

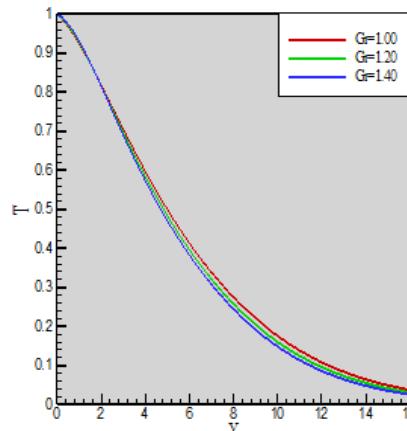
$t = 10$ .



**Fig.12** Temperature profiles for different values of  $G_r$  where

$M = 0.10, P_m = 1.00, P_r = 0.71$  and  $E_c = 0.01$  at time

$t = 30$ .



**Fig.13** Temperature profiles for different values of  $G_r$  where

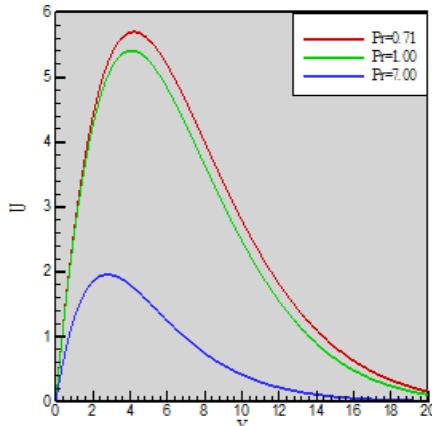
$M = 0.10, P_m = 1.00, P_r = 0.71$  and  $E_c = 0.01$  at time

$t = 40$ .

**Fig.15** Velocity profiles for different values of  $P_r$  where

$G_r = 1.00, M = 0.10, P_m = 1.00$  and  $E_c = 0.01$

at time  $t = 10$ .



**Fig.16** Velocity profiles for different values of  $P_r$  where

$G_r = 1.00, M = 0.10, P_m = 1.00$  and  $E_c = 0.01$

at time  $t = 30$ .

**Fig.17** Velocity profiles for different values of  $P_r$  where

**Fig.14** Temperature profiles for different values of  $G_r$  where

$G = 1.00, M = 0.10, P = 1.00$  and  $E = 0.01$  at time

$M = 0.10, P = 1.00$

$P = 0.71$  and  $E = 0.01$

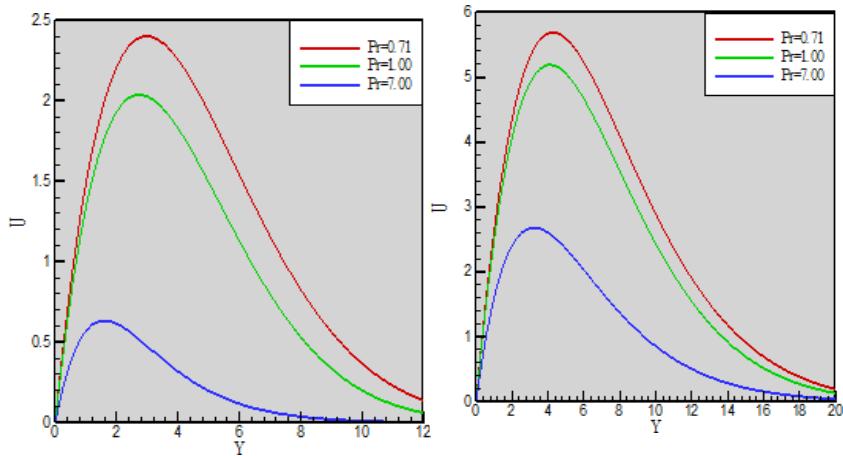
,  $m = c$

at time  $t =$

$m = , r = c$

$t = 50$ .

