Neighborhood Triple Connected Domination Number of a Graph

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ABSTRACT:
In this paper we introduce new domination parameter with real life application called neighborhood triple connected domination number of a graph. A subset S of V of a nontrivial graph G is said to be a neighborhood triple connected dominating set, if S is a dominating set and the induced subgraph <N(S)> is a triple connected. The minimum cardinality taken over all neighborhood triple connected dominating sets is called the neighborhood triple connected domination number and is denoted by γ_{ntc}. *We investigate this number for some standard graphs and find the lower and upper bounds of this number. We also investigate its relationship with other graph theoretical parameters.*

KEY WORDS: *Neighborhood triple connected domination number*

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I. INTRODUCTION

By a **graph** we mean a finite, simple, connected and undirected graph G(V, E), where V denotes its vertex set and E its edge set. Unless otherwise stated, the graph G has p vertices and q edges. **Degree** of a vertex v is denoted by d(v), the **maximum degree** of a graph G is denoted by Δ(G). A graph G is **connected** if any two vertices of G are connected by a path. A maximal connected subgraph of a graph G is called a **component** of G. The **complement** $\overline{G}$ of G is the graph with vertex set V in which two vertices are adjacent if and only if they are not adjacent in G. We denote a **cycle** on p vertices by C_p, a **path** on p vertices by P_p, and a **complete graph** on p vertices by K_p. A **wheel graph** W_n of order n, sometimes simply called an n-wheel, is a graph that contains a cycle of order n-1, and for which every vertex in the cycle is connected to one other vertex. A **tree** is a connected acyclic graph. The complete bipartite graph with partitions of order $|V_1|$=m and $|V_2|$=n, is denoted by $K_{m,n}$. A **star**, denoted by $K_{1,p-1}$ is a tree with one root vertex and p – 1 pendant vertices. A **bistar**, denoted by B(m,n) is the graph obtained by joining the root vertices of the stars $K_{1,m}$ and $K_{1,n}$.The **friendship graph**, denoted by F_n can be constructed by identifying n copies of the cycle C_3 at a common vertex. A **helm graph**, denoted by H_n is a graph obtained from the wheel W_n by joining a pendant vertex to each vertex in the outer cycle of W_n by means of an edge.

The **cartesian graph product** $G = G_1 \square G_2$, sometimes simply called the graph product of graphs G_1 and G_2 with disjoint point sets V_1 and V_2 and edge sets X_1 and X_2 is the graph with point set $V_1 \times V_2$ and $u = (u_1, u_2)$ adjacent with $v = (v_1, v_2)$ whenever $[u_1 = v_1 \text{ and } u_2 \text{ adj } v_2]$ or $[u_2 = v_2 \text{ and } u_1 \text{ adj } v_1]$. The m**-book graph** B_m is defined as the graph Cartesian product $S_{m+1} \times P_2$, where S_m is a star graph and P_2 is the path graph on two nodes.

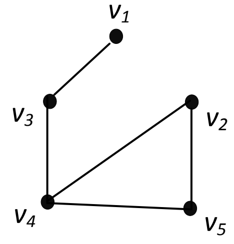
The **open neighborhood** and **closed neighborhood** of a vertex v are denoted by N(v) and N[v] = N(v) ∪ {v} respectively. If S ⊆ V, then $N(S) = \bigcup_{v \in S} N(v)$ and N[S] = N(S) ∪ S.A **cut – vertex (cut edge)** of a graph G is a vertex (edge) whose removal increases the number of components. A **vertex cut**, or **separating set** of a connected graph G is a set of vertices whose removal results in a disconnected graph. The **connectivity** or **vertex connectivity** of a graph G, denoted by κ(G) (where G is not complete) is the size of a smallest vertex cut. The **chromatic number** of a graph G, denoted by χ(G) is the smallest number of colours needed to colour all the vertices of a graph G in which adjacent vertices receive different colour. A **Nordhaus -Gaddum-type** result is a (tight) lower or upper bound on the sum or product of a parameter of a graph and its complement. Terms not defined here are used in the sense of [15].

