Dynamical Properties of Carrier Wave Propagating In a Viscous Fluid

[1,] Edison A. Enaibe , and [2,] Osafile E. Omosede
[1,2,] Department of Physics, Federal University of Petroleum Resources, P.M.B. 1220, Effurun, Delta State, Nigeria.

ABSTRACT

In this work, we superposed a 'parasitic wave' on a 'host wave' and the behavior of the constituted carrier wave is studied as it flows through an elastic uniform pipe containing a viscous fluid. The dynamical characteristics of the carrier wave as it propagate through the confined space containing the fluid is effectively studied with the use of simple differentiation technique. It is shown in this work that some wave behave parasitically when they interfere with another one, provided they are basically out of phase. The incoherent nature of their source causes the carrier wave to attenuate to zero after a specified time. The phase velocity, the oscillating radial velocity and other basic physical properties of the constituted carrier wave first increases in value before they steadily attenuate to zero. The study provides a means of using the known parameter values of a given wave to determine the basic parameters of another interfering wave which were initially not known. It is established in this study that the process of attenuation of the basic properties that constitute a physical system is not instantaneous but gradual. Since the mechanics of the resident 'host wave' would be posing a serious resistance in order to annul the destructive effect of the parasitic interfering wave.The spectrum of the decay process show exemplary behaviour at certain values of the raising multiplier. This behaviour is caused by the high attraction or constructive interference of the combined effects of the 'host wave' and the 'parasitic wave'.

KEY WORDS:*constituted carrier wave, 'host wave', 'parasitic wave', raising multiplier, characteristic angular velocity, group angular velocity.*

I. INTRODUCTION

If a wave is to travel through a medium such as water, air, steel, or a stretched string, it must cause the particles of that medium to oscillate as it passes [1]. For that to happen, the medium must possess both mass (so that there can be kinetic energy) and elasticity (so that there can be potential energy). Thus, the medium's mass and elasticity property determines how fast the wave can travel in the medium.

The principle of superposition of wave states that if any medium is disturbed simultaneously by a number of disturbances, then the instantaneous displacement will be given by the vector sum of the disturbance which would have been produced by the individual waves separately. Superposition helps in the handling of complicated wave motions. It is applicable to electromagnetic waves and elastic waves in a deformed medium provided Hooke's law is obeyed Interference effect that occurs when two or more waves overlap or intersect is a common phenomenon in physical wave mechanics. When waves interfere with each other, the amplitude of the resulting wave depends on the frequencies, relative phases and amplitudes of the interfering waves. The resultant amplitude can have any value between the differences and sum of the individual waves [2]. If the resultant amplitude comes out smaller than the larger of the amplitude of the interfering waves, we say the superposition is destructive; if the resultant amplitude comes out larger than both we say the superposition is constructive. The interference of one wave say 'parasitic wave' y_1 on another one say 'host wave' y_2 could cause the resident 'host wave' to undergo dampingto zero if they are out of phase. The damping process of y_2 can be gradual, over-damped or critically damped depending on the rate in which the amplitude of the host wave is brought to zero. However, the general understanding is that the combination of y_1 and y_2 would first yield a third stage called the resultant wave say y, before the process of damping sets in. In this work, we refer to the resultant wave as the constituted carrier wave or simply carrier wave.

A 'parasitic wave' as the name implies, has the ability of destroying and transforming the intrinsic constituents of the 'host wave' to its form after a sufficiently long time. It contains an inbuilt raising multiplier λ which is capable of increasing the intrinsic parameters of the 'parasitic wave' to become equal to those of the 'host wave'. Ultimately, once this equality is achieved, then all the active components of the 'host wave' would have been completely eroded and the constituted carrier wave ceases to exist. A carrier wave in this wise, is a corrupt wave function which certainly describes the activity and performance of most physical systems. Thus, the reliability and the life span of most physical systems are determined by the reluctance and willingness of the active components of the resident 'host wave' to the destructive influence of the 'parasitic wave'. Any actively defined physical system carries along with it an inbuilt attenuating factor such that even in the absence of any external influence the system will eventually come to rest after a specified time. This accounts for the non-permanent nature of all physically active matter.

If the wave function of any given active system is known, then its characteristics can be predicted and altered by means of anti-vibratory component. The activity and performance of any active system can be slowed down to zero-point 'death' by means of three factors: (i) Internal factor (ii) External factor, and (iii) Accidental Factor. The internal factor is a normal decay process. This factor is caused by aging and local defects in the constituent mechanism of the physical system. This shows that every physically active system must eventually come to rest or cease to exist after some time even in the absence of any external attenuating influence. The internal factor is always a gradual process and hence the attenuating wave function of the physical system is said to be under-damped.

The external factor is a destructive interference process. This is usually a consequence of the encounter of one existing well behaved active wave function with another. The resultant attenuating wave function under this circumstance is said to be under-damped, over-damped or critically-damped, depending on how fast the intrinsic constituent characteristics of the wave function decays to zero.The accidental factor leads to a sudden breakdown and restoration of the wave function of the physical system to a zero-point. In this case, all the active intrinsic parameters of the physical system are instantaneously brought to rest and the attenuation process under this condition is said to be critically-damped.

The initial characteristics of a given wave with a definite origin or source can best be determined by the use of a sine wave function. However, for the deductive determination of the initial behaviour of a wave whose origin is not certain, the cosine wave function can best be effectively utilized. Generally, we can use the available information of the physical parameters of a wave at any given position and time $y(x+\delta x, t+\delta t)$ to predict the nature of its source andthe initial characteristics when the position and time was $y(x,t)$.The reader should permit the lack of adequate references since no one has ever worked in this area before now.

The organization of this paper is as follows. In section 1, we discuss the nature of wave and interference. In section 2, we show the mathematical theory of superposition of two incoherent waves. The results emanating from this study is shown in section 3. The discussion of the results of our study is presented in section 4. Conclusion and suggestions for further work is discussed in section 5. The paper is finally brought to an end by a few lists of references.

