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# Information Reliability Into The Automated System of Managing

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**ABSTRACT:**
*One of the basic problems into the automated system of managing* **(***ASM) is providing the data reliability. In this paper we will give a model for calculating the data reliability into the ASM and also a model for calculating the complexity of the outputs. The specific roles in ASM have the control feedback connections (CFC-s). If we eliminate them from the oriented data flow graph we get the basic data flow graph, and using CFC-s we construct so called CFC graph.*

***KEYWORDS:*** *data, data flow graph, basic graph, simple graph, control feedback connections graph, data reliability*

## I. INTRODUCTION

The model of ASM is more often presented by oriented bond graph. The model, most commonly aliments subsystems with different functions and includes the increasing (directed from the object of managing to the decision-maker ) and decreasing (directed from decision-maker to the object of managing) connections, shown by so called control feedback connections (CFC-s). However, the CFC-s derive of the insufficient data decomposition either into the single data or into the groups of data, which in fact is really complex, but makes the model free of CFC-s. In our further discussion we will suppose that the oriented bonded graph $G(V,U,\varphi)$ which consists of CFC-s, is already constructed. For the graph $G$ with $n$ vertexes, without parallel edges and loops, we construct the adjacent matrix $A=[a_{ij}]_{n\times n}$ , for

$$a_{ij}=\begin{cases}1, & if\ (v_i,v_j)\in U,\\ 0, & \text{otherwise}\end{cases}$$

Let $G_1=G$ and $A_1=A$. All the elements on the main diagonal of the matrix $A_1$ are equal to 0. And we find the matrix $A_1^2$. If the main diagonal of the last mentioned matrix $A_1^2$ has any non-zero elements, then according to the theorem11.1 [5] the cycle with length 2 belongs to the appropriate vertex. We analyze the founded cycles and delete the edges of all feedback connections, except the control ones. Then, by the edges of CFC-s and the vertexes (identical to the ones in the graph $G_1$ ) we create a new CFC-s graph. So, we get a new graph $G_2$ , with adjacent matrix $A_2$ , and after that find the matrix $A_2^3$. If the main diagonal of the matrix $A_2^3$ consists non-zero elements, then by theorem11.1[5] the cycle with length 3 belongs to appropriate vertex.

We repeat the procedure. The edges of the new identified CFC-s we placed into the CFC graph, and the edges of each feedback connection we remove from the graph $G_2$. So, we get graph $G_3$, with adjacent matrix $A_3$. Continuing the procedure we get the matrixes $A_3^4, A_4^5,..., A_{n-1}^n$ , and on each step we remove the feedback connections and CFC-s place into the CFC-s graph.

So, we discuss about three types of graphs:

- graph $G$ called as data flow graph ,
- graph $G_n$ called as basic graph and
- CFC-graph

In our further discussion we will presume that the vertexes of data flow graph are numerated (labeled) such that if $(v_i,v_j)\in U$ , then $j>i$ for $(v_i,v_j)$ be not edge of the feedback connection, and $j<i$ for $(v_i,v_j)$ be edge of the feedback connection.

## II. RELIABILITY OF OUTPUT DATA

Let consider the graph $G(V,U,\varphi)$ which contains simple cycle $C_{mn}$, and the edge $(v_n, v_m)$ is CFC. This means that when data processing the vertex $v_m$ is input, and the vertex $v_n$ is control one. Let the probability of the event: the data from input to the vertex $v_m$ *till the input to the* $v_n$ *are not deformed,* is equal to $q_s$. Let, the probability of the event that in control vertex the data will be classified as deformed is equal to $q_n$. It means that the probability of the event: into the control vertex the data are not classified as deformed is $p_n = 1 - q_n$ and in this case they come out from $C_{mn}$ and are given to additional processing. We will find the probability of the event: *from the cycle* $C_{mn}$ *will be handed over deformed data*. Clearly, the probability that data are deformed when come into the vertex $v_n$ is given by $1 - q_s$, and $(1-q_s)p_n$ is probability that the data outgo from the cycle $C_{mn}$, and $(1-q_s)q_n$ is probability that through CFC-s the data return into the vertex $v_m$. Further, the probability that after the second data processing, the data will outgo from $C_{mn}$ is $(1-q_s)p_n[(1-q_s)q_n]$, and the probability that the data through CFC-s will be returned into the vertex $v_m$ is $[(1-q_s)q_n]^2$. Moreover, the probability that after the third data processing, the data will outgo from $C_{mn}$ is $(1-q_s)p_n[(1-q_s)q_n]^2$, and the probability that the data through CFC-s will be returned into the vertex $v_m$ is $[(1-q_s)q_n]^3$ and so one. From the other hand, the events