A subset S of V is called a **dominating set** of G if every vertex in V − S is adjacent to at least one vertex in S. The **domination number** γ(G) of G is the minimum cardinality taken over all dominating sets in G.
Many authors have introduced different types of domination parameters by imposing conditions on the dominating set [19].In[18] Paulraj Joseph et. al., introduced the concept of triple connected graphs. A graph G is said to be **triple connected** if any three vertices of G lie on a path.In[1] Mahadevan et. al., introduced the concept of triple connected domination number of a graph.A subset S of V of a nontrivial graph G is said to be an **triple connected dominating set**, if S is a dominating set and the induced sub graph <S> is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the **triple connected** domination number of G and is denoted by γ_{tc}.In[2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] G. Mahadevan et. al., introduced complementary triple connected domination number, complementary perfect triple connected domination number, paired triple connected domination number, triple connected two domination number, restrained triple connected domination number, dom strong triple connected domination number, strong triple connected domination number, weak triple connected domination number, triple connected complementary tree domination number of a graph, efficient complementary perfect triple connected domination number of a graph, efficient triple connected domination number of a graph, complementary triple connected clique domination number of a graph, triple connected.com domination number of a graph respectively and investigated new results on them.
In[21], Arumugam. S and Sivagnanam. C introduced the concept of **neighborhood connected domination in** graph.
A dominating set S of a connected graph G is called a **neighborhood connected dominating set** (ncd-set) if the induced subgraph <N(S)> is connected. The minimum cardinality of a ncd-set of G is called the **neighborhood connected domination number** of G and is denoted by $\gamma_{nc}(G)$.
Motivated by all the above results and ideas in this paper we introduce new domination parameter called **neighborhood triple connected domination number of a graph.**

II. NEIGHBORHOOD TRIPLE CONNECTED DOMINATION NUMBER OF A GRAPH

Definition 2.1 A subset *S* of *V* of a nontrivial graph *G* is said to be an *neighborhood triple connected dominating set,* if *S* is a dominating set and the induced subgraph *<N(S)>* is triple connected. The minimum cardinality taken over all neighborhood triple connected dominating sets is called the *neighborhood triple connected domination number* of *G* and is denoted by $\gamma_{ntc}(G)$. Any neighborhood triple connected dominating set with γ_{ntc} vertices is called a γ_{ntc} -set of *G*.

Example 2.2 For the graph G_1 in figure 2.1, $S = \{v_1, v_2\}$ forms a γ_{ntc} -set of G_1. Hence $\gamma_{ntc}(G_1) = 2$.



G₁

Figure 2.1 : Graph with γ_{ntc} = 2.

Real Life Application of Neighborhood Triple Connected Domination Number

Suppose we are manufacturing a product and need to distribute the products in different major cities and sub cities so that we give dealership to each city and the dealers in that city distribute our products in to the sub cities. The major cities may or may not be connected. If we draw this situation as a graph by considering the major cities and sub cities as vertices and the roadways connecting the cities as edges, the cities denote the dominating set say S of the constructed graph. If <N(S)> is triple connected in the constructed graph means the customer in the sub cities who needs our product when they did not get our product from the corresponding major cities can get our product either from the other major cities or any one of the other sub cities. And also the minimum cardinality of S minimizes the total cost. The above situation describes one of the real life application of neighborhood triple connected dominating set and neighborhood triple connected domination number of a graph.

Observation 2.3 Neighborhood triple connected dominating set (γ_{ntc} -set or ntcd set) does not exist for all graphs.

Example 2.4 For $K_{1,6}$, there does not exist any neighborhood triple connected dominating set.

Remark 2.5 Throughout this paper we consider only connected graphs for which neighborhood triple connected dominating set exists.

Observation 2.6 The complement of a neighborhood triple connected dominating set S need not be a neighborhood triple connected dominating set.

Example 2.7 For the graph G_2 in the figure 2.2, S = {v_1, v_5, v_6} is a neighborhood triple connected dominating set. But the complement V – S = {v_2, v_3, v_4} is not a neighborhood triple connected dominating set.

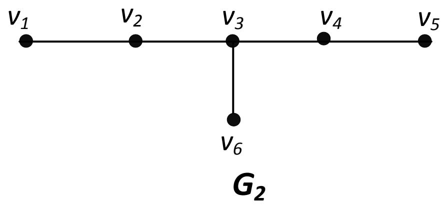


Figure 2.2

Observation 2.8 Every neighborhood triple connected dominating set is a dominating set but not conversely.

