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History of Cosmology

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1.1 NEWTONIAN COSMOLOGY: THEORETICALMODELS

Cosmologists have preferred using relativity as the basis of cosmology. Indeed, pioneering work in theoretical cosmology by Einstein, de Siter, Friedman, Lemaitre, Eddington, etc. was done with in relativistic framework. However, the level at which this text is aimed precludes the use of general relativity. We will therefore revert to Newtonian gravity on grounds of simplicity. Moreover, in 1935, E.A. Milne and W.H. McCrea showed that with suitable reinterpretation, Newtonian gravity does yield models very similar to those of relativistic cosmology. We will follow the treatment of Milne andMcCrea.

1.2 SIMPLIFYING POSTULATE

We shall use two postulates to simplify the above model construction. The first is known as the Weyl postulate and the second, the cosmological principle.

1.2.1 THE WEYLPOSTULATE

Proposed by Hermann Weyl in the early days of relativistic cosmology, this postulate states that the trajectories of a special class of observers, to be identified with galaxies, form a bundle of non-intersecting lines in space-time so that there is a unique line passing through each point in space at any giventime.

Figure:1 illustrates the special kind of motion implied by Weyl"s postulate. In the space-time diagram shown in fig. 1(b), we see the trajectories distributed in a streamlined fashion. No two members intersect. Thus, there is a unique member of the set passing through any given point in space-time. In Fig.1(a) on the other hand, the trajectories are in disordered with intersections permitted. In this case, it is not possible to identify a unique trajectory through each point. Galactic motion approximates to the idealized case of Fig. 1(b). We may identify a unique observer for each galaxy. Such observers are called _fundamental observers_.

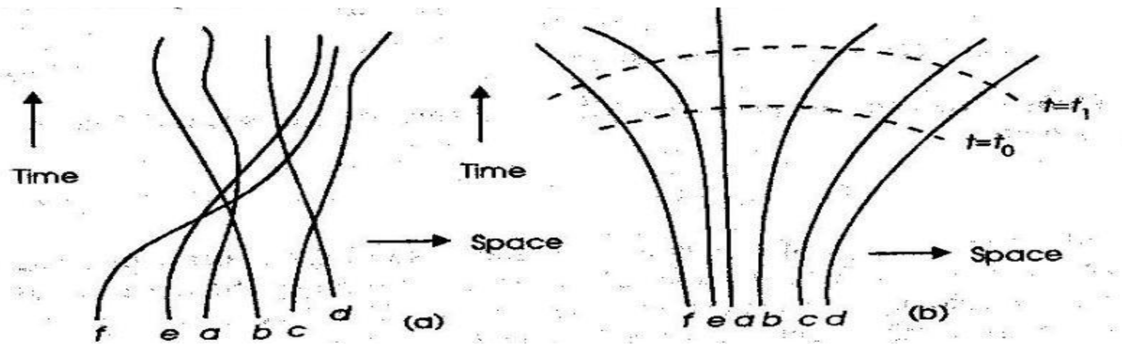


Figure: 1 shows the special kind of motion implied by Weyl"s postulate.

Thus, we may have a continuum of such trajectories of fundamental observers given in the space-time plot with Cartesian coordinates (**r**,t) as

$$\mathbf{r} = \mathbf{F}(\mathbf{t}, \mathbf{r_0}) \quad (1)$$

That is, at any given epoch t, a galaxy identified by the triplet of coordinates $\mathbf{r_0}$is at $\mathbf{r}$ given by (1). The vector function **F** is still to be determined, but it satisfies the non-intersection condition, i.e.,

$$\mathbf{F(t, r_0) = F(t, r_0') \Rightarrow r_0 = r_0'} \qquad \textbf{(2)}$$

1.2.2 THE COSMOLOGICALPRINCIPLE

This principle states that at any epoch t, the universe is homogenous and isotropic. That is, given any position in the universe and any direction in which it is viewed form that position, the large-scale aspect of the universe is the same for all fundamental observers. Let us explore one immediate consequences of this principle. At ant position **r**, the fundamental observer located there moves with a defined velocity givenby

$$\mathbf{V = dr/dt|\ r_0 = \partial F\ (t, r_0)/\partial t = G(t, r),say} \qquad \textbf{(3)}$$