$$B_i : \text{when } i-\text{th passing the deformed data outgo from } C_{mn},\; i = 1,2,3,\ldots$$

are disjoint in pairs. So, the probability of the event

$$B : \textit{from cycle } C_{mn} \textit{ allways outgo deformed data}$$

is

$$P(B) = P(\bigcup_{i=1}^{\infty} B_i) = \sum_{i=1}^{\infty} P(B_i) = (1-q_s)p_n + (1-q_s)p_n[(1-q_s)q_n] + (1-q_s)p_n[(1-q_s)q_n]^2 + \ldots$$

$$= (1-q_s)p_n\{1 + (1-q_s)q_n + [(1-q_s)q_n]^2 + \ldots\} = \frac{(1-q_s)p_n}{1-(1-q_s)q_n}.$$

According to this, the probability of the opposite event

$$\overline{B} : \textit{from the cycle } C_{mn} \textit{ do not come out deformed data}$$

is

$$q(C_{mn}) = P(\overline{B}) = 1 - P(B) = 1 - \frac{(1-q_s)p_n}{1-(1-q_s)q_n} = \frac{1-(1-q_s)q_n-(1-q_s)p_n}{1-q_n+q_sq_n} = \frac{1-(1-q_s)(q_n+p_n)}{p_n+q_sq_n} = \frac{1-(1-q_s)}{p_n+q_sq_n} = \frac{q_s}{p_n+q_sq_n}.$$

The probability of the event: the data from the input to the vertex $v_m$ till the output to the vertex $v_n$ are not deformed was denoted by $q_s$. Clearly, this probability is equal to the probability of the event: the data are not deformed in any of the vertexes $v_m, v_{m+1}, \ldots, v_{n-1}$ and having on mind that the deforming of the data is independent in each of the vertexes, we get that the events

$$D_k : \textit{the data are deformed into the vertex } v_k,\; k = m, m+1, \ldots, n-1$$

are totally independent, what actually means that the events $\overline{D}_k$, $k = m, m+1, \ldots, n-1$ are totally independent, and so

$$q_s = P(\bigcap_{k=m}^{n-1} \overline{D}_k) = \prod_{k=m}^{n-1} P(\overline{D}_k) = \prod_{k=m}^{n-1} q_k \,. \tag{1}$$

Finally, we get following

$$q(C_{mn}) = \frac{q_s}{p_n + q_sq_n} = \frac{\prod_{k=m}^{n-1} q_k}{p_n + \prod_{k=m}^{n} q_k}\,. \tag{2}$$

According to this, the following lemma is true:

**Lemma1.** If the probability that the data do not deform into $v_x$ is equal to $q_x$ and if the data deforming is independent in any vertex, then the probability $q(C_{mn})$ that from the input in the vertex $v_m$ and output in the $v_n$ from the cycle $C_{mn} : v_m, v_{m+1}, \ldots, v_{n-1}, v_n$ are get undeformed data is given by the formula (2). ■

**Definition 1.** Let $C_{mn}$ be a simple cycle in which the edge $(v_n, v_m)$ be CFC. The ratio

$$J_{KPV} = \frac{q(C_{mn})}{q_s} \tag{3}$$

is called intensity of CFC.

**Remark 1.** a) Since $q(C_{mn})$ is probability that from the cycle $C_{mn}: v_m, v_{m+1}, \ldots, v_{n-1}, v_n$ outgo the data which are not deformed, and $q_s$ is probability that from the input in the vertex $v_m$ till the output from the vertex $v_n$ the data do not deform, the formula (3) gives the power of increasing the probability of getting the reliable data when the CFC-s are replaced from the cycle $C_{mn}$ , i.e. when the edge $(v_n, v_m)$ is replaced from the cycle.

б) The equalities (1), (2) and (3) imply

$$J_{KPV} = \frac{1}{p_n + \prod_{k=m}^{n} q_k}. \tag{4}$$

в) Let $G$ be a given graph of data flow into the ASM. In that graph we identify the simple cycle $C_{mn}$ for which the vertex $v_m$ is incoming, and the vertex $v_n$ is control one. Using the formula (2) we find the probability $q(C_{mn})$ and replace the cycle $C_{mn}$ by the vertex $v_m$, which get new probability that the data are not deformed denoted as following $q_m^{'} = q(C_{mn})$. So, we get a new graph $G'$ for which into the vertex $v_m$ will entrance the same edges as entrance edges in graph $G$, and will exit the same edges which previously exit from the vertex $v_n$. We are repeating the procedure as long as necessary, i.e. till we get the graph without cycles. The graph without cycles in fact means the graph without CFC-s.