Example 2.9 For the graph G_3 in the figure 2.3, S = {v_1, v_2} is a neighborhood triple connected dominating set as well as a dominating set. For the graph H_3 in the figure 2.3, S = {v_3, v_5} is a dominating set but not a neighborhood triple connected dominating set.

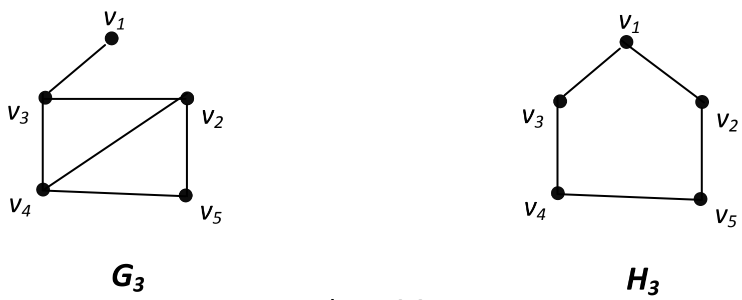


Figure 2.3

Exact value for some standard graphs:

1) For any complete graph of order $p \geq 4$, $\gamma_{ntc}(K_p) = 1$.
2) For any complete bipartite graph of order $p \geq 4$, $\gamma_{ntc}(K_{m,n}) = 2$ where m, n > 1 and m + n = p.
3) For the wheel graph of order $p \geq 4$, $\gamma_{ntc}(W_p) = 1$.

4) For the helm graph H_n of order $p \geq 7$, $\gamma_{ntc}(G) = \frac{p-1}{2}$, where $2n - 1 = p$.

Exact value for some special graphs:

1) The **diamond graph** is a planar undirected graph with 4 vertices and 5 edges as shown in figure 2.5. It consists of a complete graph K_4 minus one edge.

For any **diamond** graph G of order 4, $\gamma_{ntc}(G) = 1$.

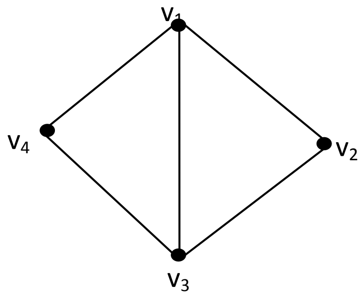
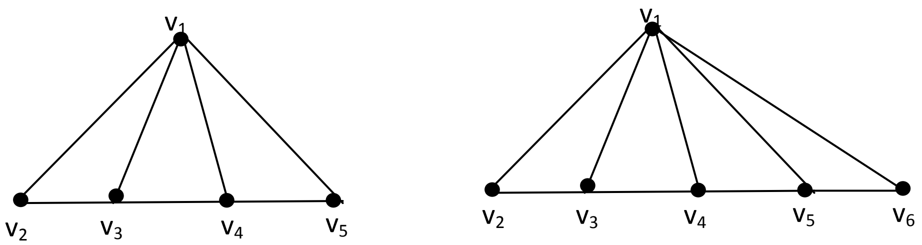


Figure 2.5

In figure 2.5 (a) $S = \{v_1\}$ is a neighborhood triple connected dominating set.

2) A Fan graph $F_{p,q}$ is defined as the graph join $\overline{K}_p + P_q$, where $\overline{K}_p$ is the empty graph on p nodes and P_q is the path graph on q nodes. The case p = 1 corresponds to the usual fan graphs.

For any **Fan** graph of order $n \geq 4$, $\gamma_{cptc}(F_{1,n-1}) = 1$.



(a) (b)

Figure 2.6

In figure 2.6 (a) $S = \{v_1\}$ is a neighborhood triple connected dominating set. And also in figure 2.6 (b) $S = \{v_1\}$ is a neighborhood triple connected dominating set.

3) The **Moser spindle** (also called the **Mosers' spindle** or **Moser graph**) is an undirected graph with seven vertices and eleven edges as shown in figure 2.7.

For the **Moser spindle** graph G, $\gamma_{ntc}(G) = 2$.

