

Path Loss Prediction by Robust Regression Methods

¹T. E. Dalkilic, ²K. S. Kula, ³B. Y. Hanci

¹Karadeniz Technical University, Faculty of Sciences, Department of Statistics and Computer Sciences,
Trabzon, Turkey.

²Ahi Evran University, Faculty of Sciences and Arts, Department of Mathematics, Kirsehir, Turkey.

³50 Kersey Crescent, RG141SZ, Speen, Newbury, UK.,

ABSTRACT.

Estimating the position of mobile terminals is an important problem for cellular networks. One of the methods of locating the mobile terminal is to use measurements of the radio path loss. This paper presents the results of robust regression methods for the prediction of path loss in a specific urban environment. Since the data set using in the application has outlier robust regression methods are used prediction of the path loss model. The performance of the path loss model which is obtained from robust regression methods are compared to Bertoni-Walfisch model, which is one of the best studied for propagation analysis involving buildings. This comparison based on the mean square error between predicted and measured values. In this study, propagation measurements were carried out at 900 MHz band in the city of Istanbul, Turkey.

KEY WORDS: Path Loss Model, Anfis, Robust Regression, Prediction

I. INTRODUCTION

Cellular mobile communication is the field of wireless communication which gets most attention and improves fast. The combination of flexibility of radio communication and digital transmission quality has an important role in the success of this system. Global Systems for Mobile Communications (GSM) has become the only global and fastest growing system standard for mobile communication in the world. Because, it is the system whose standards accepted by whole world and have the highest number of users. Communication between mobile unit and system is provided with base stations. One of the most important criteria in system design is that spread of radio sign transmitted from the transmitter antenna which is located on the base station to the mobile unit should be modeled. Walfisch and Bertoni, have study a theoretical model that encounters the effects of buildings on radio propagation. This model assumes that building heights and separation between buildings are equal [1]. Bertoni Walfish model is improved by Chrysanthou and Bertoni [2]. In the model, the effects of the difference of height and structures of buildings to the sign spread are given. In the study of Cerri G. it was studied on feed forward neural networks for path loss prediction in urban environment [3]. In the study of Ileana, neural network models for path loss prediction are comparison [4]. Xia, H.H. [5] a simplified analytical model for predicting path loss in urban and suburban environments was proposed. There are many studies on the usage of the adaptive network for parameter prediction. In the study of Chi-Bin, C., and Lee, E. S. it was studied on fuzzy adaptive network approach for fuzzy regression analysis [6]. Jhy-Shing R. J. studied on the adaptive networks based on fuzzy inference system [7]. In the study of Erbay, D.T., and Apaydin, A., adaptive network is used to parameter estimation where independent variables come from an exponential distribution [8]. Fuzzy adaptive network used for estimating the unknown parameters of regression model is based on fuzzy if-then rules and fuzzy inference system. In regression analysis, data analysis is very important. Because, every observation may be has large influence on the parameters estimates in regression model. When data set has outlier, robust M methods (Huber, Hampel, Andrews and Tukey), are used parameters estimates. In this study the path loss predictions are obtained by robust regression methods. The predictions from robust regression methods are compared with predictions from the model which is proposed by Bertoni-Walfisch path loss model. The Bertoni-Walfisch model is the most suitable theoretical model for the Cağaloğlu region, because of this model is consider the buildings database. The remainder of the paper is organized as follows. Section II introduces the measurements and the path loss models. In the Section III robust regression methods for path loss prediction are presented. In the Section IV the path loss model for real data collected from Cağaloğlu, which is urban area in Istanbul (Turkey), is obtain via robust regression methods. In Section V, a discussion and conclusion are provided.

