Logarithmic Barrier Optimization Problem Using Neural Network

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Abstract: *The combinatorial optimization problem is one of the important applications in neural network computation.The solutions of linearly constrained continuous optimization problems are difficult with an exact algorithm, but the algorithm for the solution of such problems is derived by using logarithm barrier function. In this paper we have made an attempt to solve the linear constrained optimization problem by using general logarithm barrier function to get an approximate solution. In this case the barrier parameters behave as temperature decreasing to zero from sufficiently large positive number satisfying convexity of the barrier function. We have developed an algorithm to generate decreasing sequence of solution converging to zero limit.*

Keywords: *Barrier function, weightmatrix, barrier parameter, Lagrange multiplier,s-t max cut problem, descent direction.*

I. INTRODUCTION:

Let us consider graph G=(V,E) where V=(1,2,...n) is the set of nodes and E is the set of edges. The edge between two nodes and is defined by (i,j).Let $(W_{ij})_{n \times n}$is symmetric weight matrix such that

$W_{ij}>0, \forall \epsilon E$(1.1)

Given two nodes s and t, if s<t there exist a partition U and V-U to the node set V such that W(s)=$\sum_{i \epsilon U} w_{ij}$

(1.2)

Where $j \epsilon V - U$ is maximized for s, t not belonging to the same set. This problem is called s-t max cut problem. Through the exact solution is difficult however the same can be obtained by considering a equivalent problem.

$Max: \frac{1}{4}\sum_{i=1}^{n}\sum_{j=1}^{n}(1 - x_i x_j)w_{ij}$ (1.3)

Subject to $x_s + x_t = 0 \ and x_i \epsilon \{-1,1\}, \forall i = 1,2, \dots . n$

Benson and Zhang[2]found an algorithm to get an approximate solution in this direction to solve combinatorial and quadratic optimization problem. Neural computation is combinatorial and quadraticoptimization problem due to Hopfield and Tank[10] put a foundation stone in solving such type ofproblems.Many computational model and combinatorial optimization algorithm have been developed.Aiyer,NiranjanandFallside[1]extended this idea for a theoretical study of such problem. Similar algorithm is used in solving travelling salesman problem by Durbin and Willshaw[6]and multiple

traveling salesman problem by using Neural Network, due toWacholder,Han and Mann[15].Gee,AiyerandPrager [7]made a frame work for studying optimization problem by Neural Network, where in adifferent paper Gee and Prager[7]used polyhedral combinatorics in Neural Network.Urahama[13]usedgradient projection Network for solving linear and nonlinear programming problems. Some of therelated works can be found in Bout and Miller[3],Gold and Rangaranjan [9],Gold and Mjolsness[11],Simic[12],Waugh and Westervelt[16],Walfe,Parry and Macmillan[17],Xu[18],Yuille and Kosowsky[19] etc.A systematic study of neural computational model is given by Van Den Berg[14],Clchoki and Unbehaunen[4]helps to formulate an algorithm to solve such type of problem in an efficient manner. All these workbased on deterministic annealing type and to study the global minimum of the effective energy at hightemperature and track it as temperature decreases.In this paper we have generalized the work ofDang[5] by considering an equivalent linear constrained optimizing problem with general logarithmicbarrier function.

$b(x) = -\sum_{i=1}^{n}[\ln(px_i + q) + \ln(q - px_i)]$ (1.4)

to find an approximate solution. Where the parameters p,q are positivereal. The barrier parametersbehave as temperature decreasing to zero from a sufficiently large positive number satisfying convexity of the barrier function. We have developed an algorithm for which the network converges toat least a local minimum point, if a local minimum point of the barrier problem isgenerated for adecreasing sequence of values with zero limit. Theorganization of the paper is as follows. Following the introduction in section 2, we haveconsideredlogarithm

barrier function to minimize the problem with Lagrange multiplier. In section3,we have proved the theorem byconsidering general logarithmic function. Numerical example insection4, has been presented to show the efficiency of the algorithm. Finally the paper concludedwith a remark in the section 5.