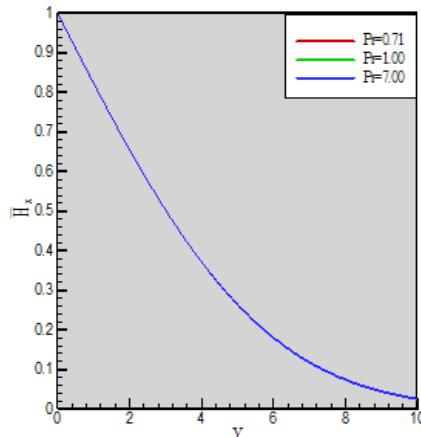
$t = 40$ .



**Fig.18** Velocity profiles for different values of  $P_r$  where

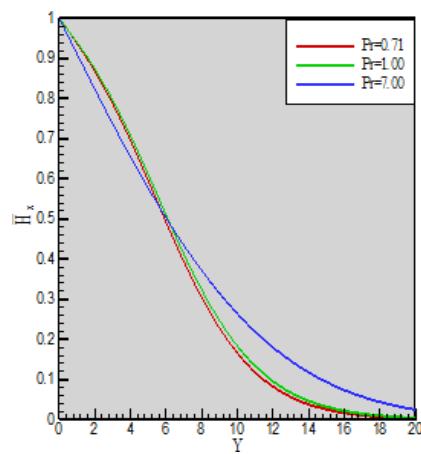
$G_r = 1.00, M = 0.10, P_m = 1.00$  and  $E_c = 0.01$  at time

$\square \square 50$ .



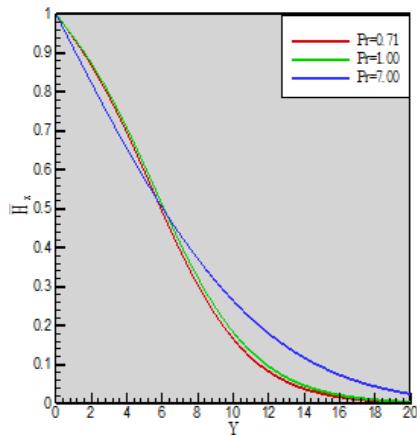
**Fig.19** Induced magnetic fields for different values of  $P_r$  where  $G_r = 1.00, M = 0.10, P_m = 1.00$  and  $E_c = 0.01$  at time  $\square \square 10$ .

**Fig.21** Induced magnetic fields for different values of  $P_r$  where  $G_r = 1.00, M = 0.10, P_m = 1.00$  and  $E_c = 0.0$  at time  $\square \square 40$ .

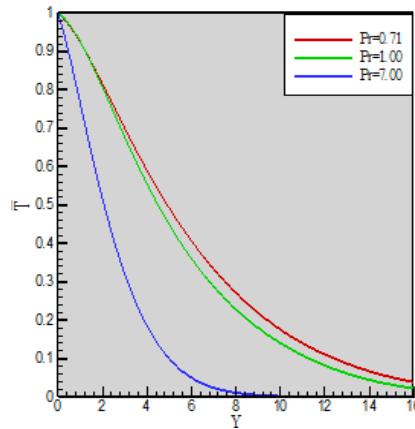


**Fig. 22** Induced magnetic fields for different values of  $P_r$  where  $G_r = 1.00, M = 0.10, P_m = 1.00$  and  $E_c = 0.01$  at time  $\square \square 50$ .

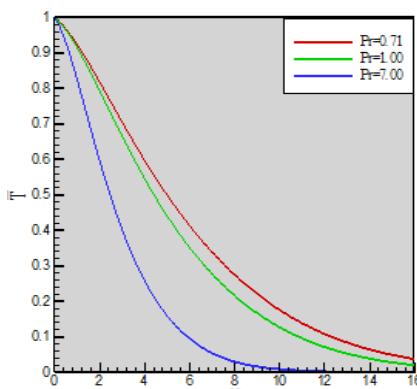
**Fig. 20** Induced magnetic fields for different values of  $P_f$  where  $G_r = 1.00, M = 0.1, P_m = 1.00$  and  $E_c = 0.01$  at time  $t = 30$ .