II. THE MEASUREMENTS AND THE PATH LOSS MODELS

To optimize the most suitable propagation model the measurements are very important. The measurement equipments consist of the transmitter and the receiver. The narrow band CW (Continuous Wave) transmitter, which can be tuned to a specific test frequency, was used together with an omni antenna. The antenna was installed on the rooftops. In order to decrease cable losses, the transmitter was located near the antenna. The receiver is a high speed GSM scanner, with Walkabout data collection software from Saftco Technologies. The measurements were carried out at an approximate speed of 40 km/h, while the receiving antenna was at a height of 1.5 m from the ground. The receiver was moved through a variety of urban environments. The measurements data was recorded every 250 m. The route length and the number of points were approximately 161 km and 644 respectively. The signal level enrollments are collected from along the streets which are between the base station antenna and the mobile station antenna. A variety of experimentally or theoretically based models have been developed to predict radio propagation in land mobile system in the literature. Walfisch and Bertoni have published a theoretical model that encounters the effects of buildings on radio propagation [1]. In this study, Bertoni-Walfisch model will be used to comparison, because of this model is take into consideration the buildings between the antennas.

2.1. A. Bertoni - Walfisch Model. Bertoni - Walfisch proposed a semi - empirical model that is applicable to propagation through buildings in urban environments. The model assumes building heights to be uniformly distributed and the separation between buildings are equal. Propagation is then equated to the process of multiple diffractions past these rows of buildings. The expression of the Bertoni-Walfisch path loss model is in dB,

$$PL_{B-W} (dB) = 89.5 + 21 \log f + 38 \log(d) - 181 \log(h_t - h_b) + A_b \tag{2.1}$$

where,

h_r , is the mobile station antenna height,

h_b , building height,

d_c , is the center-to-center spacing of the rows of the buildings,

f , is the frequency MHz

h_t , is the base station antenna height [1].

The influence of building geometry is contained in term

$$A_b = 5 \log \left[\left(\frac{d_c}{2} \right)^2 + (h_b - h_r)^2 \right] - 9 \log d_c + 20 \log \left\{ \tan^{-1} [2(h_b - h_r) / d_c] \right\} \tag{2.2}$$

III. ROBUST REGRESSION METHODS FOR PATH LOSS PREDICTION

In the determining of test statistics and coefficients, the role of each observation must be taken into consideration. The data details must also be tested, because the results of the parameter estimation may be related to an observation, and removal of this observation from the data may change the result of the analysis. This kind of observation, which has a bigger residual value than the others, is called an outlier. In the event of an outlier value, robust methods are used that are less affected than the least square method (LSM) during the estimation of the regression model. In this section, we provide definitions of robust M methods and adaptive network based fuzzy inference systems which are commonly used in the literature [9, 10, 11, 12,13].

3.1. M methods. M method is minimizing the residual function. Regression coefficients $\hat{\beta}_j$ are obtained by minimizing the sum

$$\sum_{i=1}^n \rho \left[\left(y_i - \sum_{j=1}^p x_{ij} \hat{\beta}_j \right) / d \right] \tag{3.1}$$

where $x_i = (x_{i1}, x_{i2}, \dots, x_{ip})$ are independent variables, y_i is depend variable and

$$d = \text{median} \left| r_i - \text{median} (r_i) \right| / 0.6745, r_i \text{ is the } i^{\text{th}} \text{ observed error.}$$

By taking the first partial derivative of the sum in Eq. (3.1) with respect to each $\hat{\beta}_j$ and setting it to zero, the regression coefficients are obtained from Eq. (3.2) [9, 10, 11, 12, 13].

$$\sum_{i=1}^n x_{ij} \psi \left[\left(y_i - \sum_{j=1}^p x_{ij} \hat{\beta}_j \right) / d \right] = 0 \tag{3.2}$$

3.1.1. Huber. Huber’s ρ function is defined as

$$\rho(z) = \begin{cases} \frac{z^2}{2} & |z| \leq k \\ k|z| - \frac{k^2}{2} & |z| > k \end{cases} \tag{3.3}$$

$$z = r_i / d$$

$$d = \text{median} \left| r_i - \text{median} (r_i) \right| / 0.6745$$

where k is called the tuning constant and k is set at 1.5 and r_i is the i^{th} observed error. Sometimes the numerator of d is called the median of the absolute deviations (MAD). The following function is obtained by taking the derivative of Eq. (3.3).

$$\psi(z) = \begin{cases} -k & z < -k \\ z & |z| \leq k \\ k & z > k \end{cases} \tag{3.4}$$

The function Ψ is the derivative of ρ . They are typically set up such that large residuals will be given only marginal or zero Ψ weights in Eq. (3.4). So Ψ is often labeled as "re-descending to zero" [9, 10, 11, 12, 13].