II. RESEARCH METHODOLOGY

In this work, we superposed a 'parasitic wave' with inbuilt raising multiplier λ on a 'host wave' and the combined effect of the waves is allowed to flow through a narrow pipe containing a viscous fluid. The attenuation mechanism of the carrier wave which is the result of the superposition is thus studied by means of simple differentiation technique.

2.1.Mathematical theory of superposition of waves

Let us consider two incoherent waves defined by the non - stationary displacement vectors

$$y_1 = a\beta\cos(\vec{k}\beta.\vec{r} - n\beta t - \varepsilon\beta) \quad (2.1)$$

$$y_2 = b\lambda\cos(\vec{k}'\lambda.\vec{r} - n'\lambda t - \varepsilon'\lambda) \quad (2.2)$$

where all the symbols retain their usual meanings. In this study, (2.1) is regarded as the 'host wave' whose propagation depends on the inbuilt raising multiplier $\beta(= 0 \ldots, 1)$. While (2.2) represents a 'parasitic wave' with an inbuilt raising multiplier $\lambda(= 0, 1, 2, \cdots, \lambda_{\max})$. The inbuilt multipliers are both dimensionless and as the name implies, they are capable of gradually raising the basic intrinsic parameters of both waves respectively with time.Let us superpose (2.2) on (2.1), with the hope to realize a common wave function, then

$$y = y_1 + y_2 = a\beta \cos(\vec{k}\beta \vec{r} - n\beta t - \varepsilon\beta) + b\lambda \cos(\vec{k}'\lambda \vec{r} - n'\lambda t - \varepsilon'\lambda) \qquad (2.3)$$

Suppose, we assume that for a very small parameter ζ, the below equation holds,

$$n'\lambda = \zeta + n\beta \quad (2.4)$$

$$y = a\beta \cos(\vec{k}\beta.\vec{r} - n\beta t - \varepsilon\beta) + b\lambda \cos(\vec{k}'\lambda.\vec{r} - n\beta t - \zeta t - \varepsilon'\lambda) \qquad (2.5)$$

Again in (2.5), we let

$$\varepsilon_1' = \zeta t + \varepsilon'\lambda \qquad (2.6)$$

$$y = a\beta \cos(\vec{k}\beta.\vec{r} - n\beta t - \varepsilon\beta) + b\lambda \cos(k'\lambda.\vec{r} - n\beta t - \varepsilon_1') \qquad (2.7)$$

For the purpose of proper grouping we again make the following assumption:

$$\vec{k}\beta.\vec{r} = \vec{k}'\lambda.\vec{r} = \xi \qquad (2.8)$$

$$(\vec{k}\beta - \vec{k}'\lambda).\vec{r} = \xi \qquad (2.9)$$

$$y = a\beta \cos\left((\xi - n\beta t) - \varepsilon\beta\right) + b\lambda \cos\left((\xi - n\beta t) - \varepsilon_1'\right) \qquad (2.10)$$

We can now apply the cosine rule for addition of angles to reevaluate each term in (2.10), that is,

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \quad (2.11)$$

$$y = a\beta\left\{\cos(\xi - n\beta t)\cos\beta\varepsilon + \sin(\xi - n\beta t)\sin\beta\varepsilon\right\} +$$
$$b\lambda\left\{\cos(\xi - n\beta t)\cos\varepsilon_1' + \sin(\xi - n\beta t)\sin\varepsilon_1'\right\} \qquad (2.12)$$

$$y = \cos(\xi - n\beta t)\left\{a\beta\cos\beta\varepsilon + b\lambda\cos\varepsilon_1'\right\} + \sin(\xi - n\beta t)\left\{a\beta\sin\beta\varepsilon + b\beta\sin\varepsilon_1'\right\} (2.13)$$

For technicality, let us make the following substitutions so that we can further simplify (2.14).

$$A\cos E = a\beta\cos\beta\varepsilon + b\lambda\cos\varepsilon_1' \qquad (2.14)$$

$$A\sin E = a\beta\sin\beta\varepsilon + b\lambda\sin\varepsilon_1' \ (2.15)$$

$$y = A\left\{\cos(\xi - n\beta t)\cos E + \sin(\xi - n\beta t)\sin E\right\} \qquad (2.16)$$

$$y = A\cos\left\{\xi - n\beta t - E\right\} \quad (2.17)$$

$$y = A\cos\left\{(\vec{k}\beta - \vec{k}'\lambda).\vec{r} - n\beta t - E\right\} \ (2.18)$$

The simultaneous nature of (2.15) and (2.16) would enable us to square though them and add the resulting equations term by term, that is

$$A = \sqrt{a^2\beta^2 + b^2\lambda^2 + 2ab\beta\lambda\cos\left((\beta\varepsilon - \varepsilon'\lambda) + (n\beta - n'\lambda)t\right)} \ (2.19)$$

$$y = \sqrt{a^2\beta^2 + b^2\lambda^2 + 2a\beta b\lambda\cos\left((\beta\varepsilon - \varepsilon'\lambda) + (n\beta - n'\lambda)t\right)} \times \cos\left((\vec{k}\beta - \vec{k}'\lambda).\vec{r} - n\beta t - E\right) \ (2.20)$$

Upon dividing (2.16) by (2.15), we get that

$$\tan E = \frac{a\beta\sin\beta\varepsilon + b\lambda\sin\varepsilon_1'}{a\beta\cos\beta\varepsilon + b\lambda\cos\varepsilon_1'} \qquad (2.21)$$

$$E = \tan^{-1}\left(\frac{a\beta\sin\beta\varepsilon + b\lambda\sin(\varepsilon'\lambda - (n\beta - n'\lambda)t)}{a\beta\cos\beta\varepsilon + b\lambda\cos(\varepsilon'\lambda - (n\beta - n'\lambda)t)}\right) \ (2.22)$$

