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Examining Non-Linear Transverse Vibrations of Clamped Beams Carrying N Concentrated Masses at Various Locations Using Discrete Model

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ABSTRACT

The discrete model used is an N-Degree of Freedom system made of N masses placed at the ends of solid bars connected by springs, presenting the beam flexural rigidity. The large transverse displacements of the bar ends induce a variation in their lengths giving rise to axial forces modeled by longitudinal springs causing nonlinearity. Nonlinear vibrations of clamped beam carrying n masses at various locations are examined in a unified manner. A method based on Hamilton's principle and spectral analysis has been applied recently to nonlinear transverse vibrations of discrete clamped beam, leading to calculation of the nonlinear frequencies. After solution of the corresponding linear problem and determination of the linear eigen vectors and eigen values, a change of basis, from the initial basis, i.e. the displacement basis (DB) to the modal basis (MB), has been performed using the classical matrix transformation. The nonlinear algebraic system has then been solved in the modal basis using an explicit method and leading to nonlinear frequency response function in the neighborhood of the first mode. If the masses are placed where the amplitudes are maximized, stretching in the bars becomes significant causing increased nonlinearity.

Keywords: *Concentrated masses, Discrete system, Hamilton's Principle, Nonlinear transverse constrained vibration, Spectral analysis.*

I. INTRODUCTION

An application of the discrete model developed in the works [1, 2, 3 and 4] is made here to aBernoulli beam carrying n concentrated masses at various locations and subject to geometricalnonlinear. This model focuses on the known physical phenomenon of the dynamic behavior: the stretching of the beam created nonlinearity.This study shows that the developed model is used to study successfully clamped beams with many concentrated masses simply by changing the mass matrix, with respect to those of the uniform beam defined in [1].The concentrated masses treated here are static that may be poles or benches; there are other specialized works that reflect the dynamic forces exerted by cars such traveling at different speeds on a slender bridge.

II. PRESENTATION AND NOMENCLATURE

The studied model of a beam with nconcentrated massesM_1,…..,M_i,….., and $M_n$is shown in Fig. 1:

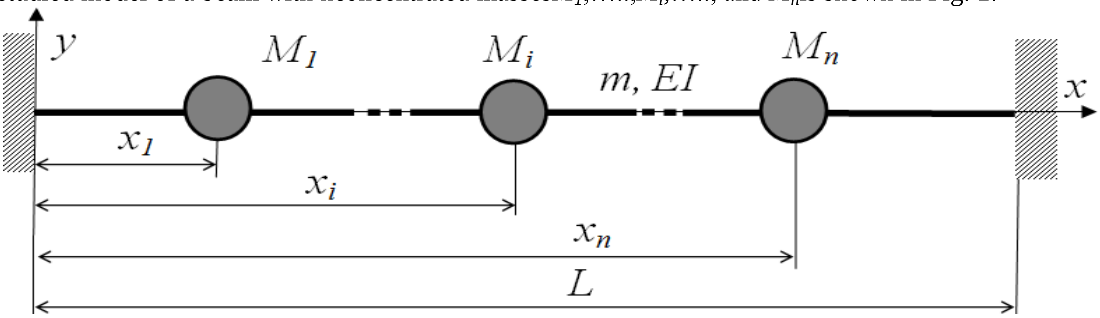

Figure1: Clamped beam with nconcentrated masses M_1, …., M_i…… and M_n

Fig. 2 shows the discrete system with *N*-dof considered in the present application, consists of *N* masses m_1+M_1, ..., m_i+M_i , ..., m_N+M_N connected by *N*+2 coiled torsion springs and *N* +2 longitudinal springs, considered in its neutral position.