3.1.2. Hampel. The Hampel Ψ function is defined as

$$\psi(z) = (\text{sign } z) \begin{cases} |z| & 0 \leq |z| \leq a \\ a & a \leq |z| \leq b \\ a \left(\frac{c - |z|}{c - b} \right) & b \leq |z| \leq c \\ 0 & c \leq |z| \end{cases} \tag{3.5}$$

The constant values are selected as $a = 1.7$, $b = 3.4$ and $c = 8.5$ in general [9, 10, 11, 12, 13].

3.1.3. Andrews. Andrews (sin estimate) Ψ function is defined as

$$\psi(z) = \begin{cases} \sin(z / k) & |z| \leq k\pi \\ 0 & |z| > k\pi \end{cases} \tag{3.6}$$

where k is taken to be 1.54 or $k = 2.1$ [9, 10, 11, 12, 13].

3.1.4. Tukey. In Tukey’s biweight estimate, the Ψ function is defined as

$$\psi(z) = \begin{cases} z(1 - (z/k)^2)^2 & |z| \leq k \\ 0 & |z| > k \end{cases} \tag{3.7}$$

where k is selected as 5 or 6 [9, 10, 11, 12, 13].

3.2. Fuzzy Inference Systems and ANFIS.

3.2.1. Fuzzy Inference Systems. The fuzzy inference system forms a useful computing framework based on the concepts of fuzzy set theory, fuzzy reasoning, and fuzzy if-then rules. The fuzzy inference system is a powerful function approximator. The basic structure of a fuzzy inference system consists of three conceptual components; a rule base, which contains a selection of fuzzy rules, a database, which defines the membership functions used in the fuzzy rules, and a reasoning mechanism, which performs the inference procedure upon the rules to derive a reasonable output. There are several different types of fuzzy inference systems developed for function approximation. In this study, the Sugeno fuzzy inference system, which was proposed by Takagi and Sugeno [14], will be used. When the input vector X is $(x_1, x_2, \dots, x_p)^T$, then the system output Y can be determined by the Sugeno inference system as

$$R^L : \text{If } (x_1 \text{ is } F_1^L, \text{ and } x_2 \text{ is } F_2^L, \dots, \text{ and } x_p \text{ is } F_p^L), \text{ Then } (Y = Y^L = c_0^L + c_1^L x_1 + \dots + c_p^L x_p),$$

Where F_j^L is fuzzy set associated with the input x_j in the L th rule and Y^L is output due to rule R^L ($L = 1, \dots, m$). The parameters used to define the membership functions for F_j^L is called the premise parameters, and c_i^L are called the consequence parameters. For a real-valued input vector $X = (x_1, x_2, \dots, x_p)^T$, the overall output of the Sugeno fuzzy inference systems a weighted average of the Y^L

$$\hat{Y} = \frac{\sum_{L=1}^m w^L Y^L}{\sum_{L=1}^m w^L} \tag{3.8}$$

where the weight w^L is the truth value of the proposition $Y = Y^L$ and is defined as

$$w^L = \prod_{i=1}^p \mu_{F_i^L}(x_i) \tag{3.9}$$

where $\mu_{F_i^L}(x_i)$ is a membership function defined on the fuzzy set F_j^L .