**Fig. 23** Temperature profiles for different values of  $P_f$  where  $G_r = 1.00, M = 0.1, P_m = 1.00$  and  $E_c = 0.01$  at time  $t = 10$ .



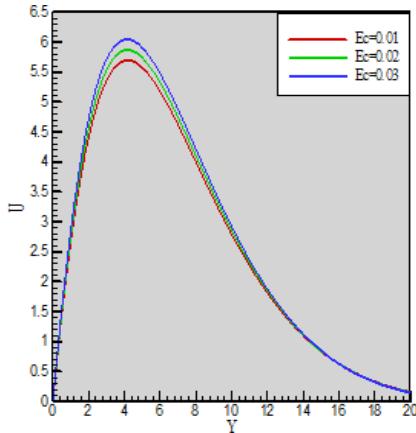
**Fig. 24** Temperature profiles for different values of  $P_f$  where  $G_r = 1.00, M = 0.1, P_m = 1.00$  and  $E_c = 0.01$  at time  $t = 30$ .



**Fig. 25** Temperature profiles for different values of  $P_f$  where

**Fig.27** Velocity profiles for different values of  $E_c$  where

$G_r = 1.00, M = 0.10, P_m = 1.00$  and  $P_f = 0.71$  attime  
 $t = 10$ .



**Fig.28** Velocity profiles for different values of  $E_c$  where

$G_r = 1.00, M = 0.10, P_m = 1.00$  and  $E_c = 0.01$  attime

$G = 1.00, M = 0.10, P = 1.00$  and  $P_f = 0.71$  attime

$t = 40$ .

r

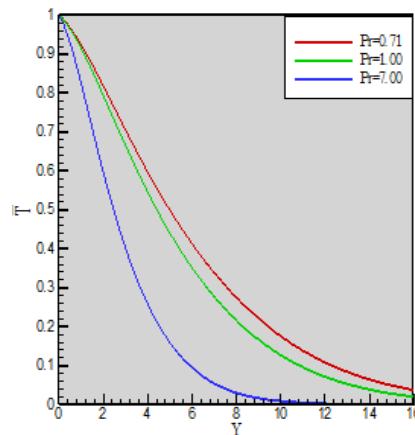
$\square \square 30$ .

, m r

**Fig.26** Temperature profiles for different values of  $P_f$  where

$G_r = 1.00, M = 0.10, P_m = 1.00$  and  $E_c = 0.01$  attime

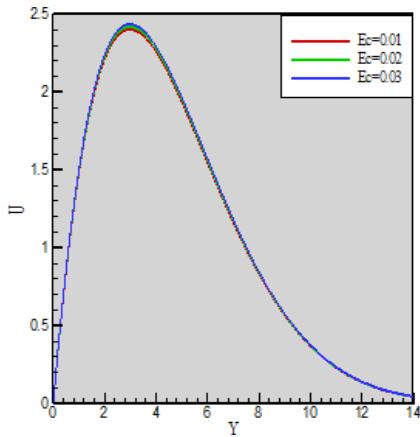
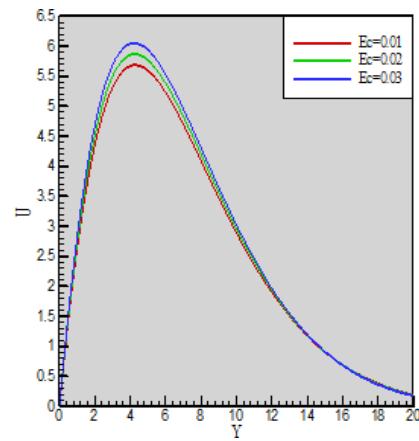
$t = 50$ .



