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Approximation To L^P Integrable Functions By Gamma Type Operators

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ABSTRACT

In this paper we studied the following modification of Gamma operators which are first introduced in [9] (see [19], [21] and [9] respectively)

$$B_m(f;x) = \int_0^\infty K_m(x,t)f(t)dt$$

where

$$K_m(x,t) = \frac{(2m+3)!}{m!\,(m+2)!} \cdot \frac{t^m x^{m+3}}{(x+t)^{2m+4}} x,t \in (0,\infty),$$

and the approximation properties of these operators. We give approximation of $B_m(f;x)$ in L^pspaces and we get upper bound for that by using the K-functional of Peetre. In this paper we use the result of A. Izgi [12].

Keywords:L^p- approximation, Gamma type operators, K-functional of Peetre, Order of approximation in L^p-spaces.

INTRODUCTION

A Korovkin type theorem for linear positive operators acting from $L^p(a,b)$ into $L^p(a,b)$ was studied in [7] and then some new result in this direction were established. Ditzian and Ivanov [5] studied Bernstein type operators and their derivatives in $L^p(0,1)$ spaces and order approximation of these operator by using theK-functional of Peetre. Direct theorems for linear combination of Szasz-Beta type operators which defined by Gupta et all [8] in L^p-approximation on positive semi axis obtained by Mahewshwari [20].

Our aim is to study approximation properties of $B_m(f;x)$operators by meansof Korovkin's theorem in L^p-spaces on $(0,B]$. Then we compute the approximation order by modulus of continuity and we give a measure smoothness using the K-functional of Peetre [24]. We obtainand estimate the L^p- distance $(1 \le p \le \infty)$ between a function f and its image by means of $B_m(f;x)$ which is given in (1).

II. PRELEMINARIES

The following operators given by Izgi and Buyukyazycy [9].

$$B_m(f;x) = \frac{(2m+3)!\,x^{m+3}}{m!\,(m+2)!} \int_0^\infty \frac{t^m}{(x+t)^{2m+3}} f(t)dt, x > 0 \qquad (1)$$

If we choose

$$K_m(x,t) = \frac{(2m+3)!}{m!\,(m+2)!} \cdot \frac{x^{m+3}t^m}{(x+t)^{2m+4}}, x,t \in (0,\infty),$$

We can write$B_m(f;x)$ as the following form:

$$B_m(x,t) = \int_0^\infty K_m(x,t)f(t)dt.$$

For the process of these operators see [19] ,[21] and [9] respectively.

In [14] it was studied the rate of pointwise convergence of the operators $B_m(x,t)$ on the set of functions with bounded variation. These operators for bivariate functions in the weighted spaces with the following operators studied by Izgi. A.[10].

$$B_{m,n}(f(r,s)x,y) = \int_0^{\infty}\int_0^{\infty} K_m(x,r)K_m(y,r)f(r,s)drds \qquad (2)$$

and also studied $B_m(f;x)$for Voronoskaya type asymptotic approximation by Izgi. A in [11].

Now we introduce some notations which will be used in main result.

We denote by $C_b(0,\infty)$ the class of continuous and bounded functions on $(0,\infty)$by $BC(0,\infty)$the spaces of all absolutely continuous functions on $(0,\infty)$ and by $L_2^p(0,\infty)$, a subspaces of $L^p(0,\infty)$such that

$$L_2^p(0,\infty) = \{f \in L^p(0,\infty): f' \in BC(0,\infty), f'' \in L^p(0,\infty)\ for\ 1 \le p \le \infty\}.$$

The norm on the spaces$L_2^p(0,\infty)$ can be defined as

$$\|g\|_{L_2^p} = \|g\|_{L^p} + \|g^{00}\|_{L^p}$$

Or equivalently

$$\|g\|_{L_2^p} = \sum_{k=0}^{2} \left\|g^{(k)}\right\|_{L^p}$$

$$= \sum_{k=0}^{2} \left(\int \left|g^{(k)}(t)\right|^p dt\right)^{1/p}$$

$$= \|g\|_{L^p} + \|g^0\|_{L^p} + \|g^{00}\|_{L^p}$$

We consider also following K-functional of Peetre [24].

$$K_p(f;\delta) = \inf_{g \in L_2^p((0,B])} \left[\|f-g\|_{L^p(0,B]} + \delta(\|g\|_{L_2^p(0,B]})\right], \delta \ge 0$$

For $f \in L^p((0,\infty])$,using Theorem 2, we have $lim_{\delta\to\infty} K_p(f;\delta) = 0$. Therefore the K-functional gives the degree of approximation of a function $f \in L^p(0,B]$by smoother functions$g \in L_2^p((0,B])$.