II. LINEAR LOGARITHMIC BARRIER FUNCTION:

In order to find the solution of the problem first of all we will convert s-t max cut problem into alinearly constrained continuous optimization problem and then derive some theoretical results froman application of linear logarithmic barrier function.From s-t max-cut problem we have

$$min: f(x) = \frac{1}{2}x^T(W - \alpha I)x \qquad (2.1)$$

Subject to$x_i + x_t = 0$

$$x_i \epsilon \{-1,1\}, i = 1,2,3 \dots \dots n$$

where$\propto$ is any positive number and I an n×n identity matrixand

$$W = \begin{pmatrix} 0 & w_{12} \cdots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} \cdots & 0 \end{pmatrix}$$

be a symmetric weight matrix.

Let $b(x) = -\sum_{i=1}^{n}[\ln(px_i + q) + \ln(q - px_i)]$ \qquad (2.2)

which is used as barrier function –q/p<x_i<q/p ,i=1,2,……n in the objective function.

Let us define a function $h(x,\beta)$ for a positive barrier parameter β such that

$$min \quad h(x,\beta) = f(x) + \beta b(x) \qquad (2.3)$$

Subject to $x_s + x_t = 0$

Let $F = \{x: x_s + x_t = 0\}$

and$B = \left\{x: -\frac{q}{p} \le x_i \le \frac{q}{p}\right\}, i = 1,2,\dots n$

$$\frac{\partial b(x)}{\partial x_i} = -\left[\frac{p}{px_i + q} + \frac{-p}{q - px_i}\right] = -p\left[\frac{1}{px_i + q} - \frac{1}{q - px_i}\right]$$

Thus $\lim_{x_i \to (-\frac{q}{p})^+} \frac{\partial b(x)}{\partial x_i} = -\infty$, $\lim_{x_i \to (-\frac{q}{p})^+} \frac{\partial b(x)}{\partial x_i} = \infty$ \qquad (2.4)

From the definition of f(x) it is clear that $\nabla f(x) = (W + \alpha I)x$ is bounded on a set B and hence there exists an interior point x^* of B such that it is a point of local or global minimum point of $h(x,\beta)$ defined in (2.1).

Now we will develop an algorithm for approximating a solution of

Min: f(x) =$\frac{1}{2}x^T(W + \alpha I)x$

Subject to x_s+ x_t=0

$x_i \epsilon \{-1,1\}, i = 1,2,\dots n$and to find minimum point where every component equal to -1 and 1 from the solution of (2.1) at the limiting value of β which converging to zero.

Let $L(x,\lambda) = h(x,\beta) - \lambda e_{st}^T x$

$$e_{st} = (0,0,\dots.0,1,0,0\dots.0,1,0\dots.0)^T \epsilon R^n$$

For any given barrier parameter β>0 ,the first order necessary optimality condition of (2.1) says that if x is a minimum point of (2.1) then there exists a Lagrange multiplier λ satisfying $\nabla_x L(x,\lambda) = \nabla h(x,\beta) - \lambda e_{st} = 0$

$$e_{st}^T x = 0 \qquad (2.5)$$

For $i \ne s$ or t we have

$$\frac{\partial L(x,\lambda)}{\partial x_i} = \frac{\partial f(x)}{\partial x_i} - \beta\left(\frac{p}{px_i + q} - \frac{p}{q - px_i}\right) = 0$$

$$\Rightarrow \beta p\left[\frac{1}{px_i + q} - \frac{1}{q - px_i}\right] = \frac{\partial f(x)}{\partial x_i}$$

$$\Rightarrow \beta p\left[\frac{-2px_i}{q^2 - (px_i)^2}\right] = \frac{\partial f(x)}{\partial x_i}$$

$$\Rightarrow (q)^2 \frac{\partial f(x)}{\partial x_i} - (px_i)^2 \frac{\partial f(x)}{\partial x_i} = -2\beta p. px_i$$

$$\Rightarrow (px_i)^2 \frac{\partial f(x)}{\partial x_i} - 2\beta p. px_i - (q)^2 \frac{\partial f(x)}{\partial x_i} = 0$$