Hence (2.26) is the resultant wave equation which describes the superposition of the 'parasitic wave' on the 'host wave'. Equation (2.28) representsaresultant wave equation in which the effects of the constitutive waves are additive in nature.However, suppose the effects of the constitutive waves are subtractive and with the view that the basic parameters of the 'host wave' are constant with time, that is, $\beta = 1$ and leave its variation for future study, then without loss of dimensionality we can recast (2.20) and (2.22) as

$$y = \sqrt{\left(a^2 - b^2\lambda^2\right) - 2\left(a - b\lambda\right)^2\cos\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)} \times \cos\left((\vec{k} - \vec{k}'\lambda).\vec{r} - (n - n'\lambda)t - E\right) (2.23)$$

where we have redefined the amplitude an the total phase angle as,

$$A=\sqrt{\left(a^2-b^2\lambda^2\right)-2\left(a-b\lambda\right)^2\cos\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)}\qquad(2.24)$$

$$E=\tan^{-1}\left(\frac{a\sin\varepsilon-b\lambda\sin((n-n'\lambda)t-\varepsilon'\lambda)}{a\cos\varepsilon-b\lambda\cos((n-n'\lambda)t-\varepsilon'\lambda)}\right)\qquad(2.25)$$

Equation (2.25) is the constitutive carrier wave equation necessary for our study. As the equation stands, it is a 'carrier wave', in which it is only the variation in the intrinsic parameters of the 'parasitic wave' that determines the activityof thephysical system which it describes. Henceforth, we have agreed in this study, that the initial parameters of the 'host wave' are assumed to be constant and also they are initially greater than those of the 'parasitic wave'.

2.1 The calculus of the total phase angle E of the carrier wave function

Let us now determine the variation of the total phase angle E with respect to time t . Thus from (2.25),

$$\frac{dE}{dt}=\left(1+\left(\frac{a\sin\varepsilon-b\lambda\sin((n-n'\lambda)t-\varepsilon'\lambda)}{a\cos\varepsilon-b\lambda\cos((n-n'\lambda)t-\varepsilon'\lambda)}\right)^2\right)^{-1}\times\frac{d}{dt}\left(\frac{a\sin\varepsilon-b\lambda\sin((n-n'\lambda)t-\varepsilon'\lambda)}{a\cos\varepsilon-b\lambda\cos((n-n'\lambda)t-\varepsilon'\lambda)}\right)(2.26)$$

$$\frac{dE}{dt}=\left\{\frac{\left(a\cos\varepsilon-b\lambda\cos((n-n'\lambda)t-\varepsilon'\lambda\right)^2}{\left(a\cos\varepsilon-b\lambda\cos((n-n'\lambda)t-\varepsilon'\lambda)\right)^2+\left(a\sin\varepsilon-b\lambda\sin((n-n'\lambda)t-\varepsilon'\lambda)\right)^2}\right\}\times$$

$$\frac{d}{dt}\left(\frac{a\sin\varepsilon-b\lambda\sin\left((n-n'\lambda)t-\varepsilon'\lambda\right)}{a\cos\varepsilon-b\lambda\cos\left((n-n'\lambda)t-\varepsilon'\lambda\right)}\right)\qquad(2.27)$$

After a lengthy algebra (2.27) simplifies to

$$\frac{dE}{dt}=Z\quad(2.28)$$

where we have introduced a new variable defined by the symbol

$$Z=(n-n'\lambda)\left(\frac{b^2\lambda^2-ab\lambda\cos\left((\varepsilon+\varepsilon'\lambda)-(n-n'\lambda)t\right)}{a^2+b^2\lambda^2-2ab\lambda\cos\left((\varepsilon+\varepsilon'\lambda)-(n-n'\lambda)t\right)}\right)\qquad(2.29)$$

as the characteristic angular velocity of the carrier wave. It has the dimension of *rad./s.*Also the variation of the total phase angle with respect to the wave number is given by

$$\frac{dE}{d(k-k'\lambda)}=t\frac{d}{d(k-k'\lambda)}(n-n'\lambda)\left(\frac{b^2\lambda^2-ab\lambda\cos\left((\varepsilon+\varepsilon'\lambda)-(n-n'\lambda)t\right)}{a^2+b^2\lambda^2-2ab\lambda\cos\left((\varepsilon+\varepsilon'\lambda)-(n-n'\lambda)t\right)}\right)(2.30)$$

2.2 Evaluation of the group angular velocity (ω_g) of the carrier wave function

The group velocity is a well-defined but different velocity from that of the individual wave themselves. This is also the velocity at which energy is transferred by the wave [3]. When no energy absorption is present, the velocity of energy transport is equal to the group velocity [4]. The carrier wave function is amaximum if the spatial oscillatory phase is equal to 1. As a result

$$\cos\left(r\left((k-k'\lambda)\cos\theta+(k-k'\lambda)\sin\theta\right)-(n-n'\lambda)t-E\right)=1\,(2.31)$$

$$r\left(\cos\theta+\sin\theta\right)-t\frac{d}{d(k-k'\lambda)}(n-n'\lambda)-\frac{dE}{d(k-k'\lambda)}=0\quad(2.32)$$

$$r\left(\cos\theta+\sin\theta\right)=t\frac{d}{d(k-k'\lambda)}(n-n'\lambda)+\frac{dE}{d(k-k'\lambda)}\quad(2.33)$$

$$r\left(\cos\theta+\sin\theta\right)=t\frac{d}{d(k-k'\lambda)}(n-n'\lambda)+t\frac{d}{d(k-k'\lambda)}(n-n'\lambda)\left(\frac{b^2\lambda^2-ab\lambda\cos\left((\varepsilon+\varepsilon'\lambda)-(n-n'\lambda)t\right)}{a^2+b^2\lambda^2-2ab\lambda\cos\left((\varepsilon+\varepsilon'\lambda)-(n-n'\lambda)t\right)}\right)$$