3.2.2. ANFIS. The Adaptive-Network Based Fuzzy Inference System (ANFIS) is a neural network architecture that can solve any function approximation problem. An adaptive network is a multilayer feed forward network in which each node performs a particular function on incoming signals as well as a set of parameters pertaining to this node and it has five layers [15],[16]. Fuzzy rule number of the system depends on numbers of independent variables and class or fuzzy sets number forming independent variables. When independent variable number is indicated with p , if level number belonging to each variable is indicated with l_i ($i = 1, \dots, p$) fuzzy rule number is indicated with

$$L = \prod_{i=1}^p l_i \tag{3.10}$$

The detail of ANFIS which is used in the path loss prediction is located in [17].

3.2.3. An algorithm to path loss prediction. In this study, the path loss prediction problem has a three-dimensional input. One of them is comes from Gaussian distribution and the others are come from exponential distribution. Because of this condition, there will be used two different membership function, one of them is named Gaussian membership function whose parameters can be represented by parameter set $\{v_h, \sigma_h\}$ and the other one is produced for the inputs which are come from exponential distribution in this study, by the membership function suggested by Erbay, T.D. and Apaydin, A. [8]. The membership function has one parameter which is represented by $\{v_h\}$. The optimal membership function for the exponential distribution function is obtained in the shape of

$$\mu(x_i) = \begin{cases} 2ce^{-\frac{x_i}{v}} & \text{if } x_i > a(c)_i \\ 1 & \text{if } x_i \leq a(c)_i \end{cases} \tag{3.11}$$

where c ($c < 1$) is a constant element and v is a distribution parameter which is called a priori parameter. In the data set derived from the exponential distribution, the limit of the data belonging to the cluster with one membership degree is dependent on the fixed element c and the parameter v , which indicates the distribution. This limit, given with $a(c)$, is described by,

$$a(c) = \max\{0, v \ln(2(1 - c))\} \tag{3.12}$$

and the optimal membership function for Gaussian distrubition is

$$\mu_{F_h}(x_i) = \exp \left[- \left(\frac{x_i - v_h}{\sigma_h} \right)^2 \right] \tag{3.13}$$

Where $\{v_h\}$ is center and $\{\sigma_h\}$ is spread of fuzzy cluster.

The steps of the proposed algorithm for predicted path loss model are as follows:

Step 0: Optimal class numbers related to data set belonging to independent variables are determined. Optimal value of class number l_i , ($l_i=2, l_i=3, \dots, l_i=\max$) can be obtained by minimizing fuzzy clustering validity function S [18]. This function is expressed by

$$S = \frac{\frac{1}{n} \sum_{i=1}^{l_i} \sum_{j=1}^n (\mu_{ij})^m \|v_i - x_j\|^2}{\min_{i \neq j} \|v_i - v_j\|^2} \tag{3.14}$$

where, μ_{ij} are fuzzy membership, v_i cluster center, n observation numbers and m fuzziness index.

Step 1: Priori parameters are determined. Spreading is determined intuitively according to the space in which input variables gain value and to the fuzzy class numbers of variables gain value and to the fuzzy class numbers of variables Center parameters are based on the space in which variables gain value and fuzzy class number and it is defined with

$$v_i = \min(X_i) + \frac{\max(X_i) - \min(X_i)}{(l_i - 1)}(i - 1), \quad i = 1, \dots, p \tag{3.15}$$

Step 2: w^L weights are calculated which are used to form matrix B to be used in counting posteriori parameter set by Eq. (3.11) and Eq. (3.13). The \bar{w}^L sets are the normalizations of the sets which is indicated with w^L .