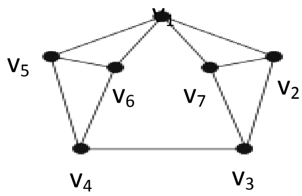


Figure 2.7

In figure 2.7, $S = \{v_3, v_6\}$ is a neighborhood triple connected dominating set.

4) The **Bidiakis cube** is a 3-regular graph with 12 vertices and 18 edges as shown in figure 2.8.

For the **Bidiakis cube** graph G, $\gamma_{ntc}(G) = 4$.

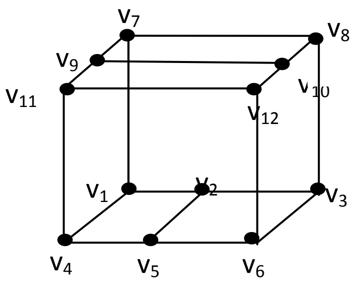


Figure 2.8

In figure 2.8, S = {v_1, v_6, v_{10}, v_{12}} is a neighborhood triple connected dominating set.

5) The **Franklin graph** a 3-regular graph with 12 vertices and 18 edges as shown below in figure 2.9.

For the **Franklin graph** G, $\gamma_{ntc}(G) = 4$.

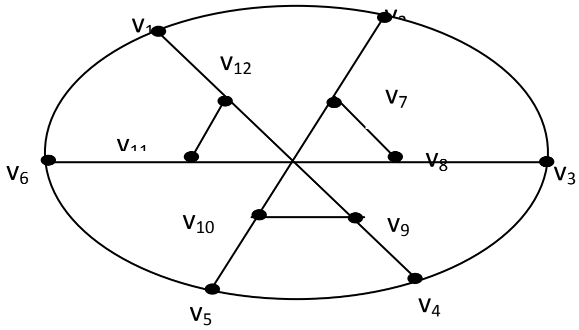


Figure 2.9

In figure 2.9, S = {v_4, v_5, v_7, v_{12}} is a neighborhood triple connected dominating set.

6) The **Frucht graph** is a 3-regular graph with 12 vertices, 18 edges, and no nontrivial symmetries as shown below in figure 2.10. It was first described by Robert Frucht in 1939.

For the **Frucht graph** G, $\gamma_{ntc}(G) = 3$.

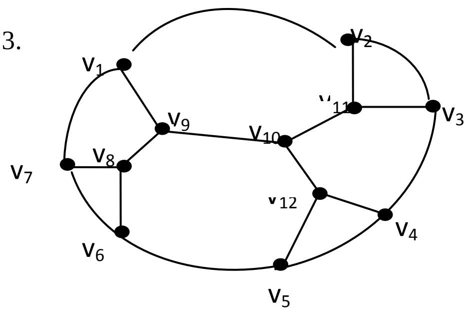


Figure 2.10

In figure 2.10, S = {v_2, v_8, v_{12}} is a neighborhood triple connected dominating set.

7) The **Dürer graph** is an undirected cubic graph with 12 vertices and 18 edges as shown below in figure 2.11. It is named after Albrecht Durer.

For the **Dürer graph** G, $\gamma_{ntc}(G) = 4$.

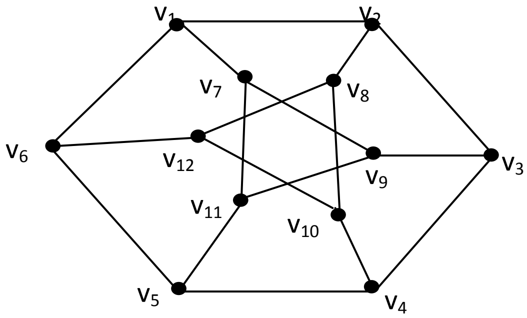


Figure 2.11

In figure 2.11, S = {v_5, v_7, v_8, v_9} is a neighborhood triple connected dominating set.

8) The **Wagner graph** is a 3-regular graph with 8 vertices and 12 edges, as shown in figure 2.12, named after Klaus Wagner. It is the 8-vertex Mobius ladder graph. Mobius ladder is a cubic circulant graph with an even number ‘n’ vertices, formed from an n- cycle by adding edges connecting opposite pairs of vertices in the cycle.