(2.34)

$$v_g = \frac{r}{t} = \frac{d}{d(k-k'\lambda)}\frac{(n-n'\lambda)}{(\cos\theta+\sin\theta)}\left\{1+\left(\frac{b^2\lambda^2 - ab\lambda\cos\left((\varepsilon+\varepsilon'\lambda)-(n-n'\lambda)t\right)}{a^2+b^2\lambda^2-2ab\lambda\cos\left((\varepsilon+\varepsilon'\lambda)-(n-n'\lambda)t\right)}\right)\right\} \quad (2.35)$$

$$v_g = \frac{r}{t} = \frac{d\omega_g}{d(k-k'\lambda)} \quad (2.36)$$

which is the basic expression for the group angular velocity, where

$$\omega_g = \frac{(n-n'\lambda)}{(\cos\theta+\sin\theta)}\left(\frac{a^2+2b^2\lambda^2-3ab\lambda\cos\left((\varepsilon+\varepsilon'\lambda)-(n-n'\lambda)t\right)}{a^2+b^2\lambda^2-2ab\lambda\cos\left((\varepsilon+\varepsilon'\lambda)-(n-n'\lambda)t\right)}\right) \quad (2.37)$$

is the group velocity of the carrier wave which has the dimension of *radian/s*. Although, ω_g and Z has the same dimension, but where Z depends on time, ω_g is dependent upon the spatial frequency or wave number(k).

2.3 Evaluation of the phase velocity (v_p)of the carrier wave.

The phase velocity denotes the velocity of a point of fixed phase angle [4]. At any instant of the wave motion the displacements of other points nearby change also and there will be one of these points, at $x+\delta x$ say, where the displacement $y(x+\delta x, t+\delta t)$ is equal to the original displacement $y(x,t)$ at point x. Now from(2.23) the carrier wave is a maximum when the spatial oscillatory phase is equal to one.

$$\cos\left((\vec{k}-\vec{k}'\lambda).\vec{r} - (n-n'\lambda)t - E\right) = 1 \quad (2.38)$$

$$(\vec{k}-\vec{k}'\lambda) = (k-k'\lambda)_x i + (k-k'\lambda)_y j + (k-k'\lambda)_z k \quad (2.39)$$

$$\vec{r} = xi + yj + zk \quad (2.40)$$

If we assume that the motion is constant in the z-direction and the wave vector mode is also the same for both x and y plane, then (2.40) becomes

$$\vec{r} = r\cos\theta\, i + r\sin\theta\, j \quad (2.41)$$

where $\theta = \pi - (\varepsilon - \varepsilon'\lambda)$ is the variable angle between y_1 and y_2, please see appendix for details.Hence

$$\cos\left((k-k'\lambda)r\cos\theta + (k-k'\lambda)r\sin\theta - (n-n'\lambda)t - E\right) = 1 \quad (2.42)$$

$$\left((k-k'\lambda)r\cos\theta + (k-k'\lambda)r\sin\theta - (n-n'\lambda)t - E\right) = 0 \quad (2.43)$$

$$\left((k-k'\lambda)\cos\theta + (k-k'\lambda)\sin\theta\right)dr - (n-n'\lambda)dt - \frac{dE}{dt}dt = 0 \quad (2.44)$$

$$\left((k-k'\lambda)\cos\theta + (k-k'\lambda)\sin\theta\right)dr - (n-n'\lambda)dt - Zdt = 0 \quad (2.45)$$

$$\left((k-k'\lambda)\cos\theta + (k-k'\lambda)\sin\theta\right)dr = \left((n-n'\lambda)+Z\right)dt \quad (2.46)$$

$$v_p = \frac{dr}{dt} = \left(\frac{\left((n-n'\lambda)+Z\right)}{(k-k'\lambda)(\cos\theta+\sin\theta)}\right) \quad (2.47)$$

This has the dimension of *m/s*. Since our argument is equally valid for all values ofr, (2.47) tells us that the whole sinusoidal wave profile move to the left or to the right at a speed v_p.

2.4 Evaluation of the oscillating angularfrequency ($\dot{\theta}$) of the carrier wave.

The variation of the spatial oscillatory phase of the carrier wave with respect to time gives theoscillating frequency ($\dot{\theta}$). Hence, from (2.43)

$$\left((k-k'\lambda)r\cos\theta + (k-k'\lambda)r\sin\theta - (n-n'\lambda)t - E\right) = 0 \quad (2.48)$$

$$(k-k'\lambda)r\left(\frac{d}{dt}(\cos\theta+\sin\theta)\right) - (n-n'\lambda) - \frac{dE}{dt} = 0 \quad (2.49)$$

$$(k-k'\lambda)r\left(\frac{d\cos\theta}{d\theta}\frac{d\theta}{dt} + \frac{d\sin\theta}{d\theta}\frac{d\theta}{dt}\right) - (n-n'\lambda) - Z = 0 \quad (2.50)$$

$$(k-k'\lambda)r\left(\cos\theta\dot{\theta} - \sin\theta\dot{\theta}\right) = \left((n-n'\lambda)+Z\right) \quad (2.51)$$

$$v_\theta = \dot{\theta} = \left(\frac{\left((n-n'\lambda)+Z\right)}{(k-k'\lambda)\,r(\cos\theta - \sin\theta)} \right) \quad (2.52)$$

The unit isper second (s^{-1}). Thus because of the tethered nature of the elastic pipe the carrier wave can only possess oscillating radial velocity and not oscillating angular velocity.

