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Efficiencies of Nearest Neighbour Balanced Block Designs using first order Correlated Models for three Treatments

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ABSTRACT
Efficiencies of Nearest Neighbour Balanced Block Design using Auto-regressive Moving Average models have investigated by Santharam and Ponnuswamy (1995). Nearest Neighbour Balanced Block Designs (NNBD) are widely used in biological and industrial experiment. Ruban Raja and Santharam (2012,2013) investigated MV- Optimality of Nearest Neighbour Balanced Block Designs using first order and second order correlated models for three and five treatments. In this paper we have investigated the efficiencies based on average variance, generalized variance and Mini- max variance of Nearest Neighbour Balanced Block Design using first order correlated models for three treatments.

Keywords: Nearest Neighbour Balanced Block Designs, Average variance, Generalized variance, and Mini- max variance.

I. INTRODUCTION

Universal optimality of NNBD using ARMA model has been introduced by Ponnuswamy and Santharam (1995). Efficiencies of NNBD using ARMA models have also been studied by Santharam and Ponnuswamy (1995). Uddin, N. (2008) constructed MV – optimality of block design for 3 treatments in $b = 3n + 1$ blocks of each size 3 and under the assumption that the blocks behave independently but there is a correlation among the observations with the same block according to AR (1) model. Optimal block design for three treatments when observations are correlated was introduced by Uddin, N (2008a) and MV – optimal block designs for correlated errors were constructed by Uddin, N (2008b). So we have considered efficiency of NNBD using first order and second order correlated models for three treatments when $\rho = 0.1, 0.2, \dots, 0.9$. In this paper we have compared the efficiencies of NNBD over regular block design using the average variance, generalized variance and Mini-max variance for NNBD when the error term ε given in the NNBD model follows AR(1),MA(1) and ARMA(1,1) models.

II. MODEL

A block design d is defined here as an allocation of v treatments to bk experimental units which are arranged into b blocks each having k units.
We assume the following model

$$Y_d = 1_{3b}\mu + Z\beta + X_d\tau + \epsilon \text{ with } cov(\epsilon) = \sigma^2 \Sigma \quad , \qquad (2.1)$$

where, Y_d = block order $3b \times 1$ column vector of observed response obtained from a design d,
1_{3b} = $3b \times 1$ column vector of ones
τ = 3×1 vector of treatment effect
X_d = $3b \times 3$ plot-treatment design matrix
β = vector of fixed block effects
$Z = I_b \otimes 1_3$ plot-block incident matrix.

III. NEAREST NEIGHBOUR BALANCED BLOCK DESIGN WHEN ERROR STRUCTURE FOLLOWS FIRST ORDER CORRELATED MODELS

First order correlated models are considered in this section for the error structure ε given in the NNBD model (2.1) follows the first-order autoregressive, moving average model and autoregressive and moving average model.

If the errors within a block follow an first order autoregressive model (AR(1)) with the parameter ρ (where ρ is the correlation between the observations in the adjacent plots) then $\Sigma = (1-\rho^2)^{-1} I_b \otimes H_k$ where I_b is a b X b identity matrix and H_k is a k x k matrix of the form

$$H_k = \begin{bmatrix} 1 & \rho & \rho^2 & \dots & \rho^{k-1} \\ \rho & 1 & \rho & \dots & \rho^{k-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho^{k-1} & \rho^{k-2} & \rho^{k-3} & \dots & 1 \end{bmatrix}.$$

If the errors within a block follow first order moving average model MA(1) then $\Sigma = I_b \otimes N_k$, where I_b is a b x b identity matrix and the k x k matrix,

$$N_k = \begin{bmatrix} 1+\rho^2 & \rho & \dots & 0 \\ \rho & 1+\rho^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1+\rho^2 \end{bmatrix}.$$

If the errors within a block follow an ARMA model ARMA (1,1) then $\Sigma = I_b \otimes J_k$, where I_b is a b x b identity matrix and the k x k matrix,

$$J_k = \begin{bmatrix} r_0 & r_1 & r_2 & \dots & r_{k-1} \\ r_1 & r_0 & r_1 & \dots & r_{k-2} \\ r_2 & r_1 & r_0 & \dots & r_{k-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{k-1} & r_{k-2} & r_{k-3} & \dots & r_0 \end{bmatrix},$$

where $r_0 = \frac{1+2\rho_1\rho_2+\rho_2^2}{1-\rho_1^2}$, $r_1 = \frac{\rho_1(1+\rho_2^2)+\rho_2(1+\rho_1^2)}{1-\rho_1^2}$, $r_k = \rho_1 r_{k-1}$, for $k \geq 2$.