For the **Wagner graph** G, $\gamma_{ntc}(G) = 3$.

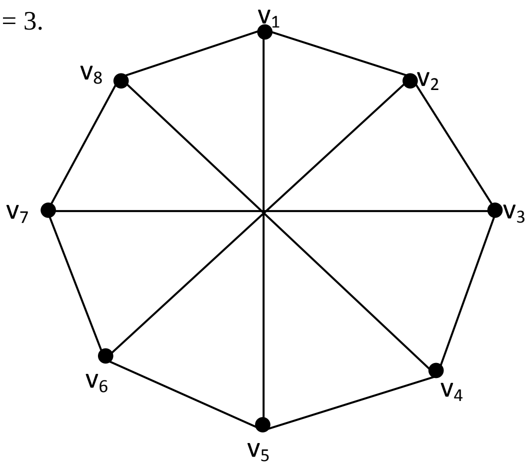
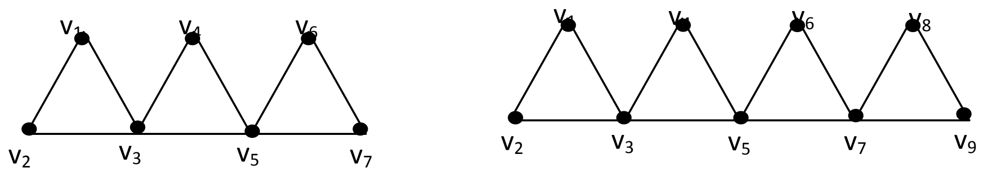


Figure 2.12

In figure 2.12, S = {v_2, v_3, v_4 } is a neighborhood triple connected dominating set.

9) The **Triangular Snake** graph is obtained from a path $v_1, v_2, \ldots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1,2,\ldots,n-1$ and denoted by mC_3 (where m denotes the number of times the cycle C_3) snake as shown in figure 2.13.

For the **Triangular Snake** G, $\gamma_{ntc}(G) = \left\lceil \frac{2m}{3} \right\rceil$.



$\gamma_{ntc}(3C_3) = 2$ $\gamma_{ntc}(4C_3) = 3$

(a) (b)

Figure 2.13

In figure 2.13 (a) S = {v_3, v_5} is a neighborhood triple connected dominating set. In figure 2.13 (b) S = {v_3, v_5, v_7} is a neighborhood triple connected dominating set.

10) The **Herschel graph** is a bipartite undirected graph with 11 vertices and 18 edges as shown in figure 2.14, the smallest non Hamiltonian polyhedral graph. It is named after British astronomer Alexander Stewart Herschel.

For the **Herschel graph** G, $\gamma_{ntc}(G) = 3$..

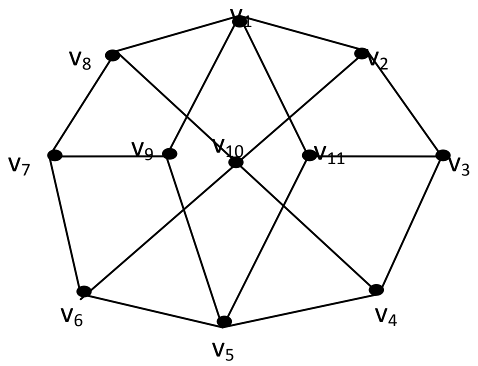


Figure 2.14

In figure 2.14, S = {v_9, v_{10}, v_{11}} is a neighborhood triple connected dominating set.

11) Any cycle with a pendant edge attached at each vertex as shown in figure 2.15 is called **Crown graph** and is denoted by C_n^+.

For the **Crown graph**, $\gamma_{ntc}(C_n^+) = n$.

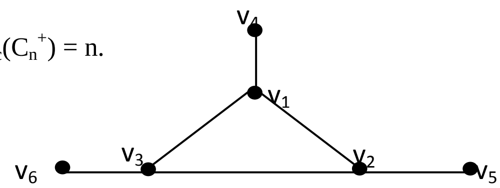


Figure 2.15

In figure 2.15 S = {v_4, v_5, v_6} is a neighborhood triple connected dominating set.