2.5 Evaluation of the radial velocity (v_r)of the carrier wave.

The variation of the spatial oscillatory phase of the carrier wave with respect to time gives the radial velocity (v_r). Hence, from (2.48)

$$(k-k'\lambda)\frac{dr}{dt}(\cos\theta + \sin\theta) - (n-n'\lambda) - \frac{dE}{dt} = 0 \quad (2.53)$$

$$(k-k'\lambda)\,\dot{r}\left((\cos\theta + \sin\theta)\right) - (n-n'\lambda) - Z = 0 \quad (2.54)$$

$$v_r = \dot{r} = \left(\frac{\left((n-n'\lambda)+Z\right)}{(k-k'\lambda)(\cos\theta + \sin\theta)} \right) \quad (2.55)$$

This has the same unit as the phase velocity which is m/s.

2.6 Evaluation of the velocity of the 'carrier wave'

Letus now evaluate the velocity with which the entire carrier wave function moves with respect to time. This has to do with the product differentiation of the non-stationary amplitude and the spatial oscillatory cosine phase.

$$v = \frac{dy}{dt} = (a-b\lambda)^2\,(n-n'\lambda)\sin\left((n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\times$$
$$\left((a^2-b^2\lambda^2) - 2(a-b\lambda)^2\cos\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right)^{-\frac{1}{2}}\times$$
$$\cos\left((k-k'\lambda)\,r\,(\cos\theta+\sin\theta) - (n-n'\lambda)t - E\right) +$$
$$\left((a^2-b^2\lambda^2) - 2(a-b\lambda)^2\cos\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right)^{\frac{1}{2}}\times$$
$$\left\{(n-n'\lambda) + Z - V_r\,(k-k'\lambda)(\cos\theta+\sin\theta) - V_\theta\,(k-k'\lambda)\,r\,(\cos\theta+\sin\theta)\right\}\times$$
$$\sin\left((k-k'\lambda)\,r\,(\cos\theta+\sin\theta) - (n-n'\lambda)t - E\right) \quad (2.56)$$

$$v = (a-b\lambda)^2\,(n-n'\lambda)\sin\left((n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda)\right)\times$$
$$(a^2-b^2\lambda^2)^{-\frac{1}{2}}\left(1 - \frac{2(a-b\lambda)^2\cos\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)}{(a^2-b^2\lambda^2)}\right)^{-\frac{1}{2}}\times$$
$$\cos\left((k-k'\lambda)\,r\,(\cos\theta+\sin\theta) - (n-n'\lambda)t - E\right) +$$
$$(a^2-b^2\lambda^2)^{\frac{1}{2}}\left(1 - \frac{2(a-b\lambda)^2\cos\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)}{(a^2-b^2\lambda^2)}\right)^{\frac{1}{2}}\times$$
$$\left\{(n-n'\lambda) + Z - V_r\,(k-k'\lambda)(\cos\theta+\sin\theta) - V_\theta\,(k-k'\lambda)\,r\,(\cos\theta+\sin\theta)\right\}\times$$
$$\sin\left((k-k'\lambda)\,r\,(\cos\theta+\sin\theta) - (n-n'\lambda)t - E\right) \quad (2.57)$$

Upon using Binomial expansion on the fractional terms and stopping at the second term we get

$$v = \frac{(a-b\lambda)^2\,(n-n'\lambda)}{(a^2-b^2\lambda^2)^{\frac{1}{2}}}\sin\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\times$$
$$\cos\left((k-k'\lambda)\,r\,(\cos\theta+\sin\theta) - (n-n'\lambda)t - E\right) +$$
$$\frac{(a-b\lambda)^4(n-n'\lambda)}{(a^2-b^2\lambda^2)^{\frac{3}{2}}}\cos\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\times\cos\left((k-k'\lambda)\,r\,(\cos\theta+\sin\theta)-(n-n'\lambda)t-E\right)+$$

$$(a^2-b^2\lambda^2)^{\frac{1}{2}}\left\{(n-n'\lambda)+Z-V_r(k-k'\lambda)(\cos\theta+\sin\theta)-V_\theta(k-k'\lambda)r(\cos\theta+\sin\theta)\right\}\times$$

$$\sin\big((k-k'\lambda)r(\cos\theta+\sin\theta)-(n-n'\lambda)t-E\big)-\frac{(a-b\lambda)^2}{(a^2-b^2\lambda^2)^{\frac{1}{2}}}\cos\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\times$$

$$\left\{(n-n'\lambda)+Z-V_r(k-k'\lambda)(\cos\theta+\sin\theta)-V_\theta(k-k'\lambda)r(\cos\theta+\sin\theta)\right\}\times$$

$$\sin\big((k-k'\lambda)r(\cos\theta+\sin\theta)-(n-n'\lambda)t-E\big)\,(2.58)$$

For the velocity of the constituted carrier wave to be maximum we have to ignore all the oscillating phases, so that

$$v_m=\frac{(a-b\lambda)^4(n-n'\lambda)+(a^2-b^2\lambda^2)^2(n-n'\lambda)}{(a^2-b^2\lambda^2)^{\frac{3}{2}}}\qquad(2.59)$$

The unit is m/s.