IV. INFORMATION MATRIX

Experimental designs are evaluated using statistical criteria. It is known that the least squares estimator minimizes the variance of mean – unbiased estimators (under the condition of the Gauss – Markov theorem). In the estimation theory for statistical models with one real parameter, the reciprocal of the variance of an ("efficient") estimator is called the "Fisher Information" for that estimator. Because of this reciprocity, minimizing the variance corresponds to maximizing information. When the statistical model has several parameters, however, the mean of the parameter – estimator is a vector and its variance is a matrix. The inverse matrix of the variance- matrix is called the information matrix.

The information matrix of $\hat{\tau}$ is the inverse of the dispersion matrix and it is given by

$$C = X'V^{-1}X - (X'V^{-1}Z)(Z'V^{-1}Z)^{-1}(Z'V^{-1}X) \ .$$

The matrix C (Gill and Shukla, 1985) is called the information matrix of a design for treatment parameters. To emphasize the dependence of information matrix on design, we write it as C_d for $d \in \Delta$.The above matrix is utilized by several authors (Martin and Eccleston, 1991; Jin and Morgan, 2008; Gill and Shukla, 1985; Kunert, 1987; Santharam and Ponnnuswamy, 1995, 1996, 1997; Uddin, 2008a, 2008b) in their investigation of various optimal and highly efficient designs.

V. COMPARISON OF EFFICIENCY OF NNBD OVER RBD

In this Section, we have investigated the behavior of some estimator of ρ, using nearest neighbor balanced block design and Regular block design with the following true parameters:

$\rho = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ and $\sigma_\varepsilon{}^2 = 1$.

Also consider $t = 5$ and $b = 7$;

The parameter $\sigma_\varepsilon{}^2$ was estimated based on the fixed effect method of estimation of ρ.

The estimates of $\sigma_\varepsilon{}^2$ based on NNBD and RBD were compared using the following three measures.

5.1 Average Variance Comparison

Consider the measure

$$R_A = \frac{\sigma_\varepsilon{}^2{}_{(RBD)} \; \sum_{i=1}^{t-1} \gamma_{RBD}^{-1}(i)}{\sigma_\varepsilon{}^2{}_{(NNBD)} \; \sum_{i=1}^{t-1} \gamma_{NNBD}^{-1}(i)},$$

where ${\sigma_\varepsilon}^2{}_{(RBD)}$ denotes the estimate of ${\sigma_\varepsilon}^2$ based on RBD and ${\sigma_\varepsilon}^2{}_{(NNBD)}$ denotes the estimate of ${\sigma_\varepsilon}^2$ based on NNBD and $\gamma_{d(i)}{}'s$ are nonzero eigen values of the information matrix. The above measure R_A compares the average variance of elementary treatment contrast when analysed by RBD and NNBD. The ratio ${\sigma_\varepsilon}^2{}_{(RBD)} / {\sigma_\varepsilon}^2{}_{(NNBD)}$ could mask the genuine efficiency of NNBD. Therefore the ratio

$$R_H = \frac{\sum_{i=1}^{t-1} \gamma_{RBD}^{-1}(i)}{\sum_{i=1}^{t-1} \gamma_{NNBD}^{-1}(i)}$$

of harmonic means will also be considered as an index of efficiency.

5.2 Generalized Variance Comparison

Another way to compare RBD and NNBD is the ratio

$$R_G = \left[\frac{{\sigma_\varepsilon}^2{}_{(RBD)}}{{\sigma_\varepsilon}^2{}_{(NNBD)}}\right]^{t-1} \prod_{i=1}^{t-1} \gamma_{NNBD}(i)\ \gamma_{RBD}^{-1}(i)$$

of generalised variances of t-1 orthogonal treatment contrasts estimated under RBD and NNBD.
It may be noted that R_G is very sensitive to the ratio ${\sigma_\varepsilon}^2{}_{(RBD)} / {\sigma_\varepsilon}^2{}_{(NNBD)}$.
We therefore, consider the ratio

$$R_D = \prod_{i=1}^{t-1} \gamma_{NNBD}(i)\ \gamma_{RBD}^{-1}(i)$$

which gives a better comparison of RBD over NNBD.

5.3. Mini – Max Variance Comparison

A property closely related to the E-optimality of a design is the closeness of variances of treatment contrast. This closeness is measured by the ratio of the smallest nonzero Eigen value to the largest Eigen value of the information matrix. Note that this ratio is independent of ${\sigma_\varepsilon}^2$.
For comparing NNBD and RBD, we taken the ratio

$$R_E = \frac{\gamma_{NNBD}(1)}{\gamma_{NNBD}(t-1)} \times \frac{\gamma_{RBD}(t-1)}{\gamma_{RBD}(1)}$$