At any epoch, v can be a function of **r** only because, at each point of space there is a unique fundamental observe. Now imagine three observers at $\mathbf{r}_1$, $\mathbf{r}_2$ and at **r**=0. The observer at **r**=0 finds that the velocities of the first two observersare

$$\mathbf{v_1 = G\ (t, r_1),\ v_2 = G\ (t, r_2)} \qquad \textbf{(4)}$$

Hence, viewed by the first of these observers, the second has the velocity

$$\mathbf{v_2 - v_1 = G(t, r_2) - G(t,r_1)} \qquad \textbf{(5)}$$

with respect to him. However, by the cosmological principle, the observer at **r**=0 has no special status. Thus seen by the observer at $\mathbf{r}_1$, the velocity of the second observer should be the same function of their relative vector ($\mathbf{r}_2$- $\mathbf{r}_1$) as in (1). Thatis,

$$\mathbf{v_2 - v_1 = G(t, r_2 - r_1)} \qquad \textbf{(6)}$$

Combining (5) and (6) we get

$$\mathbf{G(t, r_2 - r_1) = G(t, r_2) - G(t,r_1)} \qquad \textbf{(7)}$$

We see that the most form of **G**(t, **r**) is given by the tensor relation

$$\mathbf{G_{\mu\nu}(t, r) = \Sigma\nu A_{\mu\nu}r_\nu\ ;\ \lambda,\ \mu = 1,2,3} \qquad \textbf{(8)}$$

Where **r** = (r_μ) is the triple of Cartesian coordinates describing the position vector of a typical fundamental observer. The magnitude of **r** will be denoted by r. The tensor $\mathbf{A}_{\mu\nu}$ is of second rank tensor it depends on t only. Since the universe looks isotropic from any point, $\mathbf{A}_{\mu\nu}$can‟thave any fundamental direction associated with it. It can therefore only have the isotropic form

$$\mathbf{A_{\mu\nu} = H(t)\delta_{\mu\nu}} \qquad \textbf{(9)}$$

Where **H**(t) is so far an undetermined function of t. From (3) we therefore get

$$\mathbf{v} = \mathbf{H}(t)\mathbf{r} \qquad \textbf{(10)}$$

This is nothing but the velocity-distance relation obtained by Hubble! Thus Hubble‟s law is consistent with our postulate of homogeneity and isotropy : we do not enjoy any „special status‟ by being at **r** =0, say .
We can use (10) complete the integration of the differential equation(3) by writing

$$\mathbf{r} = S(t)\, \mathbf{r}_0 \qquad (11)$$

with

$$\dot{S}/S = H(t) \qquad (12)$$

The overhead dot differentiates the quantity with respect to t. We denote thee present epoch by t0 and write $H_0 = H(t_0)$. The factor S is often called the scale factor as it scales the distances with epoch. Imagine a triangle with vertex coordinates $\mathbf{a}_0$, as S(t) $\mathbf{a}_0$, S(t)$\mathbf{b}_0$ and S(t)$\mathbf{c}_0$with $S(t_0)$ =1. If S(t) increases with t, our triangle is expanding. As we shall discover shortly, this happens to be thesituation.