$$\Rightarrow px_i = \frac{2\beta p \pm \sqrt{4\beta^2 p^2 - 4\frac{\partial f(x)}{\partial x_i}\left(-q^2 \frac{\partial f(x)}{\partial x_i}\right)}}{2\frac{\partial f(x)}{\partial x_i}}$$

$$\Rightarrow x_i = \frac{\beta p \pm \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i}\right)^2}}{p \frac{\partial f(x)}{\partial x_i}} \qquad (2.6)$$

and for i=s or t

$$\frac{\partial L(x, \lambda)}{\partial x_i} = \left(\frac{\partial f(x)}{\partial x_i} - \lambda\right) - \beta \left(\frac{p}{px_i + q} - \frac{p}{q - px_i}\right) = 0$$

We derive $x_i = \dfrac{\beta p \pm \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - \lambda\right)^2}}{p\left(\frac{\partial f(x)}{\partial x_i} - \lambda\right)}$

Let $d_i(x, \lambda) = \begin{cases} \dfrac{\beta p \pm \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i}\right)^2}}{p\left(\frac{\partial f(x)}{\partial x_i}\right)}, & i \neq sort \\[6mm] \dfrac{\beta p \pm \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - \lambda\right)^2}}{p\left(\frac{\partial f(x)}{\partial x_i} - \lambda\right)}, & i = sort \end{cases} \qquad (2.7)$

for i=1,2,…n and $d(x, \lambda) = (d_1(x, \lambda), d_2(x, \lambda), \ldots d_n(x, \lambda))^T$ from the above discussion it shows that $(d_1(x, \lambda) - x$ is a descent direction of $L(x, \lambda)$ if $-\frac{q}{p} < x < \frac{q}{p}$ .Before proving our main theorem we will prove the following lemma.

2.1. Lemma: Assume that $-\frac{q}{p} < x < \frac{q}{p}$

(i) If $d_i(x, \lambda) - x_i < 0$ then $\frac{\partial}{\partial x_i}\left(L(x, \lambda)\right) > 0$

(ii)If $d_i(x, \lambda) - x_i > 0$ then $\frac{\partial}{\partial x_i}\left(L(x, \lambda)\right) < 0$

(iii)If $d_i(x, \lambda) - x_i = 0$ then $\frac{\partial}{\partial x_i}\left(L(x, \lambda)\right) = 0$

(iv)If $d_i(x, \lambda) - x_i \neq 0$ then $\nabla_x L(x, \lambda)^T (d(x, \lambda) - x) < 0$

(v)If $e_{st}^T(d(x, \lambda) - x) = 0$ and $d(x, \lambda) - x \neq 0$ then $\nabla h(x, \beta)^T (d(x, \lambda) - x) < 0$

Proof:Beforeproving these lemma we need to prove that

$\left(\frac{\partial L(x,\lambda)}{\partial x_i}\right) > 0$ if $d_i(x, \lambda) - x_i < 0$

Let $v_i = \begin{cases} 0, & i \neq sort \\ \lambda, & i = sort \end{cases}$

$$i = 1,2, \ldots . n$$

When

$$\frac{\partial f(x)}{\partial x_i} - v_i \neq 0$$

$$\frac{\beta p \pm \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}}{p\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)} = \frac{\left\{\beta p + \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right\}\left\{\beta p - \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right\}}{p\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)\left\{\beta p - \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right\}}$$

$$= \frac{\beta^2 p^2 - \beta^2 p^2 - q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}{p\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)\left\{\beta p - \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right\}}$$

$$= \frac{-q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}{p\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)\left\{\beta p - \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right\}}$$

$$= \frac{-q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}{p\left(\beta p - \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right)} \qquad (2.8)$$

Similarly $\dfrac{\beta p - \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}}{p\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)} = \dfrac{\beta^2 p^2 - \beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}{p\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)\left\{\beta p + \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right\}} = \dfrac{-q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}{\left\{\beta p + \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right\}}$ (2.9)

For $o < x_i < \frac{q}{p}$ it is clear that

$$p^2 x_i^2 - q^2 = -(px_i + q)(q - px_i) < 0$$

and that for any point $x, \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)$ can only be zero ,negative or positive.