Remember that the second order integral modulus of smoothness is given by

$$\vartheta_{2,p}(f,\delta) = \sup_{0 \le h \le \delta} \|f(x+h) - 2f(x) + f(x-h)\|_{L^p(0,B]}(I_h)$$

For an $f \in L^p(0,B]$, where $I_h$indicates that the L^p-norm is taken over the interval $[h, B-h]$.

It is also known that there are constants $c_1 > 0, c_2 > 0$, independent of *f* and *p* such that

$$c_1\vartheta_{2,p}\left(f;\delta^{1/2}\right) \le K_p(f;\delta) \le min(1,\delta)\,\|f\|_{L^p(0,B]} + 2c_2\vartheta_{2,p}\left(f;\delta^{1/2}\right) \qquad (3)$$

We need the following properties of $B_m(f;x)$which where shown in [9]:

For any $p \in N,\ p \le m+2$

$$B_m(t^p;x) = \frac{(m+p)!\,(m+2-p)!}{m!\,(m+2)!}x^p \qquad (4)$$

It follows from (4) that

$$B_m(1;x) = 1 \quad (5)$$

$$B_m(t;x) = x - \frac{x}{m+2} \quad (6)$$

$$B_m(t^2;x) = x^2 \quad (7)$$

The following equalitiesare hold by (5), (6) and (7):

$$B_m((t-x)^2;x) = \frac{2}{m+2}x^2 \quad (8)$$

$$\sup_{x\in(0,B]} B_m((t-x)^2;x) = \frac{2}{m+2}B^2 \quad (9)$$

Theorem1: Let $f \in C_b(0,\infty)$.Then for a real number B>0, the limit relation

$$\lim_{m\to\infty} B_m(f;x) = f(x)$$

holds uniformly on (0, B]

Proof: Using (6), (7) and (8) we see that:

$$\|B_m(1;x) - 1\|_{C(0,B]} = 0$$

$$\|B_m(t;x) - x\|_{C(0,B]} = \max_{x\in(0,B]} \frac{x}{m+2} \le \frac{B}{m+2} \to 0, \qquad (m\to\infty)$$

by P.P. Korovkin theorem [17], the proof of Theorem 1 is completed.

III. MAIN RESULTS FOR THE APPROXIMATION IN LP-SPACES

In this section, we prove theorems of Korovkin type for approximation in the norm of the space $L^p(0,B]$, $1 \le p \le \infty$, of integrable functions whose first derivatives belong to the class absolutely continuous functionson (0, ∞) and second derivatives belong to the class$L^p(0,\infty)$and we will give a rate of convergence using the K-functional of Peetre [24];

It is easily to see that,

$$\int_0^\infty K_m(x,t)dt = 1, \qquad and \qquad \int_0^\infty K_m(x,t)dx = \frac{m+3}{m} \le 4 \quad (10)$$

Thus $B_m(f;x)$exists for all $f \in L^p(0,\infty)$and for every fixed m. (see [18] cf. 31 Theorem of Orlicz). According to Lusinos theorem, if $f \in L^p(0,B]$then there exists a function $g \in C(0,B]$ such that for any $\varepsilon > 0$

$$\varphi(\{x|f(x) \ne g(x)\}) = \varepsilon, \quad (11)$$

Now we give the following theorem for the approximation in the L^p spaces, p ≥ 1.

Theorem 2: Let $f \in L^p(0,\infty)$and B be a fixed derivative point in (0, ∞) such that the condition,

$$\frac{|f(t) - f(x)|}{|t-x|} \le M, \quad x \in (0,B], t \in (B,\infty) \quad (12)$$

holds with the constant M. Then

$$\|B_m f - f\|_{L^p(0,B]} \to 0, \ \ (m \to \infty).$$

Prrof: By (11)

$$\|g - f\|_{L^p(0,B]} < \varepsilon \qquad (13)$$

holds.