1.3 COSMOLOGOCAL MODELS

We now introduce dynamics into our framework to calculate the form of S(t). The first and simplest class of models involves "dust" as the main component of the universe. By dust we mean pressure less fluid, no random component built into it. Thus we have a typical fluid element containing density ρ of matter with a bulk velocity **v**, given by the Hubble law

$$\mathbf{v}=H(t)\mathbf{r}, H(t)= \dot{S}/S \qquad (13)$$

the continuity equation of fluid mechanics then gives

$$\partial\rho/\partial t + \mathrm{div}\,(\rho\mathbf{v}) = 0$$

But, from (13), div **v** = 3H(t) while $\nabla\rho = 0$; which leadsto

$$\partial\rho/\partial t + 3(\dot{S}/S)\rho = 0$$

i.e., $$\rho S^3 = \text{constant} = \rho_0 S_0^3 \text{(say)} \qquad (14)$$

This is the density dilution during adiabatic expansion. Next we consider the Euler equations for fluid dynamics :

$$\rho\, [\partial\mathbf{v}/\partial t + (\mathbf{v}.\nabla)\mathbf{v}] = -\nabla p + \rho\mathbf{F} \qquad (15)$$

where p is the pressure and **F** is the external force per unit mass on the fluid element. In our case it is gravitational and satisfies the relation

$$\nabla.\mathbf{F} = -4\pi G\rho \qquad (16)$$

Substituting (13) in (15) with p =0, we get

$$\{\dot{H}r + H^2 r\} = \mathbf{F} \qquad (17)$$

Taking the divergence of this relation and using the fact that ∇.**r**= 3, We get $$H + H^2 = -\frac{4\pi G p}{3} \tag{18}$$

With **H** = S/S and ρ given by the (14), we finally get the following differential equation for S(t) :

$$\frac{S}{S} - \frac{4\pi G\rho_0 {S_0}^3}{3} \cdot \frac{1}{S^3}$$

i.e., $$S = -\frac{4\pi G\rho_0 {S_0}^3}{3S^2} \tag{18-A}$$

This equation can be easily integrated by multiplied by 2$S$to give

$$S^2 = \frac{8\pi G\rho_0 {S_0}^3}{3}\frac{1}{S} - kc^2 \tag{19}$$

This give us a one-parameter family of differential equation, the parameter k being a positive, zero or negative number (k is dimensionless since we have used the constant c to match the velocity dimension of S). The three kinds of solutions arising out of (19) are shown in the fig:
2. We will briefly discuss these cases.

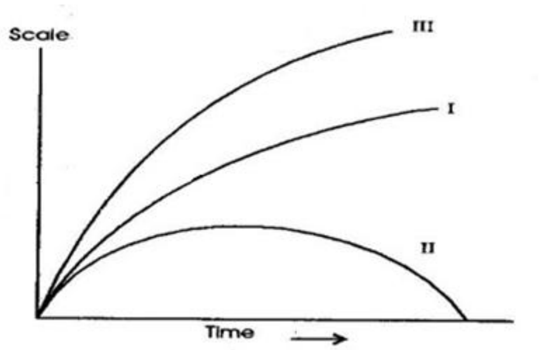


Figure: 2 The three classes of cosmological solutions are denoted by three typical S(t) curves labelled I, II and III.

Case 1 (k = 0): Here (19) has a simple power-law solution

$$S(t) = \left(\frac{t}{t_0}\right)^{2/3} S_0 \tag{20}$$

With the Hubble constant given by

$$H(t) = \frac{S(t)}{S(t)} = \frac{2}{3t}, \quad H_0 = \frac{2}{3t_0} \tag{21}$$

And the matter density by

$$\rho = \frac{3H^2}{8\pi G} \equiv \rho_c \tag{22}$$

For reasons to be discussed later, ρ_c is called the critical density. This model was jointly advocated by Einstein and de Sitter in 1932, and is called the Einstein-de Sitter model.

Case 2 (k > 0): In this case S(t) has a maximum value given by

$$S_{max} = \frac{8\pi G\rho_0 S_0^3}{3kc^2} \quad (23)$$

The universe thereafter contacts. The density at any epoch is given by

$$\frac{\rho_0 S_0^3}{S^3} = \frac{3}{8\pi G}\left(\frac{\dot{S}^2}{S^2} + \frac{kc^2}{S^2}\right) = \Omega\rho_c \quad (24)$$