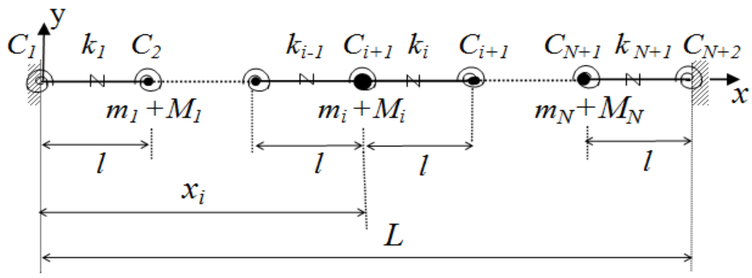

Figure 2: Discrete system with several degrees of freedom (N -dof), modelling a clamped beam with n concentrated masses

Where the stiffness of the torsional representing the flexural rigidity and the longitudinal springs of the beam are [1]:

$$C_i = \frac{EI}{l} \qquad\qquad k_i = \frac{ES}{l} \qquad (1)$$

(2) determines the location coordinates of the n masses:

$$i = x_i / l \qquad\qquad i = 1.....n \qquad (2)$$

The choice of N is very important, it is chosen so that the indices i(locations coordinates of the n masses are natural whole numbers). All locations must coincide with one of the nodes of the discrete system. $N>n$, for nodes that do not receive the concentrated masses,M_s= 0. With this consideration N-n concentrated masses (M_1, M_2 ,......, M_N) Msare equals to zero.

III. FORMULATION DIMENSIONLESS

The following equations link the dimensional values for dimensionless values (with an asterisk):

$$y^* = \frac{y}{r} \qquad (3)$$

$$\omega^* = \frac{\omega L^2}{\sqrt{\dfrac{EI}{\rho S}}} \qquad (4)$$

$$\eta_i = \frac{x_i}{L} \qquad i = 1.......N \qquad (5)$$

$$\alpha_i = \frac{\text{concentrated mass}}{\text{total mass of the beam}} = \frac{M_i}{m} = \frac{M_i}{\rho S L} \qquad i = 1.......N \qquad (6)$$

Where ρ is the density of the beam, E Young's modulus, α_i le ratio of the concentrated mass i to the total mass of the beam, and η_i non dimensional location of the concentrated mass i.

IV. FORMULATION IN THE NONLINEAR CASE

Our method is to apply the Hamilton's principle in the modal basis [1], we achieve a system of nonlinear (7) which can be written as a system of nonlinear differential equations:

$$3a_i a_j a_k \overline{b}_{ijkr} + 2a_i \overline{k}_{ir} - 2a_i (\omega^{\text{nl}}_{\text{disc}})^2 \overline{m}_{ir} = 0 \qquad i, j, k, r = 1, \ldots.., N \qquad (7)$$

We solve this equation using the explicit method in the Modal basis: The explicit formulation is based on an approximation which consists on assuming, when the first nonlinear mode shape is under examination, that the contribution vector $\{\mathbf{a}\}^T = [a_1 a_2 .....a_N]$ can be written as $\{\mathbf{a}\}^T = [a_1 \varepsilon_2 ...\varepsilon_i ..\varepsilon_N]$ with ε_i for i= 2 to N, being small compared to a_1 ($a_1^2 \varepsilon_v = 0 \quad \varepsilon_i \varepsilon_j \varepsilon_k \approx 0 \quad a_1 \varepsilon_i \varepsilon_j \approx 0$). So that the only remaining term $a_1^3 \overline{b}_{111r}$, leads to:

$$\left(\overline{k}_{jr} - (\omega^{\text{nl}}_{\text{disc}})^2 \overline{m}_{jr}\right) \varepsilon_j + \frac{3}{2} a_1^3 \overline{b}_{111r} = 0 \quad \text{for } r = 2, \ldots, N \qquad (8)$$

The contributions $\{a_{1\text{disc}} \; \varepsilon_1 \, \varepsilon_2 \cdots \varepsilon_N\}$ in the modal basis are calculated by [1]:

$$\varepsilon_r = \frac{\frac{3}{2} a_1^3 \overline{b_{111r}}}{((\omega_{disc}^{nl})^2 \overline{m_{rr}} - \overline{k_{rr}})} \quad \text{for} \quad r = 2,\ldots\ldots,N \tag{9}$$

The nonlinear frequency is calculated by:

$$(\omega_{disc}^{nl})^2 = \frac{\overline{k_{11}}}{\overline{m_{11}}} + \frac{3}{2}\frac{\overline{b_{1111}}}{\overline{m_{11}}} a_1^2 \tag{10}$$

After resolving the equation in modal basis we calculate the amplitudes in displacement basis:

$$\left[A_1 \; A_2 \ldots\ldots A_N\right]^T = [\varphi]^{-1}\left\{a_1 \;\; \varepsilon_1 \;\; \varepsilon_2 \ldots \varepsilon_N\right\} \tag{11}$$

The amplitude is the maximum of ($A_1 \, A_2 \ldots . A_N$). We plot the dimensionless frequency curve as a function of dimensionless amplitude.