Case 1: Considering $\left(\frac{\partial f(x)}{\partial x_i} - v_i\right) = 0$ then $d_i(x, \lambda) = 0$

From $d_i(x, \lambda) - px_i < 0$,we obtain $0 < x_i$

Thus $(q - px_i) < (px_i + q)$

So, $-\left(\frac{p}{px_i + q} - \frac{p}{q - px_i}\right) > 0$

and $\frac{\partial}{\partial x_i}\left(L(x, \lambda)\right) = \frac{\partial f(x)}{\partial x_i} - v_i - \beta\left(\frac{p}{px_i + q} - \frac{p}{q - px_i}\right) > 0$

Case II: Let $\left(\frac{\partial f(x)}{\partial x_i} - v_i\right) < 0$

From $d_i(x, \lambda) - x_i < 0$

Using (2.8) and (2.9) we have

$$0 < \left\{x_i + \frac{\beta p + \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}}{p\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}\right\}\left\{x_i + \frac{\beta p - \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}}{p\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}\right\}$$

$$= \left\{x_i - \frac{q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}{p\left\{\beta p - \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right\}}\right\}\left\{x_i - \frac{q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}{p\left\{\beta p + \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right\}}\right\}$$

$= x_i^2 - \dfrac{q^2 x_i \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}{p\left\{\beta p - \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right\}} - \dfrac{q^2 x_i \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}{p\left\{\beta p + \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right\}} + \dfrac{q^4 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}{p^2\left\{\beta^2 p^2 - \beta^2 p^2 - q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2\right\}}$

$= x_i^2 - \dfrac{q^2 x_i \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}{p}\left\{\dfrac{1}{\beta p - \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}} + \dfrac{1}{\beta p + \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}}\right\} - \dfrac{q^2}{p^2}$

$= x_i^2 - \dfrac{q^2 x_i \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}{p}\left\{\dfrac{\beta p + \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2} + \beta p - \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}}{\beta^2 p^2 - \beta^2 p^2 - q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right\} - \dfrac{q^2}{p^2}$

$$= x_i^2 - \left\{\frac{q^2 x_i \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}{p}\right\}\left\{\frac{2\beta p}{-q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}\right\} - \frac{q^2}{p^2} = x_i^2 - \frac{q^2}{p^2} + \frac{2\beta x_i}{\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)} = \frac{p^2 x_i^2 - q^2}{p^2} + \frac{2\beta x_i}{\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}$$

$$\Rightarrow (p^2 x_i^2 - q^2)\left(\frac{\partial f(x)}{\partial x_i} - v_i\right) + 2\beta x_i p^2 < 0$$

$$\Rightarrow \frac{\partial f(x)}{\partial x_i} - v_i + \frac{2\beta p^2 x_i}{p^2 x_i^2 - q^2} > 0$$

$$because \frac{\partial f(x)}{\partial x_i} - v_i < 0$$

$\Rightarrow \frac{\partial f(x)}{\partial x_i} - v_i - \beta\left[\frac{p}{px_i + q} - \frac{p}{q - px_i}\right] > 0$

$$\Rightarrow \frac{\partial L(x, \lambda)}{\partial x_i} > 0$$

Case III: If we have

$$0 > \left\{ x_i + \frac{\beta p + \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}}{p\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)} \right\} \left\{ x_i + \frac{\beta p - \sqrt{\beta^2 p^2 + q^2 \left(\frac{\partial f(x)}{\partial x_i} - v_i\right)^2}}{p\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)} \right\} = x_i^2 - \frac{q^2}{p^2} + \frac{2\beta x_i}{\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}$$

$$= \frac{p^2 x_i^2 - q^2}{p^2} + \frac{2\beta x_i}{\left(\frac{\partial f(x)}{\partial x_i} - v_i\right)}$$

Thus $0 > (p^2 x_i^2 - q^2)\left(\frac{\partial f(x)}{\partial x_i} - v_i\right) + 2\beta x_i p^2$

Therefore $0 < \frac{\partial f(x)}{\partial x_i} - v_i + \frac{2\beta p^2 x_i}{p^2 x_i^2 - q^2}$

$$= \frac{\partial f(x)}{\partial x_i} - v_i - \beta \left[\frac{p}{px_i + q} - \frac{p}{q - px_i}\right] = \frac{\partial L(x, \lambda)}{\partial x_i}$$

Using this result the rest of this Lemma can be proved similarly.