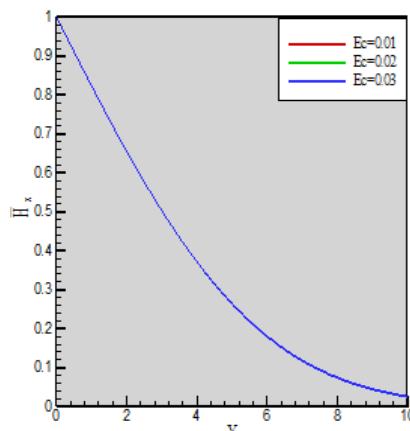
**Fig.29** Velocity profiles for different values of  $E_c$  where

$G_r = 1.00, M = 0.10, P_m = 1.00$  and  $P_f = 0.71$  attime

$t = 40$ .

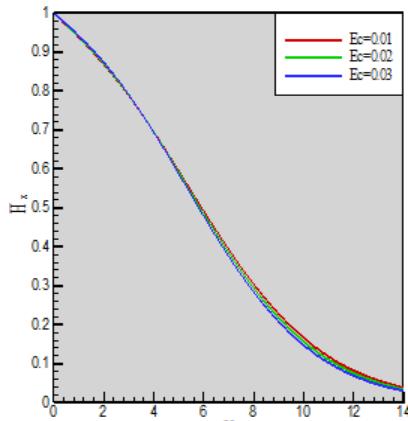


**Fig.30** Velocity profiles for different values of  $E_c$  where  
 $G_r = 1.00, M = 0.10, P_m = 1.00$  and  $P_f = 0.71$  at time  
 50.



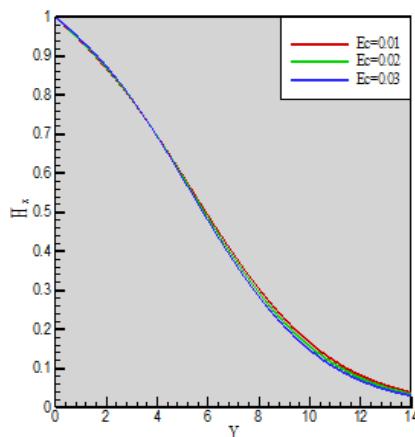
**Fig.31** Induced magnetic fields for different values of  $E_c$  where  $G_r = 1.00, M = 0.10, P_m = 1.00$  and  $P_f = 0.71$  at time 10.

**Fig.33** Induced magnetic fields for different values of  $E_c$  where  $G_r = 1.00$ ,  $M = 0.1$ ,  $P_m = 1.00$  and  $P_f = 0.71$  at time  $t = 40$ .

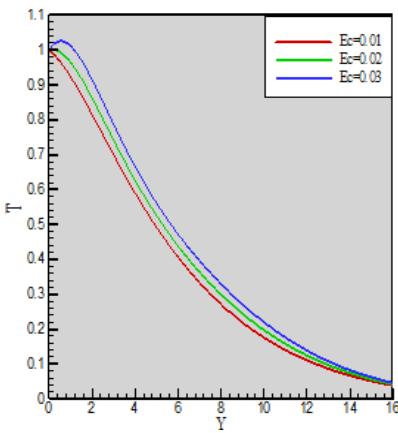


**Fig. 34** Induced magnetic fields for different values of  $E_c$  where  $G_r = 1.00$ ,  $M = 0.1$ ,  $P_m = 1.00$  and  $P_f = 0.71$  at time  $t = 50$ .

**Fig.32** Induced magnetic fields for different values of  $E_c$  where  $G_r = 1.00$ ,  $M = 0.1$ ,  $P_m = 1.00$  and  $P_f = 0.71$  at time  $t = 30$ .



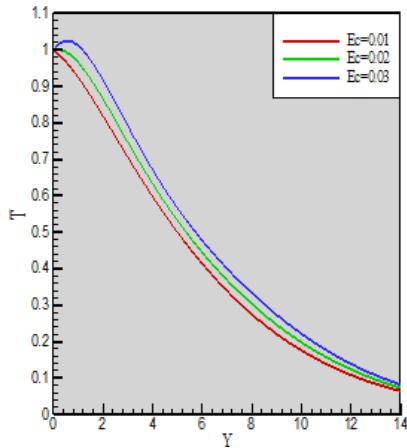
**Fig.35** Temperature profiles for different values of  $E_c$  where  $G_r = 1.00$ ,  $M = 0.1$ ,  $P_m = 1.00$  and  $P_f = 0.71$  at time  $t = 10$ .