## III. COMPLEXITY OF CREATING OUTPUT

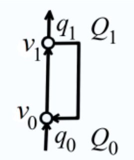
The term of complexity of creating output in terms of the input may be variously defined. For example, as a measure of complexity may be taken:

- the average duration of creating the output
- the average number of operations required to create the output etc.

Clearly, if there is no any cycle in the graph, i.e. there is no CFC-s, then for calculating the complexity of creating the output is enough to find the sum of complexity of each vertex in the graph which in fact form the path from input to the discussed output. But, if the graph consists of CFC-s this procedure for finding the complexity of output does not work.

Let consider the simple cycle shown in picture 1, and there is no any data processing in CFC-s. $Q_0$ denotes the complexity of creating the output from the vertex $v_0$, and $Q_1$ denotes the complexity of data processing into the vertex $v_1$. According to Lemma 1 the probability that output data are not deformed is given by:

$$q(C_{01}) = \frac{q_0}{p_1 + q_0 q_1}. \tag{5}$$



Picture 1

When first going through the cycle, the complexity of creating the output is $Q^{(1)} = Q_0 + Q_1$ and is realized by probability $p^{(1)} = 1 - (1 - q_0)q_1$. When second going through the cycle, the complexity of creating the output is $Q^{(2)} = 2(Q_0 + Q_1)$ and is realized by probability $p^{(2)} = [1 - (1 - q_0)q_1][(1 - q_0)q_1]$, when third going through the cycle, the complexity of creating the output is $Q^{(3)} = 3(Q_0 + Q_1)$ and is realized by probability $p^{(3)} = [1 - (1 - q_0)q_1][(1 - q_0)q_1]^2$ and so one. So, we find discrete random variable

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $Q$ | $Q^{(1)}$ | $Q^{(2)}$ | $Q^{(3)}$ | … | $Q^{(k)}$ | $Q^{(k+1)}$ | … |
| $p$ | $p^{(1)}$ | $p^{(2)}$ | $p^{(3)}$ | … | $p^{(k)}$ | $p^{(k+1)}$ | … |

with mathematical expectation $E(Q)$, which get the expected value of complexity of creating the output from the cycle presented at picture 1. So,

$$E(Q) = \sum_{i=1}^{\infty} Q^{(i)} p^{(i)} = (Q_0 + Q_1)[1-(1-q_0)q_1] + 2(Q_0+Q_1)[1-(1-q_0)q_1][(1-q_0)q_1] +$$

$$+ 3(Q_0+Q_1)[1-(1-q_0)q_1][(1-q_0)q_1]^2 + \ldots$$

$$= (Q_0+Q_1)[1-(1-q_0)q_1]\{1 + 2(1-q_0)q_1 + 2[(1-q_0)q_1]^2 + \ldots\}$$

$$= (Q_0+Q_1)\frac{1-(1-q_0)q_1}{[1-(1-q_0)q_1]^2} = \frac{Q_0+Q_1}{1-(1-q_0)q_1} = \frac{Q_0+Q_1}{1-q_1+q_0q_1} = \frac{Q_0+Q_1}{p_1+q_0q_1},$$

i.e.

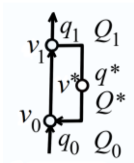
$$E(Q) = \frac{Q_0+Q_1}{p_1+q_0q_1}. \qquad (6)$$

So, we proved the following lemma.

**Lemma 2.** Let $Q_0$ and $p_0$ be the complexity of creating output from the vertex $v_0$ and the probability that data are not deform into the vertex $v_0$, respectively, $Q_1$ and $q_1$, be the complexity of data processing into the vertex $v_1$ and probability that data are not deformed into the vertex $v_1$, respectively. Then the expected value of complexity of creating output from the cycle shown in picture 1 is given by the formula (6). ■

**Remark 2.** If the graph consists simple cycle shown in picture 1, then using formulas (5) and (6) we get the probability $q(C_{01})$ and the expected value of complexity of creating the output from the cycle, after what we replace the cycle $C_{01}$ with the vertex $v_0$. Then, the vertex $v_0$ get the new probability that the data are not deformed $q_0^{'} = q(C_{01})$ and the new complexity of creating the output $Q_0^{'} = E(Q)$. So, we get a new graph in which into the vertex $v_0$ go in the same edges which were incoming for $v_0$ in the origin graph, and outgo the same edges which were outgoing for the vertex $v_1$.