Where,

$$\Omega = 1 + \frac{kc^2}{\dot{S}^2} > 1 \quad (25)$$

Thus the density parameter Ω exceeds unity for models of this type. We may also introduce the so –called deceleration parameter q defined by

$$q = -\frac{1}{H^2}\frac{\ddot{S}}{S} \quad (26)$$

From (18) we see that

$$q = -\frac{1}{H^2}\left(-\frac{4\pi G\rho_0 S_0^3}{3S^3}\right) = \frac{4\pi G\rho}{3H^2} = \frac{1}{2}\Omega \quad (27)$$

These definitions can be applied to cosmology of class 1, giving Ω =1, q =1/2. For other models, Ω and q are time-dependent and we will denote them by Ω_0 and q_0, the present value of thisparameters.

Case 3 (k < 0): In this case, as for class 1, S steadily increases from zero. We also have and (25) and (27) holding here but now Ω <1 and q <1/2.

If ρ=0 (Ω =0), we have a linearly expanding model

$$S = S_0\frac{t}{t_0} \quad (28)$$

E.A. Milne had arrived at this model in his kinematic relativity. Hence it is sometimes called the Milne model

It is clear from the above discussion that the critical density (Ω =1) separates the ever expanding models (k < 0) form those where the expansion eventually stops and gives way to contraction (k > 0). Hence, the $\rho = \rho_c$ and Ω =1 case is critical in dynamical behavior, but there k has a further signification in terms of the geometry of the space is given by t = constant. For k > 0the space is closed (the surface of a four-dimensional hyper sphere), while for k< 0 the space is open. Thusρ_c is called the closure density; e.g. for $\rho > \rho_c$ the space is closed, for $\rho < \rho_c$ it isopen.

The dynamical feature of Ω is understood within the Newtonian framework in terms of „escape velocity" the relation (19) may be rewritten in the form

$$\frac{1}{2}\dot{S}^2 - \frac{4\pi G\rho}{3S} = -\frac{1}{2}kc^2 \quad (29)$$

For a unit sphere ($\mathbf{r}_0 = 1$), the radius R = $\mathbf{r}_0$S = S and $1/2\dot{R}^2 = 1/2\dot{S}^2$ is the kinetic energy of outward motion of a particle of unit mass comoving with the surface of the sphere. Similarly - 4πGρ/3S = -4πGρ/3R is the potential energy of that particle. Thus, $-kc^2/2$ is the total energy of the particle. The particle „escape" to infinity if k < 0, is trapped if k>0 and is on the borderline for k =0. The expanding universe behaves likewise!

The three types of models described here are commonly known as Friedman models as they were first obtained in 1922-24 by Alexander Friedmann. Friedman's work was, however, in relativistic cosmology. Despite the differences between the Newtonian and relativistic theory of gravity, it is something of a surprise that formally the models derived here by Newtonian methods are the same as the Friedmann models. Even the redshift formula of Newtonian methods agrees with the relativisticformula!

1.4 THE COSMOLOGICALCONSTANT

IN 1917 Einstein had attemped to obtain within the framework of general relativity the theoretical model of a static universe. In this he, at first, did not succeed. The reason is appaent from our dynamical equation (18) which does not admit a solution with S=0, S=0, S =constamt. To get round the diifficulty, Einstein added an extra term called the „λ –term‟ to his equations, where λ is the constant known commonly as a cosmological constant.

In 1917, nebular redshift not regarded as universally established (remember Hubble‟s constant came in 1929); soEinstein‟s desire to have a static model is understandable. The aditional term he introduced had negligible ffect on terrestial; or even galactic gravity: it became significant only at the cosmological level. We shall shortly see why. Later, when the expanding universe concept gained currency and the 1922 models of Friedmann became relevant, Einstien realized that the λ-term was not necessary after all. He therforeretrcted it as „the gretest blunder‟ in his life. Nevertheless the term has survived largely because several astronomers and physicist have found it attractive for various reasons. We will therefore briefly discuss it here even though we are using Newtonian framework.