V. RESULTS IN FOR FIVE CONCENTRATED MASSES

5.1 Linear Case

A computer program has been written, allowing any case of linear or nonlinear vibrations of a *N*-dof carrying *n* masses to be examined in a systematic and unified manner.

Fig.3 shows beam loaded uniformly with five equal concentrated masses and equidistant(see numerical valuesfor linear vibrations in Table 1 case 2, and Fig.9 for nonlinear vibrations).

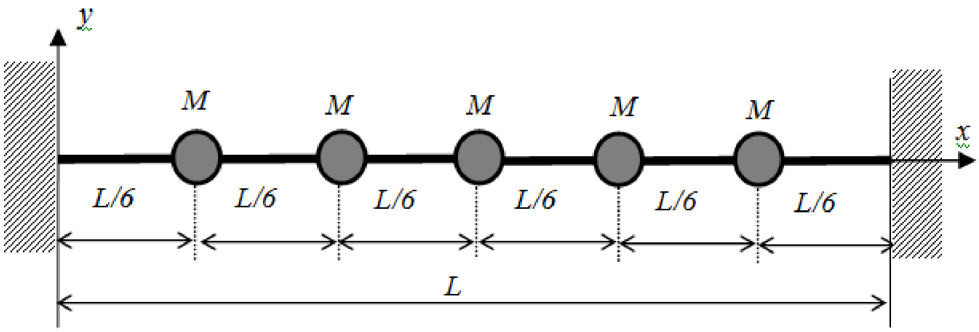

Figure 3: Beam loaded uniformly with five equal and equidistant concentrated masses.

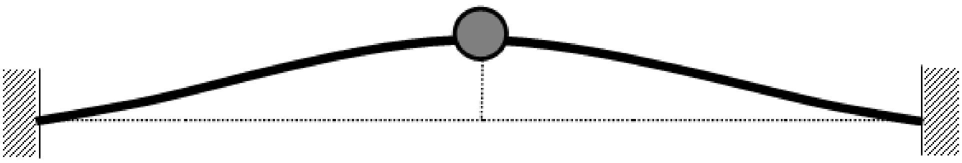

Figure 4: First mode of beam loaded with all concentrated masses grouped in the middle of the beam

In this case of charging, the first linear frequency changes a lot and the nonlinearity increasesseenumerical values for linear vibrations inTable 1 case 3, andFig.10 for non linear vibrations. In this same caseof chargingthe second linear frequency don't changes (Fig.5).

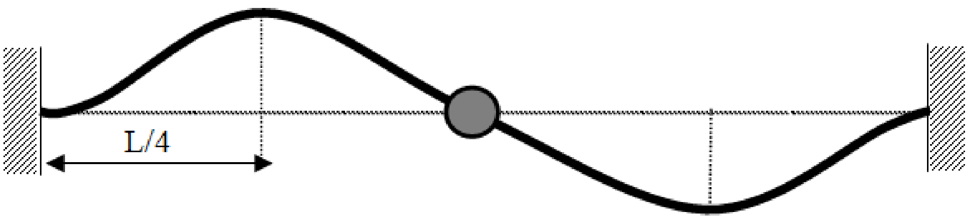

Figure 5: Second modeof beam loadedwith all concentrated masses grouped in the middle of the beam.

If we charged the beam in maximum of amplitude like illustred in Fig.6, the second linear frequency changes a lot see Table 1 case 4.

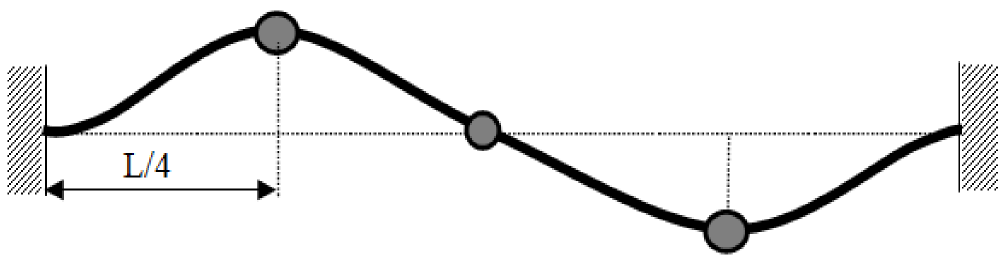

Figure 6: Second mode of beam loaded in maximum of amplitude.