From Theorem1, $\|B_m g - g\|_{C(0,B]} \to 0 \ \ (m \to \infty)$.Thus for Ɛ > 0 there exists an $m_0 \in N$such that for all $m > m_0$.

$$\|B_m g - g\|_{C(0,B]} < \varepsilon.$$

Now we can write that

$$B_m(f;x) - f(x) = \int_0^\infty K_m(x,t)\big(f(t) - f(x)\big)dt$$

$$= \int_0^B K_m(x,t)\big(f(t) - f(x)\big)dt + \int_B^\infty K_m(x,t)\big(f(t) - f(x)\big)dt$$

$$= E_1(x) + E_2(x) \qquad (14)$$

$$|E_1(x)| \le \int_0^B K_m(x,t)\big(f(t) - f(x)\big)dt$$

$$\le \int_0^B K_m(x,t)|f(t) - g(t)|dt + \int_0^B K_m(x,t)|g(t) - g(x)|dt + \int_0^B K_m(x,t)|g(x) - f(x)|dt$$

$$\le E_{11}(x) + E_{12}(x) + E_{13}(x) \qquad (15)$$

For sufficiently large *m* by (13)

$$\|E_{11}(x)\|_{L^p(0,B]} \le \left(\int_0^B |f(t) - g(t)|^P dt\right)^{\frac{1}{p}} < \varepsilon. \qquad (16)$$

Now, we evaluate $\|E_{12}(x)\|_{L^p(0,B]}$. Since g is a continuous function in (0, B] we can write well known inequality

$$|g(t) - g(x)| < \varepsilon + \frac{2M_1(t-x)^2}{\delta^2},$$

where δ > 0 and M_1 constant such that $|g(x)| < M_1$. Then

$$E_{12}(x) = \int_0^B K_m(x,t)|f(t) - g(t)|dt < \varepsilon\int_0^\infty K(x,t)dt + \frac{2M_1}{\delta^2}\int_0^\infty (t-x)^2 K(x,t)dt$$

and by (9)

$$E_{12}(x) < \varepsilon + \frac{2M_1}{\delta^2}\frac{2}{2+m}B^2$$

Since $\frac{2B^2}{2+m} \to 0$as $m \to \infty$, for a large m

$$\|E_{12}(x)\|_{L^p(0,B]} < C\varepsilon, \qquad (17)$$

where C is a positive constant, If$|t-x| < \varphi$ then$|g(t) - g(x)| < \varepsilon$, hence

$$\int_0^\infty K_m(x,t)|g(t) - g(x)|dt < \varepsilon.$$

By (14) we have

$$\|E_{13}(x)\|_{L^p(0,B]} < \varepsilon. \qquad (18)$$

Thus for larg m,

$$\|E_1(x)\|_{L^p(0,B]} < \varepsilon. \qquad (19)$$

Consider $E_2(x)$ using the condition (12) and Holder's inequality, we get

$$|E_2(x)| \le \int_B^\infty \left|K_m(x,t)\big(f(t) - f(x)\big)\right|dt$$

$$\le M\int_B^\infty K_m(x,t)|t-x|dt$$

$$\le M\sqrt{\int_B^\infty K_m(x,t)|t-x|^2dt}\ \sqrt{\int_0^\infty K_m(x,t)dt}$$

$$\le M\sqrt{\delta_m}.$$

where $\delta_m = \frac{2B^2}{m+2}$ (see (9)),

Thus,

$$\|E_2(x)\|_{L^p(0,B]} \le M\sqrt{\delta_m}B^{\frac{1}{p}} \qquad (20)$$

and therefore (13), (19) and (20)

$$\|B_m f - f\|_{L^p(0,B]} \le \left(\varepsilon + M\sqrt{\delta_m}B^{\frac{1}{p}}\right) \qquad (21)$$

Holds for $x \in (0,B]$ and for sufficiently large m. Thus the proof is completed.

IV. RATE OF CONVERGENCE

We use Lemma 1 to establish the degree of approximation with (3). Namely, we first approximate $f \in L^p(0,B]$by $f \in L_2^p((0,B])$and then use Lemma 1, the J.J. Swetits and definition the K-functional and (4). Also see [2], [4] and [22] for this method.