Now if we will consider all $-\frac{q}{p} < x < \frac{q}{p}$, $d(x, \lambda) - x = 0$ $if and only if \nabla_x L(x, \lambda) = 0$

Since $-\frac{q}{p} < d(x, \lambda) < \frac{q}{p}$ then $d(x, \lambda) - x$ satisfies the desired property and when searching for a point in $(x, \lambda) - x$, the constraint $-\frac{q}{p} < x < \frac{q}{p}$ is always satisfied automatically if the step length is a number between $-\frac{q}{p} and \frac{q}{p}$.

$$Let P(\lambda) = -\left(\frac{q\left(\frac{\partial f(x)}{\partial x_s} - \lambda\right)}{\beta p + \sqrt{\beta^2 p^2 \left(2\left(\frac{\partial f(x)}{\partial x_s} - \lambda\right)\right)^2}} + \frac{q\left(\frac{\partial f(x)}{\partial x_s} - \lambda\right)}{\beta p + \sqrt{\beta^2 p^2 + q\left(\frac{\partial f(x)}{\partial x_s} - \lambda\right)}} \right)$$

Given any point $x \epsilon F$ with $-\frac{q}{p} < x < \frac{q}{p} for d(x, \lambda) - x$ to become a feasible descent direction as $h(x, \beta) = f(x) + \beta b(x)$ and we want to solve $p(\lambda) = 0$

(2.10)

$\lambda(x) = \frac{1}{2q}\left(\frac{\partial f(x)}{\partial x_s} + \frac{\partial f(x)}{\partial x_i}\right)$ is a solution of (2.10)

Since $\{\nabla f(x)/x \epsilon \beta\}$ is bounded hence $\{\lambda(x) : x \epsilon B\}$ is bounded. Using the feasible descent direction $d(x, \lambda(x)) - x$ and $\lambda(x) = \frac{1}{2q}\left(\frac{\partial f(x)}{\partial x_s} + \frac{\partial f(x)}{\partial x_s}\right)$ for updating Lagrange multiplier λ we have developed an algorithm for approximating a solution of

$$min: f(x) = \frac{1}{2}x^T(W - \alpha I)x$$

subject to $x_s + x_t = 0$

$$x_i \epsilon \{-1, 1\}, i = 1, 2, \dots n$$

Let $\beta_m, m = 1, 2, \dots$ be any given sequence of positive numbers such that $\beta_1 > \beta_2 > \cdots$ and $\lim_{m \to \infty} \beta_m = 0$. The value of β_1 should be sufficiently large so that $h(x, \beta_1)$ is convex over $-\frac{q}{p} < x < \frac{q}{p}$.

Let x^0 be an arbitrary non zero interior point of B satisfying $e_{st}^T x^0 = 0$. For m=1,2,... starting at x^{m-1} , we can generate the feasible descent direction $d(x, \lambda(x)) - x$ to search for better feasible interior point x^m satisfying $d(x^m, \lambda(x^m)) = 0$

III. MAIN RESULTS

Using the above lemmas we can prove the following main theorem.

Theorem: For $\beta = \beta_k$ every limit point of $x^k, k = 1, 2, \dots$ generated by $x^{x+1} = x^k + \mu_k(d(x^k, \lambda(x^k)) - x^k)$ is a stationary point of $min\ h(x, \beta) = f(x) + \beta b(x)$ subject to $x_s + x_t = 0$

Proof: $\beta_k, k = 1, 2, \dots$ be any given sequence of positive numbers such that $\beta_1 > \beta_2 > \cdots$ and $\lim_{k \to \infty} \beta_k = 0$. The value of $\beta 1$ should sufficiently large so that $h x, \beta$ is convex over $-qp < x < qp$.