2.7 Evaluation of the energy attenuation equation

In natural systems, we can rarely find pure wave which propagates free from energy-loss mechanisms. But if these losses are not too serious we can describe the total propagation in time by a given force law $f(t)$ .The propagating carrier wave which takes its course from the origin of the elastic pipe is affected by two factors: (i) the damping effect of the mass of the surrounding fluid (ii) the damping effect of the dynamic viscosity of the elastic walls of the pipein response to the wave propagation. Let us consider a carrier wave propagating through an elastic pipe of a given elasticity Q ,if the fluid in the pipe has a mass m and viscosity μ , the dissipationof the carrier wave-energy if the fluid is Newtonian, would obey the equation

$$f(t)=\mu\frac{d^2y}{dt}+Q\,y^2\,(2.60)$$

$$f(t)dt=2\,\mu\,y\,d\,y+Q\,y^2dt\,(2.61)$$

For the carrier wave to have a maximum value then the spatial oscillating part is ignored such that

$$y_m^2=\left(a^2-b^2\lambda^2\right)-2\left(a-b\lambda\right)^2\cos\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\,(2.62)$$

$$f(t)\,dt=2\mu\,y_m\,dy_m+Q\Big(a^2-b^2\lambda^2-2\left(a-b\lambda\right)^2\cos\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\Big)dt\,(2.63)$$

$$\int f(t)\,dt=2\mu\int y_m\,dy_m+Q\int\Big((a^2-b^2\lambda^2)-2\left(a-b\lambda\right)^2\cos\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\Big)dt\,(2.64)$$

$$impulse=2\mu\,y_m^2+Q\left((a^2-b^2\lambda^2)t-\frac{2\left(a-b\lambda\right)^2\sin\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)}{(n-n'\lambda)}\right)(2.65)$$

$$impulse\times v_m=2\mu\,y_m^2\times v_m+Q\times v_m\left((a^2-b^2\lambda^2)t-\frac{2\left(a-b\lambda\right)^2\sin\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)}{(n-n'\lambda)}\right)(2.66)$$

$$Energy\,E=2\mu\,y_m^2v_m+Q\,v_m\left(\frac{(a^2-b^2\lambda^2)(n-n'\lambda)t-2\left(a-b\lambda\right)^2\sin\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)}{(n-n'\lambda)}\right)(2.67)$$

The unit is in Joulesor kgm^2s^{-2}or Nm. However, in this work we assume the dynamic viscosity μ of the fluid mediumwhere the carrier wave is propagating as $\mu=0.004\,N\,s\,m^{-2}$ and the elasticity of the wall of the narrow pipe $Q=1.9048\times10^{-6}\;kg\,m^{-1}s^{-2}$.

2.8 Determination of the 'parasitic wave' parameters (b , n' , ε' and k')

Now, suppose we assign arbitrary values for the parameters of the 'host wave' say: $a = 0.00524m$, $n = 0.000882rad./s$, $\varepsilon = 0.6109rad.$ (35^0) and $k = 166rad./m$, then we can precisely determine the basic parameters of the 'parasitic wave' which were initially not known before the interference,using the below method.The gradual deterioration in the physical system under studyshows that after a sufficiently long period of time all the active constituents of theresident 'host wave' would have been completely eroded by the destructive influence of the 'parasitic wave', on the basis of this argument the below relation holds.

$$a - b\lambda = 0 \quad \Rightarrow 0.00524 = b\lambda$$
$$n - n'\lambda = 0 \quad \Rightarrow 0.000882 = n'\lambda$$
$$\varepsilon - \varepsilon'\lambda = 0 \quad \Rightarrow 0.6109 = \varepsilon'\lambda \} \text{ -------------------- } (2.68)$$
$$k - k'\lambda = 0 \quad \Rightarrow 166 = k'\lambda$$

Upon dividing the sets of relations in (2.68) with one another with the view to firsteliminate λ we get

$$\Rightarrow 5.94104n' = b$$
$$\Rightarrow 0.0085775\varepsilon' = b$$
$$\Rightarrow 0.000031566k' = b$$
$$\Rightarrow 0.00144377\varepsilon' = n' \} \text{ ---------------------}(2.69)$$
$$\Rightarrow 0.00000531325k' = n'$$
$$\Rightarrow 0.00368k' = \varepsilon'$$

A more realistic and applicable relation is when: $0.0085775\varepsilon' = 0.000031566k'$, from which based on simple ratio

$$\varepsilon' = 0.0000316rad.$$
$$k' = 0.00858rad./m \} \text{ -------------------- } (2.70)$$
$$n' = 4.56 \times 10^{-8} rad./s$$
$$b = 2.71 \times 10^{-7} m$$

Any of these values of the 'parasitic wave' shall produce a corresponding approximate value of lambda $\lambda = 19332$upon substituting them into (2.68). Note that for the interest of uniformity and anticipated complications we are using the minimum value which is 19332. Hence the interval of the multiplier is $0 \le \lambda \le 19332$.Thus, so far, we have determined the basic intrinsic parameters of both the 'host wave' and those of the 'parasitic wave' both contained in the carrier wave.

2.9 Determination of the attenuation constant (η)

Attenuation is a decay process. It brings about a gradual reduction and weakening in the initial strength of the basic parameters of a given physical system. In this study, the parameters are the amplitude (a), phase angle (ε), angular frequency (n) and the spatial frequency (k). The dimension of the attenuation constant (η)is determined by the system under study. However, in this work, attenuation constant is the relative rate of fractional change (FC) in the basic parameters of the carrier wave. Thereare 4 (four) attenuating parameters present in the carrier wave. Now, if a , n , ε , k represent the initial basic parameters of the 'host wave' that is present in the carrier wave and $a - b\lambda$, $n - n'\lambda$, $\varepsilon - \varepsilon'\lambda$, $k - k'\lambda$ represent the basic parameters of the 'host wave' that survives after a given time. Then, the FC is

$$\sigma = \frac{1}{4} \times \left(\left(\frac{a - b\lambda}{a} \right) + \left(\frac{\varepsilon - \varepsilon'\lambda}{\varepsilon} \right) + \left(\frac{n - n'\lambda}{n} \right) + \left(\frac{k - k'\lambda}{k} \right) \right) \quad (2.71)$$

$$\eta = \frac{FC|_{\lambda=i} - FC|_{\lambda=i+1}}{unit\, time(s)} = \frac{\sigma_i - \sigma_{i+1}}{unit\ time\,(s)} \qquad (2.72)$$

The dimension is *per second* (s^{-1}). Thus (2.72) gives $\eta = 0.0000517s^{-1}$ for all values of $\lambda\, (i = 0, 1, 2, \ldots, 19332)$.