**Fig.36** Temperature profiles for different values of  $E_c$  where

value of  $G_r$  increases. At  $\tau=50$  the induced magnetic field  $G_r \square 1.00, M \square 0.10, P_m \square 1.00$  and  $P_f \square 0.71$  at time  $\square 30$ .

profile shows identical graph as  $\tau = 40$ . Also as the value of  $\tau$  increases the velocity profiles also increase up to  $\tau=40$ . But at  $\tau=50$  the velocity profiles show same graph as  $\tau=40$ . The temperature profile (**Figs.11 to 14**) remains same as the value of  $G_r$



increases at  $\tau=10$ . But for  $\tau=30, 40$  and  $50$  the temperature profile decreases as the value of  $G_r$

increases.

**Fig.37** Temperature profiles for different values of  $E_c$  where

$G_r = 1.00, M = 0.10, P_m = 1.00$  and  $P_f = 0.71$  at time

$\tau = 40$ .

At  $\tau = 50$  the temperature profile shows identical graph as  $\tau = 40$ .

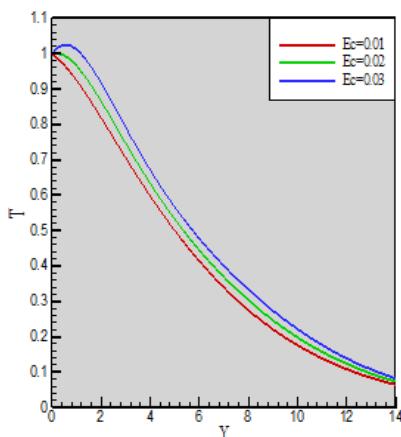
Also as the value of  $\tau$  increases the temperature profiles increase upto  $\tau = 40$ . But at  $\tau = 50$  the temperature profile shows same graph as  $\tau = 40$ .

As the value of  $Pr$  increases the velocity profile (**Figs. 15 to 18**) decreases at  $\tau = 10, 30$  and  $40$ . But at  $\tau = 50$  the velocity profile shows same graph as  $\tau = 40$ . On the other hand, as the value of  $f$  increases the velocity profile also increases upto  $\tau = 40$ . But at  $\tau = 50$  the velocity profile shows same graph as  $\tau = 40$ .

The induced magnetic field profile (**Figs. 19 to 22**) remains same as the value of  $Pr$  increases at  $\tau = 10$ . But for  $\tau = 30, 40$  and  $50$  the induced magnetic field profile initially decreases and after certain amount of  $Y$  it increases as the value of  $Pr$  increases. At  $\tau = 50$  the induced magnetic field profile shows identical graph as  $\tau = 40$ . Also as the value of  $\tau$  increases the induced magnetic field profile also increases upto  $\tau = 40$ . But at  $\tau = 50$  the induced magnetic field profile shows same graph as  $\tau = 40$ .

As the value of  $Pr$  increases the temperature profile (**Figs. 23 to 26**) decreases at  $\tau = 10, 30$  and  $40$ . But at  $\tau = 50$  the temperature profile shows same graph as  $\tau = 40$ . On the other hand, as the value of  $f$  increases the temperature profile also increases upto  $\tau = 40$ . But at  $\tau = 50$  the temperature profile shows same graph as  $\tau = 40$ .

**Fig.38** Temperature profiles for different values of  $E_c$



where

As the value of  $E_c$

increases the velocity profile (**Figs. 27 to**

$G_r = 1.00, M = 0.10, P_m = 1.00$  and  $P_f = 0.71$  at time

$\tau = 50$ .