Let x^0 be an arbitrary non zero interior point of B satisfying $e_{st}^T x^0 = 0$ for k=1,2,... starting from x^{k-1}, we derive the feasible descent direction $d(x^k, \lambda(x^k)) - x^k = 0$, if $\|d(x^k, \lambda(x^k)) - x^k\| < 1$

When β is sufficiently small so that a feasible solution which every component equal to either -1 or 1 can be generated by rounding off x^k.

Let $\beta_{k+1} = \theta \beta_k$

$$x^{k+1} = x^k + \mu_k(d(x^k, \lambda(x^k) - x^k))$$

Satisfying $h(x^{k+1}, \beta_k) = \min_{\mu[0,1]} h[x^k + \mu(d(x^k, \lambda(x, k) - x^k, \beta_k)]$

Let $a_{max} = \max_{1 \leq i \leq n} \max_{x \epsilon B} \left(\frac{\partial f(x)}{\partial x_i} - v_i(x) \right)$

with $x^{min} = \left(x_1^{min}, x_2^{min}, \dots x_n^{min} \right)^T$, $x^{max} = (x_1^{max}, x_2^{max}, \dots x_n^{max})^T$, $\mu[0,1]$

$$x^{min} = min \left\{ x_i^0, \frac{-a_{max}}{\beta p + \sqrt{\beta^2 p^2 + \alpha_{max}^2}} \right\}$$

$$x_i^{max} = max \left\{ x_i^0, \frac{-a_{max}}{\beta p + \sqrt{\beta^2 p^2 + \alpha_{max}^2}} \right\}, i = 1,2, \dots n$$

Since $\{\nabla f(x): x \epsilon B\}$ and $\{\lambda(x): x \epsilon B\}$ are bounded then a_{max} is bounded.

Thus $-\frac{p}{q} < x^{min} \leq d(x, \lambda(x)) \leq x^{max} < \frac{p}{q}$

For any $x \epsilon B$ and $x^{min} \leq x^k \leq x^{max}$, $k = 1,2, \dots$

Therefore no limit of $x_i^k, k = 1,2 \dots$ is equal to either -1 or 1 for i=1,2,…n

From the above lemma we obtain $d(x^k, \lambda(x^k) - x^k)$ is a feasible solution of $min\, h(x, \beta) = f(x) + \beta b(x)$ subject to $x_s + x_t = 0$

Let $X = \{x \epsilon F: x^{min} \leq x^k \leq x^{max}\}$

and $\Omega = \{x \epsilon X: d(x, \lambda(x)) - x = 0\}$, for any $x \epsilon X$

Let $A(x) = \{x + \mu^*(d(x, \lambda(x)) - x\}$

$\mu^* \epsilon [0,1], h(x + \mu^*(d(x, \lambda(x)) - x), \beta) = min_{\mu \epsilon [0,1]} h(x + \mu(d(x, \lambda(x)) - x), \beta)$

Now we will prove A(x) is closed at every point $x \epsilon X - \Omega$

Let $\overline{x}$ be any arbitrary point of $X - \Omega$

Let $x^r \epsilon X - \Omega$, $r = 1,2, \dots$ be a sequence convergent to $\overline{x}$ and $y^r \epsilon A(\overline{x})$, r=1,2,…n be a convergent sequence converges to $\overline{y}$.

To prove that $A(\overline{x})$ is closed, we need to show that $\overline{y} \epsilon A(\overline{x})$.