2.10 Determination of the time (t)

We used the information provided in section 2.9, to compute the various times taken for the carrier wave to attenuate to zero. The maximum time the carrier wave lasted as a function of the raising multiplier λ is also calculated from the attenuation equation shown by (2.72). The reader should note that we have adopted a slowly varying regular interval for the raising multiplier since this would help to delineate clearly the physical parameter space accessible to our model. However, it is clear from the calculation that the different attenuating fractional changes contained in the carrier wave are approximately equal to one another.We can now apply the attenuation time equation given below.

$$\sigma = e^{-(2^{\gamma}\eta t)/\lambda} \quad (2.73)$$

$$t = -\left(\frac{\lambda}{2^{\gamma}\eta}\right)\ln\sigma \quad (2.74)$$

where γ is the functional index of any physical system under study and here we assume $\gamma = 3$.The equation is statistical and not a deterministic law. It gives the expected basic intrinsic parameters of the 'host wave' that survives after time t. Clearly, we used (2.74) to calculate the exact value of the decay time as a function of the raising multiplier.In this work, we used table scientific calculator and Microsoft excel to compute our results. Also the GNUPLOT 4.6 version was used to plot the corresponding graphs.

III. PRESENTATION OF RESULTS

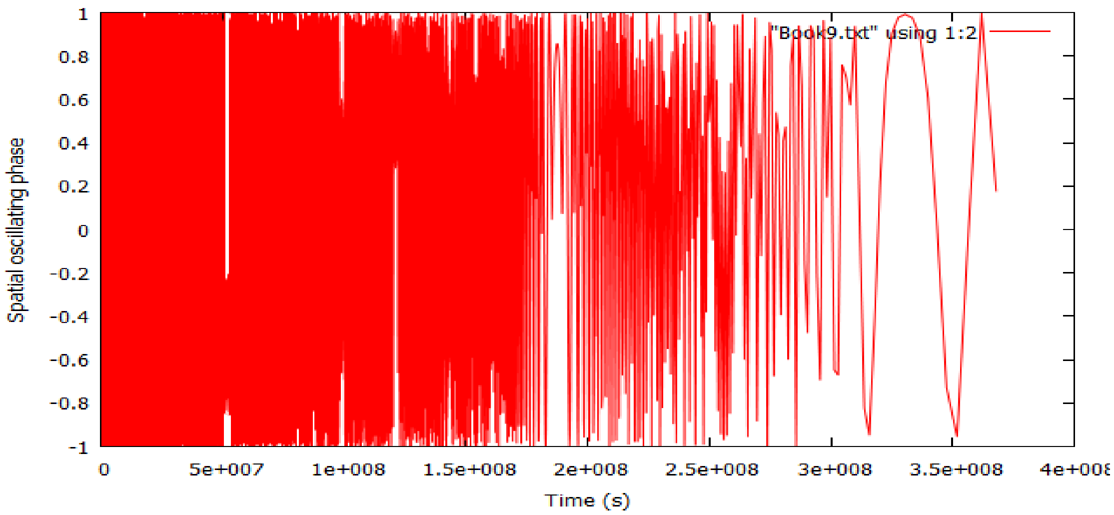

Fig. 3.1. This **r**epresentsthe graph of thespatial oscillating phase of the carrier wave against time t .

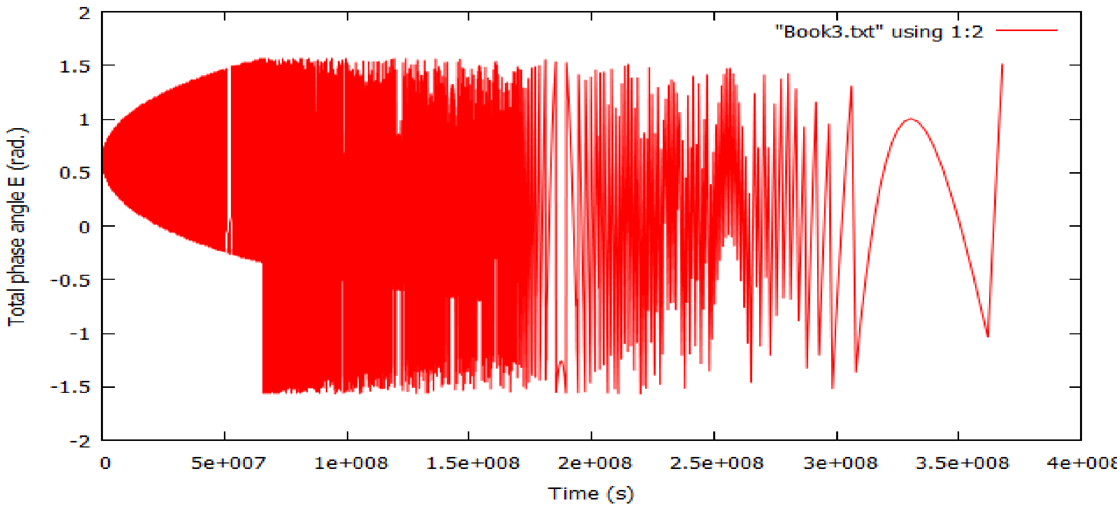

Fig. 3.2. This **r**epresentsthe graph of the total phase angle E of the carrier wave against time t .