The following lemma gives upper bound of approximation of $B_m f$to $f$in $L^p(0,B](m \to \infty)$with help of $\|f\|_{L^p}$ and δ_m. Also it helps the prove of Theorem 3.

Lemma 1: Let $f \in L_2^p(0,\infty)$and f satisfies the condition (13). For all sufficiently large m,

$$\|B_m f - f\|_{L^p(0,B]} \le C_p\left(\|f\|_{L_2^p(0,B]}\right)\delta_m$$

where C_p is a positive constant and independent of *f* and *m*.

Proof: Now we assume that, p > 1 and $x \in (0, B]$. Since $f \in L_2^p(0, \infty]$ using Taylor Theorem, we can write that,

$$f(t) - f(x) = f^0(x)(t - x) + \int_0^t (t - r) f''(r) dr.$$

Applying operator B_m on both side we get

$$B_m(f(t) - f(x); x) = f'(x) B_m(t - x; x) + B_m\left(\int_x^t (t - r) f''(r) dr\, ; x\right)$$
$$= U_1(x) + U_2(x). \tag{22}$$

Using (6) we get following inequality

$$\|U_1\|_{L^p(0,B]} \le C_1\left(\|f\|_{L^p(0,B]}\right) \frac{B}{m+2} \frac{1}{B^p}.$$

Now we need the Hardy-Littlewood majorante of f'' at x, which is defined to be

$$\theta_{f^{00}}(x) = \sup_{0 \le t \le x; t \ne x} \frac{1}{t - x} \int_x^t |f''(r)| dr. \tag{23}$$

Since $p > 1$ and $f \in L_2^p$, $\theta_{f^{00}}(x) \in L^p$ according to [29 Theorem 13.5] we get,

$$\int_0^B \left|\theta_{f^{00}}(x)\right|^p dx \le 2\left(\frac{p}{p-1}\right)^p \int_0^B \left|f''(x)\right|^p dx, \tag{24}$$

By using (9) and (23) then we obtain on (0, B]

$$|U_2(x)| \le B_m\left(|t - x| \int_x^t \left|f''(r)\right| dr; x\right)$$
$$\le \theta_{f^{00}}(x) B_m((t - x)^2); x)$$
$$\le \theta_{f^{00}}(x) \delta_m. \tag{25}$$

Then, when we use (24) in above inequality (25)

$$\|U_2\|_{L^p(0,B]} \le 2^{\frac{1}{p}}\left(\frac{p}{p-1}\right) \|f''\|_{L^p(0,B]} \delta_m.$$

Since $\frac{B}{m+2} \le \delta_m$ we obtain that,

$$\|U_1\|_{L^p(0,B]} + \|U_2\|_{L^p(0,B]} \le \left[C_1 B^{\frac{1}{p}} + 2^{\frac{1}{p}}\left(\frac{p}{p-1}\right)\right]\left(\|f\|_{L^p(0,B]} + \left\|f''\right\|_{L^p(0,B]}\right) \delta_m$$
$$\le C_p\left(\|f\|_{L^p(0,B]} + \left\|f''\right\|_{L^p(0,B]}\right) \delta_m$$

where $C_p = \left[C_1 B^{\frac{1}{p}} + 2^{\frac{1}{p}}\left(\frac{p}{p-1}\right)\right]$

Let p = 1

$$\int_0^B |f'(x)||B_m(t - x); x| dx \le C_2\left(\|f\|_{L^1(0,B]} + \left\|f''\right\|_{L^1(0,B]}\right) \frac{B^2}{m+2} \tag{26}$$

$$\int_0^B |U_2(x)| dx \le \int_0^B B_m\left(|t - x| \int_x^t |f''(r)| dr\, ; x\right) dx$$
$$\le \left\|f''\right\|_{L^1(0,B]} \int_0^B B_m((t - x)^2; x) dx$$
$$\le \left\|f''\right\|_{L^1(0,B]} B \delta_m$$
$$\le B\left(\|f\|_{L^1(0,B]} + \left\|f''\right\|_{L^1(0,B]}\right) \delta_m$$
$$\le B\left(\left\|f''\right\|_{L_2^1(0,B]}\right) \delta_m \tag{27}$$

Since $\frac{B}{m+2} \leq \delta_m$ anduse (26), (27) in (22), for $p \geq 1$we have

$$\|U_1\|_{L^p(0,B]} + \|U_2\|_{L^p(0,B]} \leq C_3\left(\|f\|_{L_2^p(0,B]}\right) B\delta_m$$

where$C_3 = 1 + C_2$.