From $x^r \epsilon X - \Omega$ and $\overline{x} \epsilon X - \Omega$ we have $d(x^r, \lambda(x^r)) - x^r \neq 0$ and $(\overline{x}, \lambda(\overline{x})) - \overline{x} \neq 0$, where $d(x, \lambda(x))$ is continous. Thus $d(x^r, \lambda(x^r))$ converges to $d(\overline{x}, \lambda(\overline{x}))$ as $r \to \infty$. Since $y^r \epsilon A(x^r)$ then there exists a number $\mu^* \epsilon [0,1]$ satisfying $y^r = x^r + \mu^* d(x^r, \lambda(x^r)) - x^r \epsilon A(x^r)$

From $(x^r, \lambda(x^r)) - x^r \neq 0$, we obtain

$$\mu_r^* = \frac{\|y^r - x^r\|}{d\|x^r, \lambda(x^r) - x^r\|}$$

And as, $r \to \infty$

$$\mu^* - \overline{\mu^*} = \frac{\|\overline{y} - \overline{x}\|}{d\|\overline{x}, \, (\lambda(\overline{x})) - \overline{x}\|}$$

with $\overline{\mu} \epsilon [0,1]$, $\overline{y} = \overline{x} + \overline{\mu^*}[d(\overline{x})(\lambda(\overline{x})) - \overline{x})]$, since $y^r \epsilon A(x^r)$

we have $h(y^r; \beta) \leq hx^r + \mu(d(x^r, \lambda(x^r)) - x^r), \beta)$ for any $\mu \in [0,1]$.

It implies that $h(\overline{y}, \beta) \leq h(\overline{x} + \mu(d(\overline{x}), (\lambda(\overline{x})) - \overline{x})), \beta)$

For any $\mu \epsilon [0,1]$, which proves that $h(\overline{y}, \beta) = min\, h(\overline{x} + \mu(d(\overline{x}), \lambda(\overline{x})) - \overline{x}), \beta)$
$$\mu \epsilon [0,1]$$

According to the definition of A(x), it follows that $\overline{y} \in A(\overline{x})$, $since X is bounded and x^k \epsilon X$, $k = 1,2, \dots \dots \dots$

Then by Bolzonoweierstrass theorem we can extract a convergent subsequence from the sequence $x^k, k = 1,2, \dots \dots \dots$

Let $x^{k_i}, i = 1,2, \dots \dots \dots$ be a convergent subsequence of the sequence $x^k, k = 1,2, \dots \dots \dots$

Let x^* be the limit point of the sequence. Clearly as $k \to \infty$, $h(x^k, \beta)$ converges to $h(x^*, \beta)$, since $h(x, \beta)$ is continuous and $h(x^{x+1}, \beta) < h(x^x, \beta)$, $k = 1,2, \dots$

Considering the sequence $x_j^{k+1}, j = 1,2, \dots$ and by (2.10)

We can write $x_j^{k+1} = x^{k_i} + \mu_{kj} (d(x^{kj}, \lambda(x^{kj}) - x^{kj})$

and $h(x_j^{k+1}, \beta) = min\, h(x^{kj} + \mu(d(x^{kj}, \lambda)(x^{kj} - x^{ki}), \beta$

According to definition of $A(x)$ we have $x_j^{k+1} \epsilon A(x^{k_i})$, $j = 1,2, \dots$ are bounded, then there exist $x_j^{k+1}, j \epsilon k$ a convergent subsequence. If $x^* \notin \Omega$ and $A(x^*)$ is closed then $x^* \epsilon A(x^*)$ and we have $h(x^*, \beta) < h(x^k, \beta)$

Which contradicts that $h(x^k, \beta)$ converges as $\to \infty$ i.e $x^* \epsilon \Omega$.

The use of logarithmic barrier function finds a minimal point for solving linearity constrained continuous optimization problem to s-t max cut problem.

IV. NUMERICAL EXAMPLE:

In order to establish the effectiveness and efficiency of the algorithm for obtaining optimization,we have solved by using MATLAB.

For our solution we have consider α= 0.000001

β_0 =1-S$_{min}$/2 with S$_{min}$being the minimum eigenvalue of W-αI.

In our computation we have taken following variables.