For avoiding the complexity here we show the variations for  $\tau = 10, 30, 40$  and  $50$ . For the change of  $G_r$ , the velocity

**30**) also increases at  $\tau = 10, 30$  and  $40$ . But at  $\tau = 50$  the velocity profile shows same graph as  $\tau = 40$ . One the other hand, as the value of  $f$  increases the velocity profile also increases upto  $\tau = 40$ . But at  $\tau = 50$  the velocity profile shows same graph as  $\tau = 40$ .

profile is shown in (**Figs. 3 to 6**). From these Fig. we see The induced magnetic field profile (**Figs. 31 to 34**) remains

that, as the value of  $G_r$

increases the velocity profile also

same as the value of  $E_c$

increases at  $\tau=10$ . But for  $\tau=30, 40$

increases at  $\tau=10, 30$  and  $40$ . But at  $\tau=50$  the velocity profile and 50 the induced magnetic field profile decreases as the

shows same graph at  $\tau=40$ . On the other hand, as the value

value of  $E_c$

increases. At  $\tau=50$  the induced magnetic field

of  $t$  increases the velocity profile also increase up to  $\tau=40$ .

But at  $\tau=50$  the velocity profile shows same graph at  $\tau=40$ . The induced magnetic field profile (**Figs. 7 to 10**) remains same as the value of  $G_r$  increases at  $\tau=10$ . But for  $\tau=30, 40$

profile shows identical graph as  $\tau=40$ . Also as the value of  $t$  increases the velocity profile also increase up to  $\tau=40$ . But at  $\tau=50$  the velocity profile shows same graph at  $\tau=40$ .

As the value of  $E_c$  increases the temperature profile (**Figs. 35 to 38**) and 50 the induced magnetic field profile decreases as the

increases at  $\tau=10, 30$  and  $40$ . But at  $\tau=50$  the

temperature profile shows same graph at  $\tau=40$ . On the other hand, as the value of  $t$  increases the temperature profile also increases up to  $\tau=40$ . But at  $\tau=50$  the temperature profile shows same graph at  $\tau=40$ .

## CONCLUSIONS

A transient heat transfer flow through a electrically conducting incompressible viscous fluid past an electrically nonconducting continuously moving semi-infinite vertical plate under the action of strong magnetic field taking into account the induced magnetic field constant heating is investigated in this work. The resulting governing system of dimensionless coupled nonlinear partial differential equations are numerically solved by an explicit finite difference method. The results are discussed for different values of important parameters as magnetic parameter, magnetic diffusivity numbers, Grashof number, Prandtl number and Eckert number. Some of the important findings obtained from the graphical representation of the results are listed herewith;

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- [8] Gribben RJ, "The hydromagnetic boundary layer in the presence of pressure gradient", Proceedings of Royal Society London, v. 287, p. 123-141, 1965.

1. The velocity increases with the increase of  $E_c$  or  $G_r$  while it decreases with the increase of  $P_r$ .

2. The magnetic induction decreases with the increase

of  $G_r$ ,  $E_c$ . Moreover, the magnetic induction

initially decreases and later increases with the

increase of  $P_r$ . Particularly, the induced magnetic

- [9] Lin and Wu, "Combined heat and mass transfer by laminar natural convection from a vertical plate with uniform heat flux and concentration", Int. J. Heat Mass Transfer, v. 32, p. 293-299, 1997.

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field is greater for heavier than lighter particles.

3. The temperature increases with the increase of  $E_c$  while it decreases with the increase of  $G_r$  or  $P_r$ . Particularly, the fluid temperature is more for air than water and it is less for lighter than heavier particles.

As the basis for many scientific and engineering applications for studying more complex vertical problems involving the flow of electrically conducting fluids, it is hoped that the present investigation of the study of applied physics of flow over a vertical surface can be utilized. In the migration of underground water or oil as well as in the filtration and water purification processes, the findings may be useful for study of movement of oil or gas and water through the reservoir of an oil or gas field. The results of the problem are also of great interest in geophysics and astrophysics in the study of interaction of the geomagnetic field with the fluid in geothermal regions.

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