VI. EFFICIENCY OF NNBD WHEN THE ERROR STRUCTURE FOLLOWS FIRST ORDER CORRELATED MODELS

Table 6.1

AR (1) Model for $b = 3n+1, n = 2;\ (v = 3, b = 7, k = 3)$

ρ_1	R_A	R_H	R_G	R_D	R_E
0.1	0.93125	1.00409	0.90296	1.00409	1.11714
0.2	0.87186	1.01554	0.83456	1.01554	1.24060
0.3	0.82362	1.03349	0.78825	1.03349	1.37130
0.4	0.78470	1.05741	0.75967	1.05741	1.51031
0.5	0.75359	1.08707	0.74590	1.08707	1.65882
0.6	0.72905	1.12244	0.74505	1.12244	1.81818
0.7	0.71006	1.16368	0.75597	1.16368	1.98996
0.8	0.69574	1.21109	0.77809	1.21109	2.17601
0.9	0.68539	1.26515	0.81131	1.26515	2.37849

Table 6.2

MA (1) model for $b = 3n+1, n = 2;\ (v = 3, b = 7, k = 3)$

ρ_1	R_A	R_H	R_G	R_D	R_E
0.1	0.94840	1.004917	0.89505	1.00491	1.12906
0.2	0.91227	1.021653	0.81459	1.02165	1.28948
0.3	0.89666	1.052099	0.76419	1.05209	1.48143
0.4	0.90323	1.095827	0.74449	1.09582	1.69964
0.5	0.92908	1.149271	0.75107	1.14927	1.93121
0.6	0.96685	1.205797	0.77524	1.20579	2.15569
0.7	1.00664	1.257204	0.80601	1.25720	2.34922
0.8	1.03930	1.296341	0.83323	1.29634	2.49196
0.9	1.05929	1.319403	0.85046	1.3194	2.57463

Table 6.3
ARMA (1) model for $b = 3n + 1, n = 2;\ (v = 3, b = 7, k = 3)$

ρ_1	ρ_2	R_A	R_H	R_G	R_D	R_E
0.1	0.1	0.91639	1.01835	0.82464	1.01835	0.79125
0.2	0.2	0.89747	1.07388	0.75005	1.07388	0.62696
0.3	0.3	0.93181	1.15383	0.75252	1.15383	0.51284
0.4	0.4	0.97765	1.22031	0.78325	1.22031	0.45225
0.5	0.5	0.98379	1.22834	0.78793	1.22834	0.44612
0.6	0.6	0.94424	1.17341	0.75983	1.17341	0.49286
0.7	0.7	0.90342	1.09636	0.74443	1.09636	0.58752
0.8	0.8	0.90100	1.03654	0.78319	1.03654	0.71914
0.9	0.9	0.94117	1.00680	0.87981	1.00680	0.86697

VII. CONCLUSIONS

We have compared NNBD over regular block design with reference to the following efficiencies, namely,

i) Average variance (R_A , R_H)
ii) Generalized variance (R_G, R_D)
iii) Mini – Max variance (R_E)

.

The following parameters $t = 3$ and $b = 7$., $\rho = 0.1(0.1)\ 0.9$.
The parameter σ^2 has been estimated based on the fixed ρ values. Average variance (R_A , R_H), Generalized variance (R_G, R_D) and Min – Max variance (R_E) have been computed for NNBD when the error term ε given in NNBD model follows first order correlated models.
The Table 6.1 reveals the efficiencies of AR(1) models with t =3, b=7, and ρ = 0.1(0.1) 0.9, there is considerable advantage in using NNBD as far as average variance (R_A and R_G), generalized variance (R_H and R_D) and Min-max variance (R_E) are concerned. The R_H, R_D and R_E show increasing efficiency values. The R_A shows decreasing efficiency values and R_G values are increasing when ρ taking values 0.1 to 0.6 and its increasing when ρ taking values from 0.7 to 0.9.
The Table 6.2 shows the efficiencies of MA(1) models with t =3, b=7, and ρ = 0.1(0.1) 0.9, there is considerable advantage in using NNBD as far as average variance (R_A and R_G), generalized variance (R_H and R_D) and Min-max variance (R_E) are concerned. The R_H, R_D and R_E show increasing efficiency values. The R_A shows decreasing efficiency values when ρ taking values from 0.1 to 0.3 and its increasing when ρ taking values 0.4 to 0.9.The R_G shows decreasing efficiency values when ρ taking values 0.1 to 0.4 and its increasing when ρ taking values from 0.4 to 0.9.
The Table 6.3 shows the efficiencies of ARMA(1,1) models with t =3, b=7, and ρ = 0.1(0.1) 0.9, there is considerable advantage in using NNBD as far as average variance (R_A and R_G), generalized variance (R_H and R_D) and Min-max variance (R_E) are concerned. The R_H , R_D shows increasing efficiency values. R_A and R_G show unstable efficiency values. The R_G shows decreasing efficiency values and R_E values are decreasing when ρ taking values from 0.1 to 0.5 and its increasing when ρ taking values from 0.6 to 0.9.

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