NI=No. of iteration

OBJM =Objective value of (1.4)

OBJU =Greatest integer value of

$$\max \quad : \frac{1}{4}\sum_{ij} w_{ij}(1 - x_{ij})$$

Subject to trace $(e_{st}e_{st}^T X) = 0$

$$X_{ii} = 1 \ , i = 1,2,\dots n$$

$x \geq 0$, RATIO=(OBJU-OBJM)/OBJU

In the computation we have considered weights w_{ij} are the random integer between 1 to 50 and theresults computed are given in the following table for the value of m=.6

Table for the numerical results for m=.6

No of Test	NI	OBJM	OBJU	RATIO
1	100	35670	35798	.03
2	150	36150	37630	.02
3	200	46260	47350	.02
4	250	20350	21450	.02
5	300	35672	36505	.02

From the above computation it shows that the ratio is nearly equal to .02 and it proves the convergence of the solution to the local minimum point and shows that the algorithm is an effective one.

V. CONCLUSION:

In this paper we have taken a general logarithmic barrier function to solve the continuous optimization problem by transforming it in to s-t max-cutproblem and developed an algorithm is basedon the barrier generalized logarithm the barrier function. The algorithm developed for generatingdecreasing sequence of solution which converges at least a local minimum point of (2.1) with everycomponent equal to either $\frac{-q}{p} to \frac{q}{p}$.

REFERENCES:

[1]. S.Aiyer,M.NiranjanandF.Fallside(1990):A theoretical investigation into the performance of theHopfieldmodel.IEEE Transaction on NeuralNetworks,I,204-205.

[2]. S.J.Benson,Y.YeandX.Zhang(1998):Mixed linear and semidefinite programming for combinatorial and quadratic optimization, Preprint.

[3]. Bout and Miller(1990):Graph partitioning using annealed networks, IEEE Transaction on NeuralNetworks,I, 192-203.

[4]. A.Cichocki and R.Unbehaunen(1993):Neural networks for optimization and signal processing, New York; Wiley.

[5]. C.Dang(2000):Approximating a solution of the s-t max cut problem with a deterministic annealing algorithm,Neural Networks 13,p.801-810.

[6]. R.DurbinandD.Willshaw(1987):An analogue approach to the travelling salesman problem usingan elastic network method, Nature 326,689-691.

[7]. A.Gee,S.AiyerandR.Prager(1993): An analytic framework for Optimizing Neural Networks.Neural Networks 6,79-97.

[8]. A.Gee and R. Prage(1994):Polyhedral combinatories and NeuralNetworks.Neural Computation6,161-180.

[9]. S.GoldandA. Rangarajan(1996):Soft assign versus soft bench-marking in combinatorial optimization. Advances in Neural information processing systems 8, 626-632.(MITpress:Cambridge,M.A.)

[10]. J.HopfieldandD.Tank(1985):Neural Computation of decision in optimization problems.Biologicalcybernetics,52,141-152.

[11]. A.Rangarajan,S.GoldandE.Mjolsness(1996):A novel optimizing network architecture with applications.Neural computation,8,1041-1060.

[12]. P.Simic(1990):Statistical mechanics as the underlying theory of elastic and neuralOptimizations.Networks,1,89-103.

[13]. K.Urahama(1996):Gradient projection network, analog solver for linearly constrained nonlinearprogramming.Neural Computation 6,1061-1073.

[14]. J.Van den Berg(1996):Neural relaxation dynamics. Ph.D. thesis Erasmus University of Rotterdam,TheNetherlands.

[15]. E.Wacholder,J.HanandR.Mann(1989):A Neural network algorithm for the multiple travelingsalesmanproblem,Biological cybernetics 61,11-19.

[16]. F.WaughandR.Westervelt(1993):Analog neural networks with local competition: Idynamicsand stability physical Review E.47,4524-4536.

[17]. W.Walfe, M.ParryandMacmillan's(1994):Hopfield style Neural Networks and the TSP, Proceedings of the 7th.IEEE International Conference on Neural Networks. New York:IEEEpress(pp.-4577-4582).

[18]. L.Xu(1994):Combinatorial optimization neural nets based on a hybird of Lagrange and transformation approaches.Proceeding of the world congress on Neural Networks, San Diego,CA.(pp.-399-404).

[19]. A.Yuille and J.Kosowsky(1994):Statistical physical algorithms that converges.NeuralComputation,6,341-356.