Step 3: On the condition that the independent variables are fuzzy and the dependent variables are crisp, a posteriori parameter set is obtained as crisp numbers in the shape of, $c_i^L = (a_i^L, b_i^L)$, $c_i^L = a_i^L$. In that condition,

$$Z = (B^T B)^{-1} B^T Y \tag{3.16}$$

equality is used for determining the a posteriori parameter set. Here B is weighted input matrix and, Y and Z defined as

$$Y = [y_1, y_2, \dots, y_n]^T$$

$$Z = [a_0^1, \dots, a_0^m, a_1^1, \dots, a_1^m, a_p^1, \dots, a_p^m]^T$$

Step 4: By using posteriori parameter set c_i^L obtained in Step 3, the system model indicated with

$$f_{4,L} = \overline{w}^L Y^L \tag{3.17}$$

Setting out from the models and weights specified in Step 2, prediction values are obtained with

$$\hat{Y} = \sum_{L=1}^m \overline{w}^L Y^L \tag{3.18}$$

Step 5: Error related to model is counted as

$$\varepsilon = \frac{1}{n} \sum_{k=1}^n \varepsilon_k^2 = \frac{1}{n} \sum_{k=1}^n (y_k - \hat{y}_k)^2 \tag{3.19}$$

If $\varepsilon < \phi$, then posteriori parameter has been obtained as parameters of models to be formed, the process is determinated. If $\varepsilon \geq \phi$, then, step 6 begins. Here ϕ is a law stable value determined by decision maker.

Step 6: Central priori parameters specified in Step 1, are updated with

$$v_i' = v_i \pm t \tag{3.20}$$

in a way that it increases from the lowest value to the highest and decreases from the highest value to the lowest. Here, t is size of step;

$$t = \frac{\max(x_{ji}) - \min(x_{ji})}{a} \quad j = 1, \dots, n \quad i = 1, \dots, p \tag{3.21}$$

and a is stable value which is determinant of size of step and therefore iteration number.

Step 7: Predictions for each priori parameter obtained by change and error criterion related to these predictions are counted with

$$\varepsilon_k = y_k - \hat{y}_k \tag{3.22}$$

The lowest of error criterion is defined. Priori parameters giving the lowest error specified, and prediction obtained via the models related to these parameters is taken as output.

IV. PREDICTION PATH LOSS MODEL

In this section, try to obtain the most suitable path loss model based on the value of signal level in 900 MHz frequency, in Cağaloğlu region. The obtained model will be compare with the Bertoni-Walfisch model. Because, this model is take into consideration the buildings database. To Cağaloğlu region, number of observation is 644, base station antenna height is 16m, route is 161km, the average of the building heights 14.656m, and the average of the center-to-center spacing of the rows of the buildings is 42.7083m. From the result of the residual analysis, the 278th,

335th, 414th, 420th, 427th, 547th, 615th, 639th and 644th, nine observations are outliers. Standardized residuals for these observations are greater than 2.5. Cağaloğlu region is urban area and it has regular building structure. The gabs between buildings along the streets are small. Figure 1

shows the histogram of the center-to-center spacing of the rows of the buildings d_c , the building height h_b , and the α , respectively Figure (1-a), Figure (1-b) and Figure (1-c), which are the independent variables used in the constitute path loss model. This is appearing from histograms, the building heights h_b , have gauss distribution and the center-to-center spacing of the rows of the buildings d_c and the propagation angle α have exponential distribution. Because of that, during the form of the path loss model by ANFIS, the membership function which is expressed in Eq. (3.11) and (3.13) is used. The algorithm which is proposed in section five was operated with a program written in MATLAB for data set from Cağaloğlu region. This data set has 745 observations. And the fuzzy rules to path loss model based on fuzzy inference system are obtained as

$$\begin{aligned}
 \hat{Y}_1 &= -7250 + 160 x_1 + 2630 x_2 + 10154 x_3 \\
 \hat{Y}_2 &= 4390 - 60 x_1 - 1750 x_2 + 13530 x_3 \\
 \hat{Y}_3 &= -12050 - 30 x_1 + 490 x_2 - 6210 x_3 \\
 \hat{Y}_4 &= 8360 + 10 x_1 - 340 x_2 + 210 x_3 \\
 \hat{Y}_5 &= -240 + 20 x_1 - 850 x_2 - 76350 x_3 \\
 \hat{Y}_6 &= 520 - 0.7 x_1 + 4301 x_2 - 18540 x_3 \\
 \hat{Y}_7 &= 3470 - 1.1 x_1 - 110 x_2 + 6060 x_3 \\
 \hat{Y}_8 &= -1500 - 0.7 x_1 + 50 x_2 + 440 x_3
 \end{aligned}
 \tag{4.1}$$

where

x_1 : the center to center spacing of the rows of the buildings (d_c),

x_2 : the building heights (h_b),

x_3 : the propagation angle (α).