12) Any path with a pendant edge attached at each vertex as shown in figure 2.16 is called **Hoffman tree** and is denoted by P_n^+.

For the **Hoffman tree**, $\gamma_{ntc}(P_n^+) = n$.

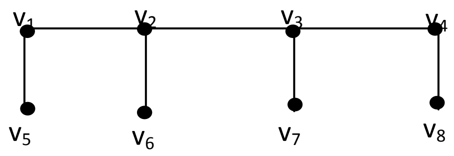


Figure 2.16

In figure 2.16 S = {v_5, v_7, v_7, v_8} is a neighborhood triple connected dominating set.

16) The **Moser spindle** (also called the **Mosers' spindle** or **Moser graph**) is an undirected graph, named after mathematicians Leo Moser and his brother William, with seven vertices and eleven edges as shown in figure 2.17.

For the **Moser spindle graph** G, $\gamma_{ntc}(G) = 3$.

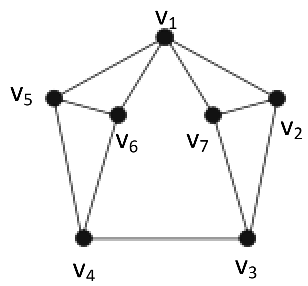


Figure 2.17

Here S = {v_3, v_6} is a neighborhood triple connected dominating set.

Theorem 2.10 For the path of order $p \geq 3$, $\gamma_{ntc}(P_p) = \left\lceil \frac{p}{2} \right\rceil$.

Proof Let $P_p = (v_1, v_2, \ldots, v_p)$. If p is even, then $S = \{v_i : I = 2k, 2k+1$ and k is odd$\}$ is a ntcd – set of P_p and if p is odd, then $S_1 = S \cup \{v_p\}$ is a ntcd – set of P_p. Hence $\gamma_{ntc}(P_p) \leq \left\lceil \frac{p}{2} \right\rceil$. Also if S is a γ_{ntc} – set of P_p, then N(S) contains all the internal vertices of P_p and hence $|S| \geq \left\lceil \frac{p}{2} \right\rceil$. Hence $\gamma_{ntc}(P_p) = \left\lceil \frac{p}{2} \right\rceil$.

Theorem 2.11 For the cycle of order p >3, $\gamma_{ntc}(C_p) = \begin{cases} \left\lceil \frac{p}{2} \right\rceil & if\ p \not\equiv 3 (mod\ 4) \\ \left\lfloor \frac{p}{2} \right\rfloor & if\ p \equiv 3(mod\ 4). \end{cases}$

Proof Let $C_p = (v_1, v_2, \ldots, v_p, v_1)$. If $p = 4k + r$, where $0 \leq r \leq 3$. Let $S = \{v_i : i = 2j, 2j+1$, j is odd and $1 \leq j \leq 2k - 1\}$. Let $S_1 = \begin{cases} S & if\ p \equiv 0(mod\ 4) \\ S \cup \{v_p\} & if\ p \equiv 1\ or\ 2(mod\ 4) \\ S \cup \{v_{p-1}\} & if\ p \equiv 3(mod\ 4). \end{cases}$

Clearly S_1 is a ntcd-set of C_p and hence $\gamma_{ntc}(C_p) \leq \begin{cases} \left\lceil \frac{p}{2} \right\rceil & if\ p \not\equiv 3(mod\ 4) \\ \left\lfloor \frac{p}{2} \right\rfloor & if\ p \equiv 3(mod\ 4). \end{cases}$

Now, let S be any γ_{ntc}*-set of* C_p. Then <S> contains at most one isolated vertex and <N(S)> = $\begin{cases} P_{p-1} & if\ p\ \not\equiv 3(mod\ 4) \\ C_p & if\ p\ \equiv 3(mod\ 4). \end{cases}$

Hence <S> $\geq \begin{cases} \left\lceil \frac{p}{2} \right\rceil & if\ p\ \not\equiv 3(mod\ 4) \\ \left\lfloor \frac{p}{2} \right\rfloor & if\ p\ \equiv 3(mod\ 4). \end{cases}$

The results follows.