Let the graph $G$ consists of a simple cycle $C: v_0, v_1, v*$, and the input is in the vertex $v_0$, the output for the cycle $C$ is formed into the vertex во темето $v_1$, and the vertex $v*$ denotes the part of CFC where the data are processing (picture 2). Let $q*$ and $Q*$, be the probability that data are not deformed into the vertex $v*$ and the complexity of creating the output from the vertex $v*$, respectively.



Picture 2

The expected value of complexity of creating the output of the cycle shown in drawing 2 is given by the formula

$$E(Q) = Q'p' + Q''p'', \qquad (7)$$

where

- $Q'$ be a complexity of creating the output when the output is formed directly after the first data passing through the vertex $v_0$,
- $Q''$ be the expected value of creating the output when the output is formed directly after the second, the third, … data passing through the vertex $v_0$,
- $p'$ be the probability of the event that data will be handed over from cycle $C$ directly after the first data passing through the vertex $v_0$, and
- $p''$ be the probability of the event that data will be handed over from cycle $C$ directly after the second, the third, … data passing through the vertex.

It is clear that,

$$Q' = Q_0 + Q_1, \quad p' = 1-(1-q_0)q_1 \text{ and } p'' = 1 - p' = (1-q_0)q_1. \qquad (8)$$

Our aim is to calculate the expected value of creating the output $Q''$. First, let note that if the data are not handed over from the cycle $C$ after the first data passing through the vertex $v_0$, the vertex $v*$ is treated as incoming vertex into the cycle, and the complexity of creating the output till the input into the vertex $v*$ is given by the $Q_0 + Q_1$ when added the complexity of creating the output after the first data passing through the vertex $v*$. Lemma 2 directly implies the following:

$$Q'' = Q_0 + Q_1 + \frac{(Q_0+Q_1)+Q*}{p_1+(q_0q_1)q*}. \qquad (9)$$

Finally, by (7), (8) and (9), about the expected value of complexity of creating the output of the cycle shown in picture 2, we get:

$$E(Q) = (Q_0 + Q_1)[1 - (1 - q_0)q_1] + [Q_0 + Q_1 + \frac{Q_0 + Q_1 + Q^*}{p_1 + q_0 q_1 q^*}](1 - q_0)q_1$$

$$= Q_0 + Q_1 + \frac{Q_0 + Q_1 + Q^*}{p_1 + q_0 q_1 q^*}(1 - q_0)q_1,$$

i.e.

$$E(Q) = Q_0 + Q_1 + \frac{Q_0 + Q_1 + Q^*}{p_1 + q_0 q_1 q^*}(1 - q_0)q_1. \qquad (10)$$

Hence, it is easy to find that the probability that undeformed data are get is:

$$q(C) = \frac{q_0(p_1 + q^* q_1)}{p_1 + q_0 q_1 q^*}. \qquad (11)$$

So, we proved the following lemma:

**Lemma 3.** Let $Q_0$ and $p_0$ be the complexity of creating output from the vertex $v_0$ and the probability that data do not deformed into the vertex $v_0$, respectively. $Q_1$ and $q_1$ denote the complexity of data processing into the vertex $v_1$ and the probability that data do not deformed into the vertex $v_1$, respectively, and $q^*$ and $Q^*$, denote the probability that data do not deformed into the vertex $v^*$ and the complexity of creating output from the vertex $v^*$. Then expected value of complexity of creating the output from the cycle shown in drawing 2 is given by formula (10), and the probability that we get the data which are not deformed is given by formula (11).

**Remark 3.** If there is a simple cycle as shown in picture 2, in the cycle, using the formulas (11) and (12) we get the probability $q(C)$ and the expected value of the complexity of creating the cycle outcome and we replace the cycle $C$ by the vertex $v_0$, and the vertex get new probability $q_0^{'} = q(C)$ that the data do not deform and new complexity of creating the output $Q_0^{'} = E(Q)$. Clearly, we get a new graph in which, in the vertex $v_0$ enter each edges which were also incoming for the origin graph, and exit each edges which were outgoing edges for the vertex $v_1$.

## REFERENCES


[1] J. A. Anderson. Diskretna matematika sa kombinatorikom, CET, Beograd, 2005.
[2] T. Harju Lecture notes in graph theory, University of Turku, 2007.
[3] N. Sarapa. Teorija vjerojatnosti, Školska knjiga, Zagreb, 2002.
[4] И. Мирчев. Графи, Неофит Рилски, Благоевград, 2001.
[5] Д. Цветковиќ. Шокароски, Р.: Основи на теорија на графови, Скопје, 1975.
[6] А. Н. Ширяев. Вероятностй, Наука, Москва, 1989.