The parameter estimation from M methods and least square method (LSM) are located in Table 1.

	LSM	Huber	Hampel	Tukey	Andrews
$\hat{\beta}_0$	57.7577	58.5896	58.3194	58.7315	58.1706
$\hat{\beta}_1$	-0.1148	-0.1237	-0.1221	-0.1248	-0.1193
$\hat{\beta}_2$	-0.1468	-0.1707	-0.1595	-0.1764	-0.1585
$\hat{\beta}_3$	-218.2631	-217.7969	-218.1587	-218.0822	-218.0103

TABLE 1. Parameter estimation from LSM and M method

The input variable number, which is according to independent variables are three and the fuzzy class number of each input variable is two, which is determinate in initial step in proposed algorithm. And then fuzzy rules number is eight from Eq. (3.10). The comparison of the predictions is based on the error criterion given with Eq. (3.22). The error related to predictions obtained via the models given with Eq. (4.1), which are formed by ANFIS, is found as

$$\varepsilon_{ANFIS} = \frac{1}{n} \sum_{k=1}^n (y_k - \hat{y}_k)^2 = 30.6540$$

The error related to predictions obtained via the model given with Eq. (2.1) which is proposed by Bertoni-Walfisch, is found as

$$\varepsilon_{B-W} = \frac{1}{n} \sum_{k=1}^n (y_k - \hat{y}_k)^2 = 112.1499$$

And the error related to predictions obtained via M methods and the LMS are found as

$$\begin{aligned}
 \varepsilon_{Huber} &= 45.9195 \\
 \varepsilon_{Hampel} &= 45.8920 \\
 \varepsilon_{Andrews} &= 45.9365 \\
 \varepsilon_{Tukey} &= 45.8728 \\
 \varepsilon_{LMS} &= 45.8572
 \end{aligned}
 \tag{4.2}$$

The graphs of errors obtained via proposed algorithm, M methods and Bertoni- Walfisch model are shown in Figure 2. In Figure (2-a), errors from Bertoni-Walfisch model, in Figure (2-b), errors from LSM, in Figure (2-c), errors from fuzzy adaptive network which is related to proposed algorithm in this work, in Figure (2-d) to (2-g) errors from Huber, Hampel, Tukey and Andrews are shown respectively.

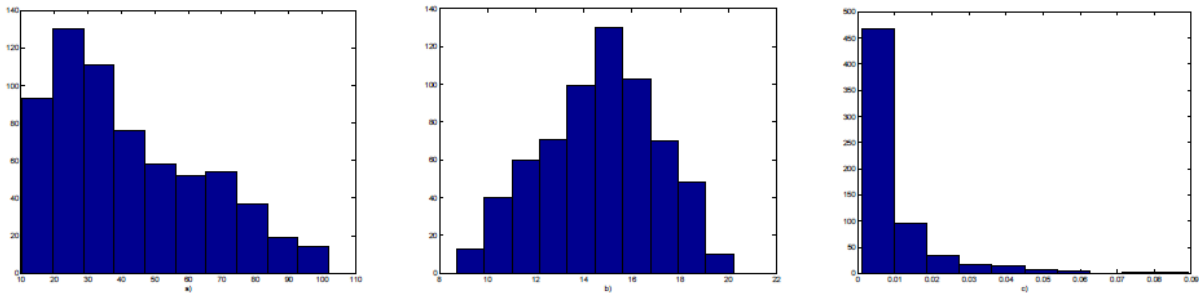


FIGURE 1. (a-b-c) Histograms of the independent variables.