Theorem 2.12 For any connected graph *G* with $p \geq 3$, we have $\left\lceil \frac{p}{\Delta+1} \right\rceil \leq \gamma_{ntc}(G) \leq p - 1$ and the bounds are sharp.

Proof Since any neighborhood triple connected dominating set is a dominating set, $\left\lceil \frac{p}{\Delta+1} \right\rceil \leq \gamma(G) \leq \gamma_{ntc}(G)$, the lower bound is attained. Also for a connected graph clearly $\gamma_{ntc}(G) \leq p - 1$. For $K_{2,3}$, the lower bound is attained and for P_3 the upper bound is attained.

Theorem 2.13 For any connected graph *G* with $p \geq 3$ vertices, $\gamma_{ntc}(G) = p - 1$ if and only if G is isomorphic to P_3, C_3 and any one of the graphs G_1 and G_2 shown in figure 2.18.

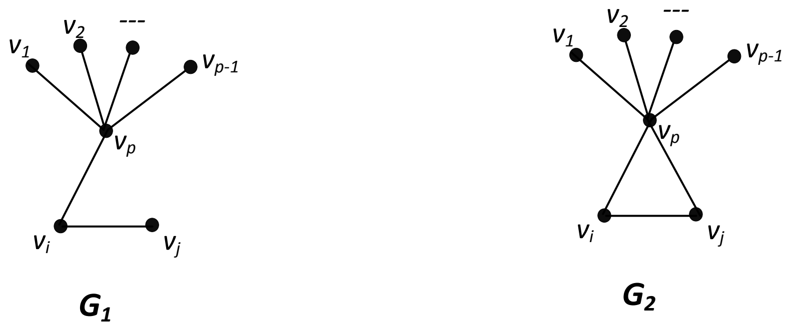


Figure 2.18

Proof Suppose G is isomorphic to any one of the graphs as stated in the theorem, then clearly, $\gamma_{ntc}(G) = p - 1$. Conversely, assume that G is a connected graph with $p \geq 3$ vertices and , $\gamma_{ntc}(G) = p - 1$. Let S = $\{v_1, v_2, \ldots, v_{p-1}\}$ be a $\gamma_{ntc}(G)$ -set. Let <V - S> = $\{v_p\}$. Since S is the neighborhood triple connected dominating set, there exists v_i in S such that v_i is adjacent to vp. Also <N(S)> is triple connected, we have v_i is adjacent to v_j for $i \neq j$ in S.

Case (i) $|N(S)| = 3$.

Then the induced subgraph <N(S)> has the following possibilities. <N(S)> = P_3 or C_3. Hence G is isomorphic to P_3, C_3 or the graphs G_1 and G_2 in figure 2.18.

Case (ii) $|N(S)| > 3$.

Then there exists atleast one v_k for $i \neq j \neq k$ in S which is adjacent to either v_i or v_j.

If v_k is adjacent to v_i, then the induced subgraph <N(S)> contains $K_{1,3}$ and hence it is not triple connected. If we increase the degrees of the vertices in S we can find a neighborhood triple connected dominating set with fewer elements than S. Hence no graph exists in this case.

If v_k is adjacent to v_j, then we can remove v_k from S and find a neighborhood triple connected dominating set of G with less than p – 1 vertices, which is a contradiction. Hence no graph exists in this case.

The Nordhaus – Gaddum type result is given below:

Theorem 2.14 Let *G* be a graph such that *G* and $\overline{G}$ have no isolates of order $p \geq 3$. Then

(i) $\gamma_{ntc}(G) + \gamma_{ntc}(\overline{G}) \leq 2p - 2$

(ii) $\gamma_{ntc}(G). \gamma_{ntc}(\overline{G}) \leq (p-1)^2$.

Proof The bound directly follows from *theorem 2.12*. For K_3, both the bounds follows.

III. RELATION WITH OTHER GRAPH THEORETICAL PARAMETERS

Theorem 3.1 For any connected graph G with $p \geq 3$ vertices, $\gamma_{ntc}(G) + \kappa(G) \leq 2p - 2$ and the bound is sharp if and only if $G \cong K_3$.