Table 1 case 5 showsthat if we chargethe beam in maximum of amplitude (Fig.7), the third linear frequency changes a lot.

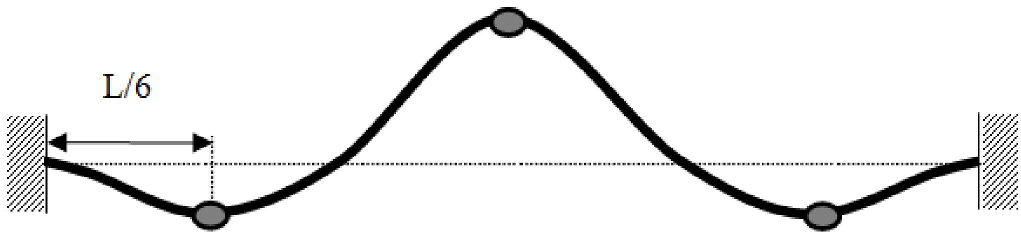

Figure 7: Third mode of beamloadedchargedinmaximum of amplitude see Table 1 case 5.

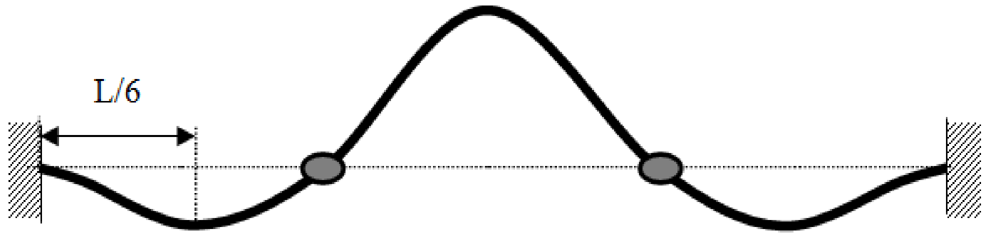

Figure8: Beam loaded in the nodes of the third mode.

In this caseof charging (Fig.8), the third linear frequency doesn't changes: it is equal of the beam without concentrated masses (Table 1 case 6).

Table 1: The first three natural frequencies of the discrete system (N=49), for $\alpha_i = 1/2$ and different cases of loading (different values of η_i)

	Case1	Case2	Case3	Case4	Case5	Case6
α_i	0	1/2	1/2	1/2	1/2	1/2
η_1	0	0.166	0.5	0.25	0.166	0.333
η_2	0	0.333	0.5	0.25	0.166	0.333
η_3	0	0.5	0.5	0.5	0.5	0.5
η_4	0	0.666	0.5	0.75	0.833	0.5
η_5	0	0.833	0.5	0.75	0.833	0.666
$\omega^{cc\,nl}_{disc\ 1}$	23.3	11.52	6.16	11.77	11.62	9.51
$\omega^{cc\,nl}_{disc\ 2}$	64.182	32.06	64.18	27.42	35.80	31.58
$\omega^{cc\,nl}_{disc\ 3}$	125.64	62.17	94.41	53.40	48.47	68.71

5.2 Nonlinear Case

Fig.9 shows the nonlinear frequenciesof a beam loaded uniformly.We note that in this case, the resonance frequencies and the effect of the nonlinearity decrease.