The λ –term corresponds to a radial force of repulsion between two masses that varies in proportion to the distance between them. Thus, fortwo particle A and B separatede by two vector r, B will be repelled by a force 1/3λ**r** per unit mass form A and vice versa. Therefore (14) gets modifiedto

$$\rho \frac{\partial v}{\partial t} + v.\nabla v = -\nabla P + \rho + \frac{1}{3}\lambda\rho r \quad (30)$$

And equation (18) changed to

$$\ddot{S} = -\frac{4\pi G\rho_0 S_0^3}{3S^2} + \frac{1}{3}\lambda S \quad (31)$$

Alternatively, we may modify equation (15) by adding λ to the right hand side. Similarly, instead of (19) we have

$$\dot{S}^2 = \frac{8\pi G\rho_0 S_0^3}{3S} - kc^2 + \frac{1}{3}\lambda S^2 \quad (32)$$

Now we see that it is possible to solve (31) and (32) with the requirement that

$$S = S_E = Constant \quad (33)$$

Writing

$$\rho_E = \frac{\rho_0 S_0^3}{S_E^3} \quad (34)$$

We get relevant equations as

$$-kc^2 + \frac{1}{3}\lambda S_E^{\,2} + \frac{8\pi G}{3}\rho_E S_E^{\,2} = 0 \qquad (35)$$

$$-\frac{4\pi G}{3}\rho_E S + \frac{1}{3}\lambda S_E = 0 \qquad (36)$$

These are easily solved to give

$$\lambda = 4\pi G\rho_E, \qquad kc^2 = S_E^2 \qquad (37)$$

In the words, k >0. The undetermined constant S_E can be fixed by setting k =1. Thus the radial size of the universe is related to the density through the fundamental constants λ and G.

In relativistic cosmology also, Einstein found the corresponding answer : that the universe is closed, with a finite volume. Einstein liked the fact that in his model the radius of the universe was determined by the density of matter in it, in a clear demonstration that the geometry of space is uniquely related to the matter occupying it.

The „Einstein universe", as the model came to be known, did not long enjoy a unique status in cosmology as its creator had hoped. In 1917, a few months after Einstein"s result, W. de Sitter found another solution of equations (31) and (32):

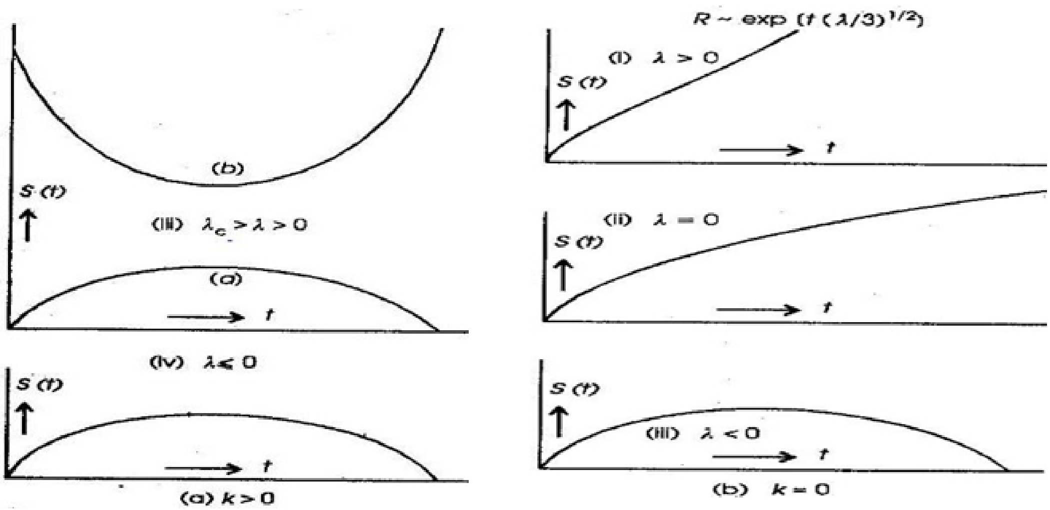
$$\propto e^{Ht}, \ \rho_0 = 0, \ k=0, \qquad (38)$$

Where

$$\lambda = H^2 \qquad (39)$$

This universe expands forever, exponentially, but is empty. The de Sitter universe describes motion without matter in contrast to the Einstein universe which has matter without motion.