Thus, the proof of Lemma 1 is completed.

Theorem 3: Let $f \in L^p(0,\infty)(1 \leq p \leq \infty)$and f satisfied the condition (12). For all sufficiently large m and $B>0$, B is a derivative point of$f$, then the following inequality

$$\|B_m f - f\|_{L^p(0,B]} \leq M_p\left[\delta_m\|f\|_{L_2^p(0,B]} + \vartheta_{2,p}(f;\delta_m)\right] \tag{28}$$

holds. Where M_p is a positive constant, independent of f and m.

Proof: For all sufficiently large m, from Lemma 1 we can write

$$\|B_m h - h\|_{L^p(0,B]} \leq \begin{cases} \left(\varepsilon + M\delta_m B^{1/p}\right)\|h\|_{L^p(0,B]}\,; h \in L^p(0,B] \\ C_p\left(\|h\|_{L_2^p(0,B]}\right)\delta_m\,;\ h \in L_2^p(0,B] \end{cases}$$

where C_p is positive constant which independent of h,m and where h satisfies (12). When $f \in L^p(0,\infty)$and $g \in L_2^p(0,\infty)$the condition (12) is satisfies then

$$\begin{aligned} \|B_m f - f\|_{L^p(0,B]} &\leq \|B_m(f-g) - (f-g)\|_{L^p(0,B]} + \|B_m g - g\|_{L^p(0,B]} \\ &\leq \left(\varepsilon + M\delta_m B^{1/p}\right)\|f-g\|_{L^p(0,B]} + C_p\left(\|g\|_{L_2^p(0,B]}\right)\delta_m \\ &\leq M_\rho\left[\|f-g\|_{L^p(0,B]} + \delta_m\left(\|g\|_{L_2^p(0,B]}\right)\right] \end{aligned}$$

where $M_\rho = max\left\{\left(\varepsilon + M\delta_m B^{\frac{1}{p}}\right), C_p\right\}$

Taking infimum over all $g \in L_2^p(0,B)$which satisfies (12) on the right hand side using the definition of the K-functional we get,

$$\|B_m f - f\|_{L^p(0,B]} \leq M_\rho \sup_{g \in L_2^p((0,B])}\left[\|f-g\|_{L^p(0,B]} + \delta_m\left(\|g\|_{L_2^p(0,B]}\right)\right]$$

Since, for a sufficiently large $m, \delta_m < 1$and from (4),

$$K_p(f;\delta_m) \leq \delta_m\|f\|_{L_p(0,B]} + 2c_2\vartheta_{2,p}(f;\delta_m^{1/2})$$

$$M_\rho K_p(f;\delta_m) \leq M_\rho\left[\delta_m\|f\|_{L_p(0,B]} + 2c_2\vartheta_{2,p}\left(f;\delta_m^{\frac{1}{2}}\right)\right]$$

We obtain (28),

$$\|B_m f - f\|_{L^p(0,B]} \leq M_p\left[\delta_m\|f\|_{L_p(0,B]} + \vartheta_{2,p}\left(f;\delta_m^{\frac{1}{2}}\right)\right].$$

Thus the proof of the Theorem 3 is completed.

REFRENCES

[1]. Agratini. O., "On Class of Linear PossitiveBivariant Operators of King, Studia Univ, Bades-Bplyai", Mathematica, L1, No:1, December 2006.

[2]. Berens H. and Devore. R. A., "Quantitative Korovkin Theorems for L^p- Spaces in Approximation Theory II (G. G Lorentz, Ed.)", Academic Press, New York, pp. 289-298,1976.

[3]. Berens.S. J., "Theorems of Korovkin Type for L^p- Spaces ", Pasific J. Math. 53, pp. 11-19,1974.

[4]. [4]Devore. A., "Degree of Approximation in Approximation Theory II (G. G Lorentz, Ed.)", Academic Press, New York, 1976, pp. 117-161. No.2, Ekim, pp. 451-460,2003.