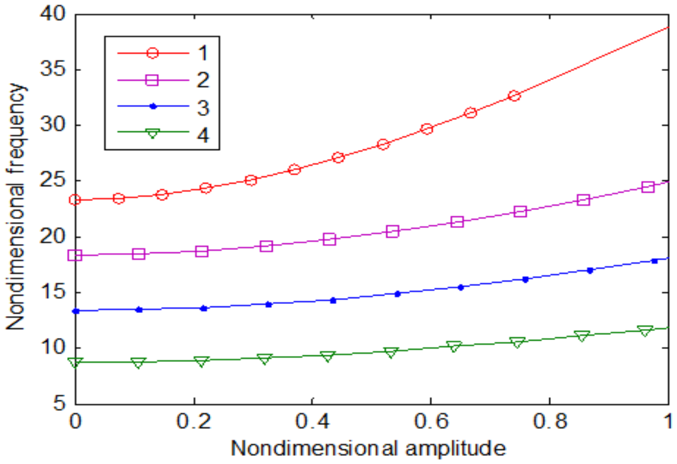

Figure 9: Frequency curves according to the amplitude corresponding to the discrete systems with N = 49 dof, η_1= 1/6, η_2 = 1/3, η_3 = 1/2,η_4 =2/3, η_5=5/6 and (**1**:α_i = 0; **2**:α_i =1/10;**3**: α_i = 1/2; **4**: α_i = 1)

Fig. 10 shows the nonlinear frequencies of a beam where the masses are placed at the bellies. We notethat in this case, the first nonlinear frequency changes much. This is due to the mass inertia effect which increases the stretching in the bars causing increased nonlinearity.

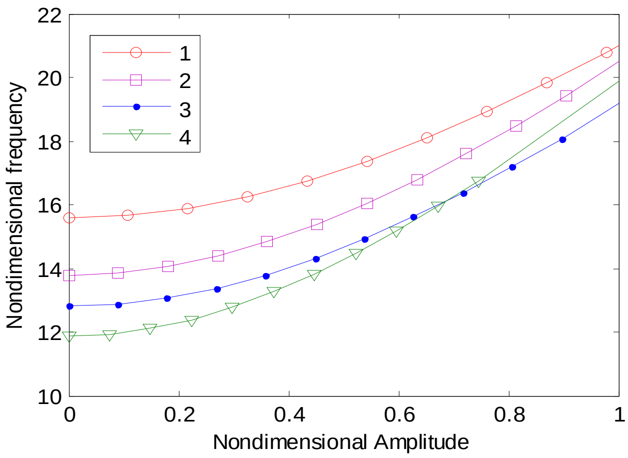

Figure10: Frequency curves according to the amplitude corresponding to the discrete systems with N = 49 dof, η_1= 1/6, η_2 = 1/3, η_3 = 1/2,η_4 =2/3, η_5=5/6 and (**1**:α_i = 1/5; **2**:α_1 =α_5=0, α_2 =α_4=2/5, α_3 =1/5;**3**: α_1 =α_5=0, α_2 =α_4=1/5, α_3 =3/5; **4**: α_1=α_2 =α_4 =α_5=0, α_3 =1)

VI. CONCLUSION

The discrete model developed and validated in the case of a continuous beam presented in [1 and 2]was applied to the beams with n concentrated masses. Linear and nonlinear vibrations were examined. This shows the effectiveness of this discrete model, its formulation and the associated program for the study of linear and nonlinear vibrations of a beam with discontinuities in the distribution of masses. The linear frequency change much when installing the masses at the bellies. This is due to the mass inertia effect which increases if the masses are placed where the amplitudes are maximized. Stretching in the bars become significant causing increased nonlinearity.

REFERENCES

[1]. A. Rahmouni, Z. Beidouri and R. Benamar, A discrete model for geometrically nonlinear transverse free constrained vibrations of beams with various end conditions, *Journal of Sound and Vibration332*, 2013, 5115–5134.

[2]. A. Rahmouni, Z. Beidouri and R. Benamar, A discrete model for the natural frequency and mode shapes of beams with various boundary conditions Marrakech-Maroc, *MATEC Web of Conferences01*, 2012; *DOI: 10.1051/matecconf/20120110015*

[3]. A. Rahmouni and R. Benamar A discrete model for geometrically nonlinear transverse free constrained vibrations of clamped beam carrying a concentrated mass at various locations,*Porto-Portugal EURODYN 2014,2093-2099, ISSN 2311-9020 ISBN 978-972-752-165-4* [4] A. Rahmouni and R. BenamarNon linear free transverse vibrations of clamped beams carrying two or three concentrated masses at various locations,*CSNDD Conference on Structural NonlinearDynamics and Diagnosis, 2016*.