What about more general solutions? Looking at (32) we see that there are basically two parameters, k and λ. For k greater than, equal to or less than 0, we have different dynamical behaviour for different λ. Figures 2.3 (a-c) illustrate them.



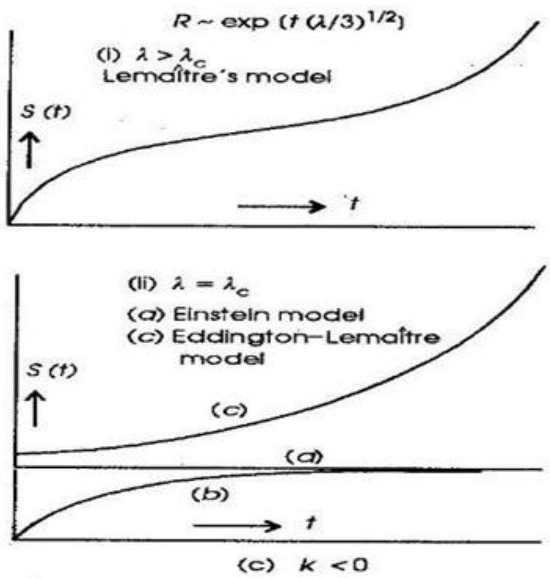


Figure: 3 The three cases --- k > 0, K < 0 --- are illustated by S(t) curves for different values of λ in hte respective figure (a), (b) and (c)

Figure 3(c) shows the interesting case advocated by Eddington and Lemaitre. In Lemaitre"s version, λ is very close to but slightly greater than the critical value for the Einstein universe. The model has the universe expanding form S=0, coasting along close to S= S_E for a considerable period and then expanding away. During the coasting period, the universe in the pseudo-Einstein state while for the asymptotic future it is in the De Sitter State. Eddington felt that an Einstein universe would be unstable and expand, being so triggered by the process for forming galaxies. Fro, if galaxies form by gravitational condensation of matter, the process is helped if the universe is static or near static rather than expanding.

1.5 SPACE-TIME SINGULARITY

The Friedmann models and the λ- cosmologies in general have the common feature that S becomes zero at some epoch. In Newtonian cosmology this implies a state of infinite density and possibly infinite temperature if we could extend our dust model to those where in pressure also matters. This is an unphysical state of affairs but it gets worse in the corresponding relativistic models wherein also the S=0. Space-time was singular at this epoch.

It is usual to call this singular epoch the of big bang, a phrase coined by Fred Hoyle, and these models are often referred to as the „big bang models" . Some general theorems tell us that under normal physical conditions the big bang-type singular situation is unavoidable in relativity. In Newtonian cosmology, the state of infinite density does not imply space-time singularity because the close relationship between physics and geometry is not resentthere.

In a steady state universe the Hubble constant

$$Ht = \frac{S}{S} \quad (40)$$

Must be a constant, so that

$$S \propto \exp H, H = Constant \quad (41)$$

The density ρ is also a constant in this universe. The PCP, however, is not able to determine ρ in terms of H and other physical properties of the universe since it lacks a quantitative dynamical theory. Bondi and Gold argued that the observations of the local universe together with the PCP are sufficient to determine the physical features of the universe anywhere at epoch without a dynamical theory.

The constancy of ρ despite expansion means that matter must be continually created at a rate

$$Q = 3H_\rho \quad (42)$$

What is the physical mechanism of creation?

Hoyle tackled this question in his independent approach to the steady state theory. He proposed a scalar creation field of cosmological nature that interacts with matter at the time of creation. The creation field has negative pressure and negative energy density. We will not go into the details of the approach here. However, the currently popular inflationary universe to be discussed in the following chapter has considerable similarity with the above picture.