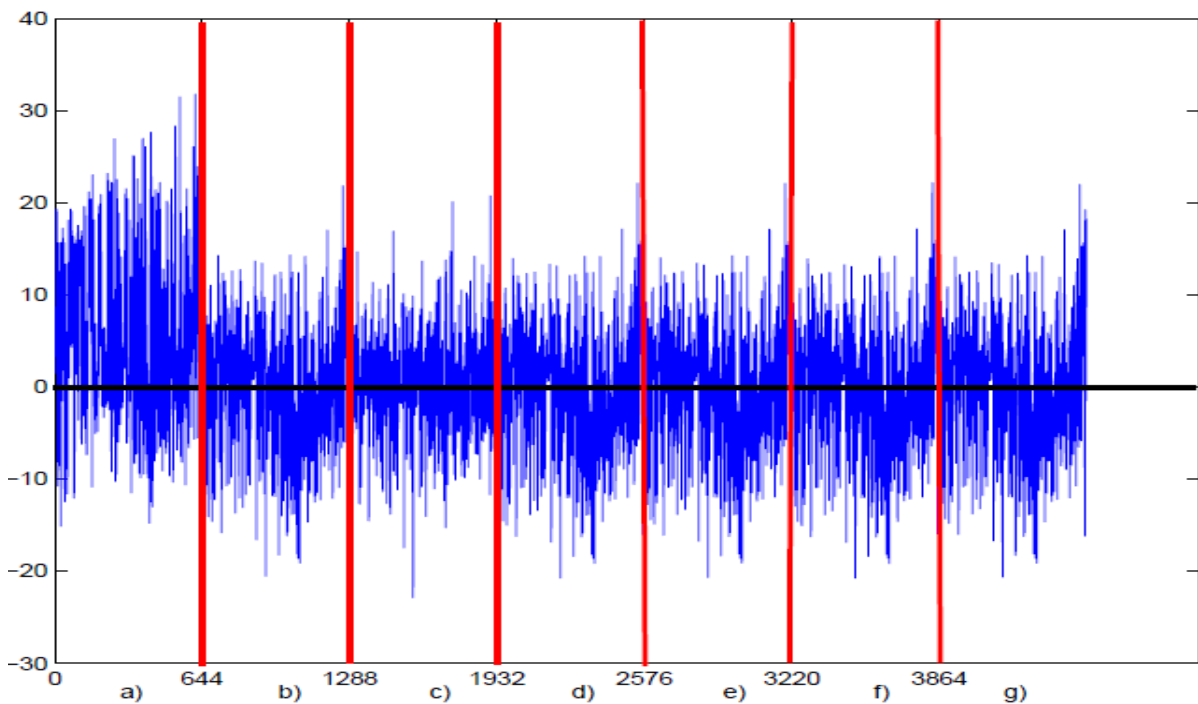


FIGURE 2. Errors from Prediction.

V. CONCLUSIONS

The path loss model prediction for the 900 MHz band is achieved depends on the measurements for Cağaloğlu which is the urban area in Istanbul. For the each measurement which are obtained from 644 different point in Cağaloğlu, the building height (h_b), center-to-center spacing of the rows of the buildings (d_c), propagation angle between base station antenna and mobile station antenna in radian (α) are counted. And they are used as the input variable in the robust regression methods. The Bertoni-Walfisch model is first model which is taking into consideration the effect of the buildings in path loss modeling. Because of the measurements are collecting from the urban are, the predictions from proposed algorithm are compared whit the predictions from Bertoni-Walfisch Model. The predictions from robust regression methods are compared with the Bertoni-Walfisch Model. And according to the indicated error criterion, which is expressed in Eq. (3.22) the errors related to the predictions that are obtained from the robust regression methods are less than the errors that are obtained from the Bertoni-Walfisch Model. The robust methods don't necessitate the equality of the heights and distance of buildings, it can be used for the different areas which have the similar characteristics of the area studied on.

Acknowledgment: The authors would like to thank to the GSM Operator Vodafone Turkey for providing the measurement equipment and locations of transmitter.

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