[5]. Ditzian Z. and Ivanov.K., "Bernstein Type Operators and Their Derivatives", J. Approx. Theory 56, pp. 72-90,1989.

[6]. Donner. K., "Korovkin Theorems in L^pSpaces", J. Funct. Anal. 42, pp.12-28,1981.

[7]. Dzyadik. V. K., "On the Approximation of Functions by Linear Positive operators and Singular Integrals", Mat. Sbornik, 70, pp. 508-517,1976 (in Russian).

[8]. Gupta V., Srivastav G. S. and Sahai A., "On Simultaneous Approximation by Szasz-Beta Operators", Soochow. J. of Math. 21(1), pp. 1-11,1995.

[9]. Izgi A. and Buyukyazici., I., "Approximation in Boundless Interval and Order of Approximation". (In Turkish), Kastamonu Egitim Dergisi, Cilt, II(2),, pp. 451-456,2003.

[10]. Izgi A., "Order of Approximation of Functions of Two Variables by New Type Gamma Operators", General Mathematics, 17, No:1, pp.23-32,2009.

[11]. Izgi A., "Voronovskaya type Asymptotic Approximation by Modified Gamma operators", Applied Mathematics and Computation, 217, pp. 8061-8067,2011.

[12]. Izgi. Aydin, "Approximation to L^p Integrable Functions by Gamma Type Operators, J. of Analysis and Computation, Vol.7, No. 2, pp. 113-124,2011.

[13]. Johnen. H., "Inequalities Connected with the Moduli of Smoothness", Math. Vesnik, 9(24), Pp.289-303,1972.

[14]. Karsli H., Gupta V. and Izgi. A., "Rate of Pointwise Convergence of a New kind of Gamma Operatorsfor Functions of Bounded Variation", Applied Mathematics Letters., 22(4), pp. 505-510,2009.

[15]. King. J. P.,"Positive Linear Operators which Preserve x^2, Acta Math. Hungar., 99 No:3, pp. 203-208,2009.
[16]. Kitto W. and Walbert. D. E., " KorovkinApproximation L^p- Spaces", Pasific J. Math. 63, pp. 153-167,1976.
[17]. Korovkin. P. P., "Linear Operators and Approximation Theory", Delhi, 1960.
[18]. Lorentz. G. G., "Bernstein Polynomials", University of Toronto Press, Toronto, 1953.
[19]. Lupas A. and Muller., M., "Approximation Seigenschaften Der Gamma Operatoren". Math. Zeit. 98, pp. 208-226 1967 (in German).
[20]. Maheshwari. P.,"Some Direct Estimates for Szasz-Beta Operators in L^p-Norm". I. J. Pure and Appl. Math. Sci. 3(1), pp. 61-72,2006.
[21]. Mazhar. S. M., "Approximation By Positive Operators On infinite Intervals", 5, Fas.2, pp.99-104,1991.
[22]. Muller. M. W., "Die Gute Der L^p-Approximation Durch Kantorovic-Polynome", Mat Z, 151, pp.243-246,1967.
[23]. Muller. M. W., "L^p-Approximationby the Method of Integral Meyer-Konig and zeller Operators", Studia Math. 63,pp. 81-88,1978.
[24]. Peetre., J., "A Theory of Interpolation of Normed Spaces Lecture Notes", Brazilia, 1963.
[25]. Rempulska L. and Tomczzak .K., "Approximationby certain Linear Operators Preserving Turk. J. Math. 32 pp. 1-11,2008.
[26]. Swetit J. J. and Wood B., "Quantative Estimates for L^p-Approximationwith Positive Linear Operators", J. Approx. Theory, 38, pp. 81-89,1983.
[27]. Swetit J. J. and Wood B., "On the Degree of L^p-Approximationwith Positive Linear Operators", J. Approx. Theory, 87, pp. 239-241,1996.
[28]. Wood B., "Degree of L^p-Approximationwith Certain Positive Convolution Operators", J. Approx. Theory, 23, pp. 534-363,1978.
[29]. Zygmund. A., "Trigonometric Series", vols. I, II. London/New York, Cambridge Univ., Press, 1968.

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