In 1933, Hoyle, G. Burbidge and the author had proposed a variation of the original steady state theory in which the universe has a scale factor of the kind

$$= e^{tP}\{1+ \alpha cos\theta(t)\} \quad (43)$$

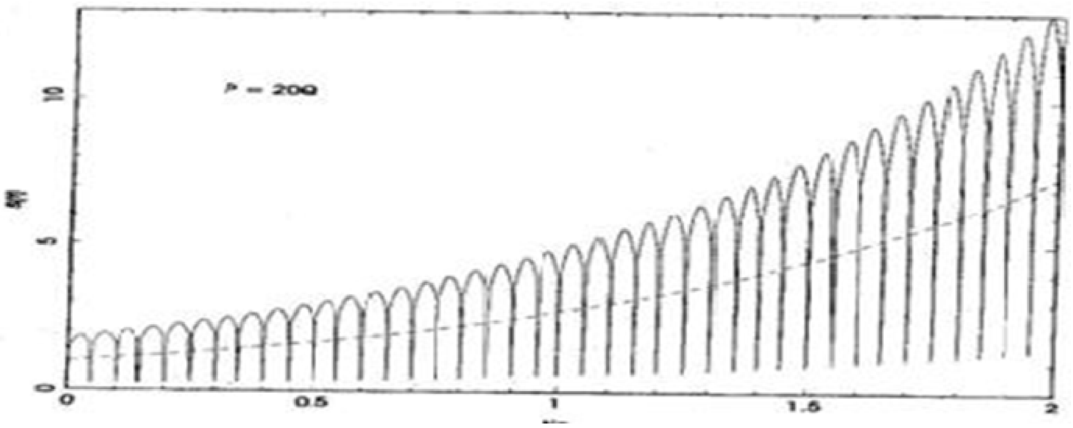


Figure: 4(a) The scalar factor for the quasi-steady state cosmology. Here we see the long termbehavior.

Here P is a constant and the constant parameter α satisfies 0 < α < 1. The function θ(t) isdeterminebythedynamicofthereationprocessandisperodicwithperiodQsuchthatθ(0) = 0, θ(Q) = 2π. This cosmology is called the quasi-staedy state cosmology.

The creaion process in the model is preodic, with a stop-go character which causes the universe to oscillate around an average that increase exponentialy with time. The characterstic period P of exponentialy is very large (say P ≈ 20Q) compared to the periodic Q of oscillation. Since $|\alpha| < 1$, the universe is non-singular. Figure 4(a) gives the long term scale factor of this cosmology. Notice that in Fig. 4(b) we have the he possibility of some sources being blueshifted.

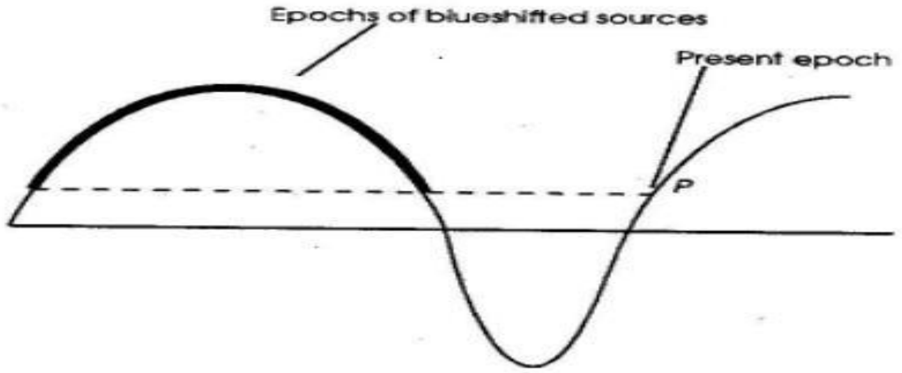


Figure: 4(b) The scalar factor for the quasi-steady state cosmology. Here we see a typical oscillation. If the present epoch is denoted by P, the thick part of the previous oscillaton denotes the blueshifted sources.

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