Proof Let G be a connected graph with $p \geq 3$ vertices. We know that $\kappa(G) \leq p - 1$ and by *theorem 2.12*, $\gamma_{ntc}(G) \leq p - 1$. Hence $\gamma_{ntc}(G) + \kappa(G) \leq 2p - 2$. If $G \cong K_3$ then clearly $\gamma_{ntc}(G) + \kappa(G) = 2p - 2$. Let $\gamma_{ntc}(G) + \kappa(G) = 2p - 2$. This is possible only if $\gamma_{ntc}(G) = p - 1$ and $\kappa(G) = p - 1$. But $\kappa(G) = p - 1$, and so $G \cong K_p$ for which $\gamma_{ntc}(G) = 1$ for $p > 3$. Hence $p = 3$ so that $G \cong K_3$.

Theorem 3.2 For any connected graph G with $p \geq 3$ vertices, $\gamma_{ntc}(G) + \Delta(G) \leq 2p - 2$ and the bound is sharp.

Proof Let G be a connected graph with $p \geq 3$ vertices. We know that $\Delta(G) \leq p - 1$ and by *theorem 2.12*, $\gamma_{ntc}(G) \leq p - 1$. Hence $\gamma_{ntc}(G) + \Delta(G) \leq 2p - 2$. For K_3, the bound is sharp.

Theorem 3.3 For any connected graph G with $p \geq 3$ vertices, $\gamma_{ntc}(G) + \chi(G) \leq 2p - 1$ and the bound is sharp if and only of $G \cong K_3$.

Proof Let G be a connected graph with $p \geq 3$ vertices. We know that $\chi(G) \leq p$ and by *theorem 2.12*, $\gamma_{ntc}(G) \leq p - 1$. Hence $\gamma_{ntc}(G) + \chi(G) \leq 2p - 1$. Suppose $G \cong K_3$, then clearly $\gamma_{ntc}(G) + \chi(G) = 2p - 1$. Let $\gamma_{ntc}(G) + \chi(G) = 2p - 1$. This is possible only if $\gamma_{ntc}(G) = p - 1$ and $\chi(G) = p$. Since $\chi(G) = p$, G is isomorphic to K_p for which $\gamma_{ntc}(G) = 1$, for $p > 3$. Hence $p = 3$, so that $G \cong K_3$.

IV. RELATION WITH OTHER DOMINATION PARAMETERS

Theorem 4.1 For any connected graph G with $p \geq 5$ vertices, $\gamma_{tc}(G) + \gamma_{ntc}(G) < 2p - 3$.

Proof Let G be a connected graph with $p \geq 5$ vertices. We know that $\gamma_{tc}(G) \leq p - 2$ for $p \geq 5$ vertices and by *theorem 2.12*, $\gamma_{ntc}(G) \leq p - 1$ for $p \geq 3$ vertices. Hence $\gamma_{tc}(G) + \gamma_{ntc}(G) \leq 2p - 3$ for $p \geq 5$ vertices. Also by *theorem 2.13*, the bound is not sharp.

Theorem 4.2 For any connected graph G with $p \geq 4$ vertices, $\gamma_{ctc}(G) + \gamma_{ntc}(G) < 2p - 4$.

Proof Let G be a connected graph with $p \geq 4$ vertices. We know that $\gamma_{ctc}(G) \leq p - 3$ for $p \geq 4$ vertices and by *theorem 2.12*, $\gamma_{ntc}(G) \leq p - 1$ for $p \geq 3$ vertices. Hence $\gamma_{ctc}(G) + \gamma_{ntc}(G) \leq 2p - 4$ for $p \geq 4$ vertices. Also by *theorem 2.13*, the bound is not sharp.

Theorem 4.3 For any connected graph G with $p \geq 5$ vertices, $\gamma_{cptc}(G) + \gamma_{ntc}(G) < 2p - 3$.

Proof Let G be a connected graph with $p \geq 5$ vertices. We know that $\gamma_{cptc}(G) \leq p - 2$ for $p \geq 5$ vertices and by *theorem 2.12*, $\gamma_{ntc}(G) \leq p - 1$ for $p \geq 3$ vertices. Hence $\gamma_{ctc}(G) + \gamma_{ntc}(G) \leq 2p - 3$ for $p \geq 5$ vertices. Also by *theorem 2.13*, the bound is not sharp.

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