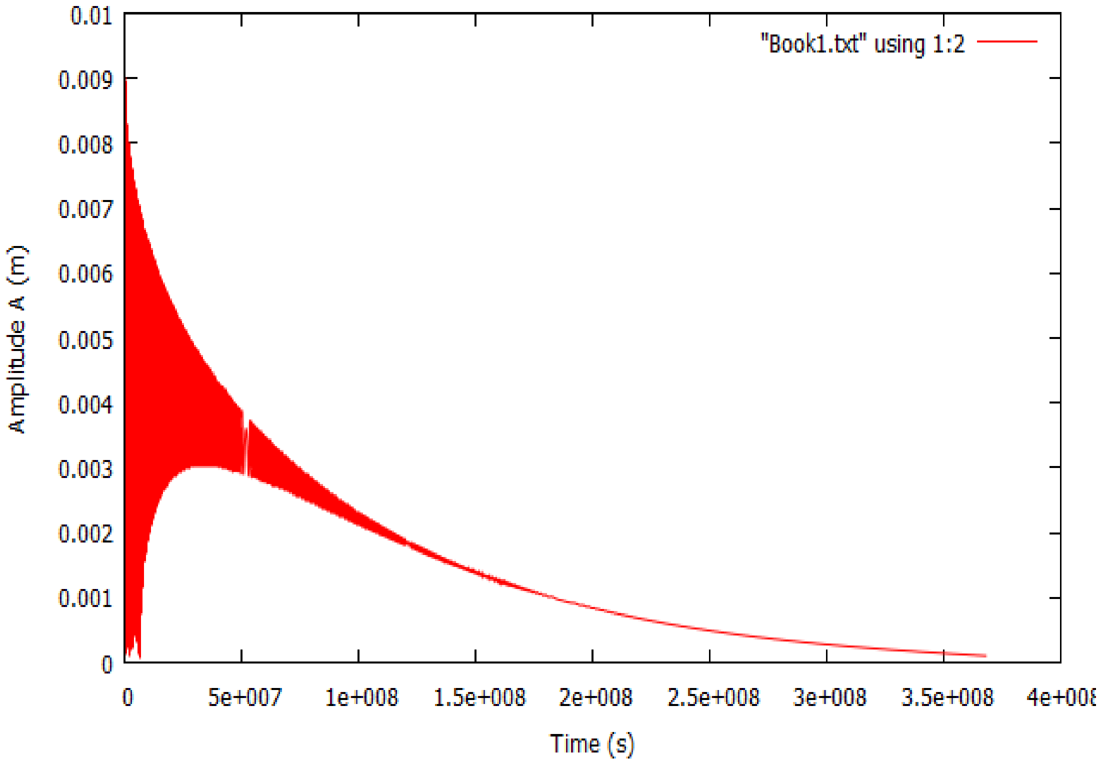

Fig. 3.3. This **r**epresentsthe graph of theamplitude A of the carrier wave against time t .

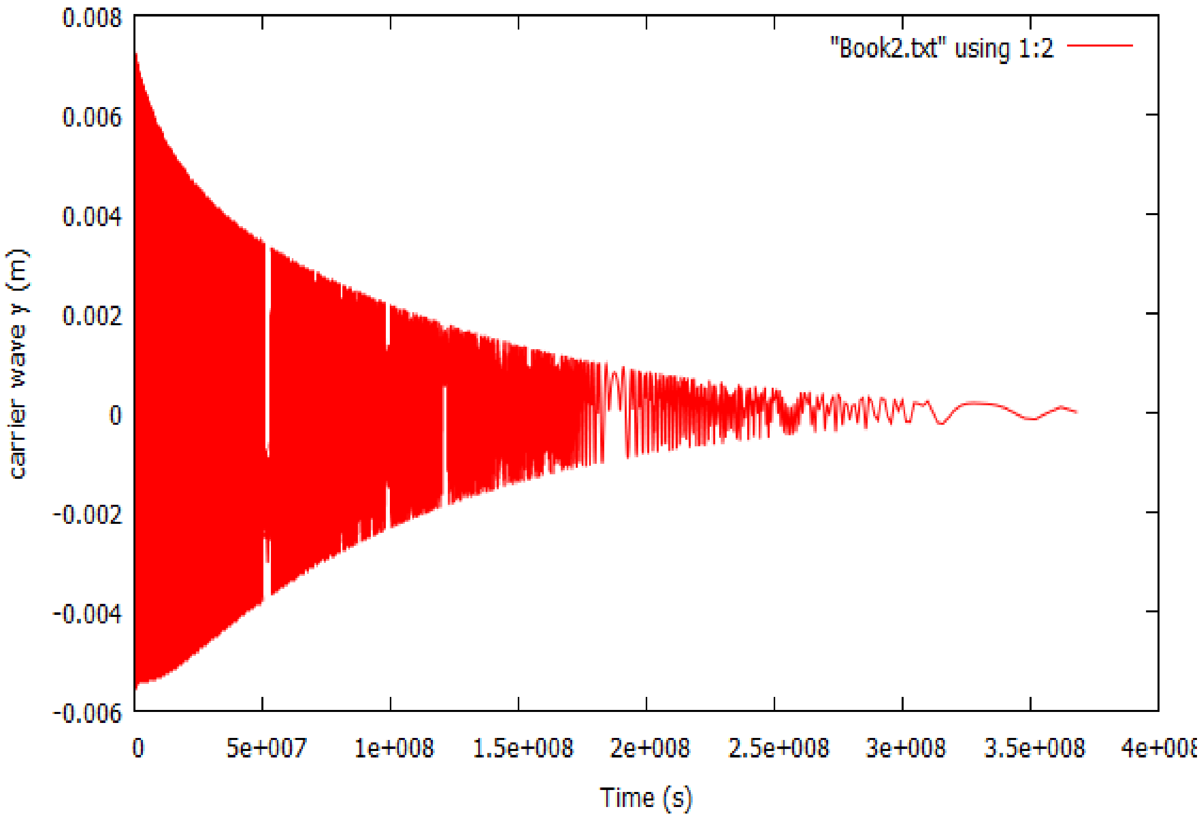

Fig. 3.4. This **r**epresents the graph ofthe carrier wave displacement against time t .

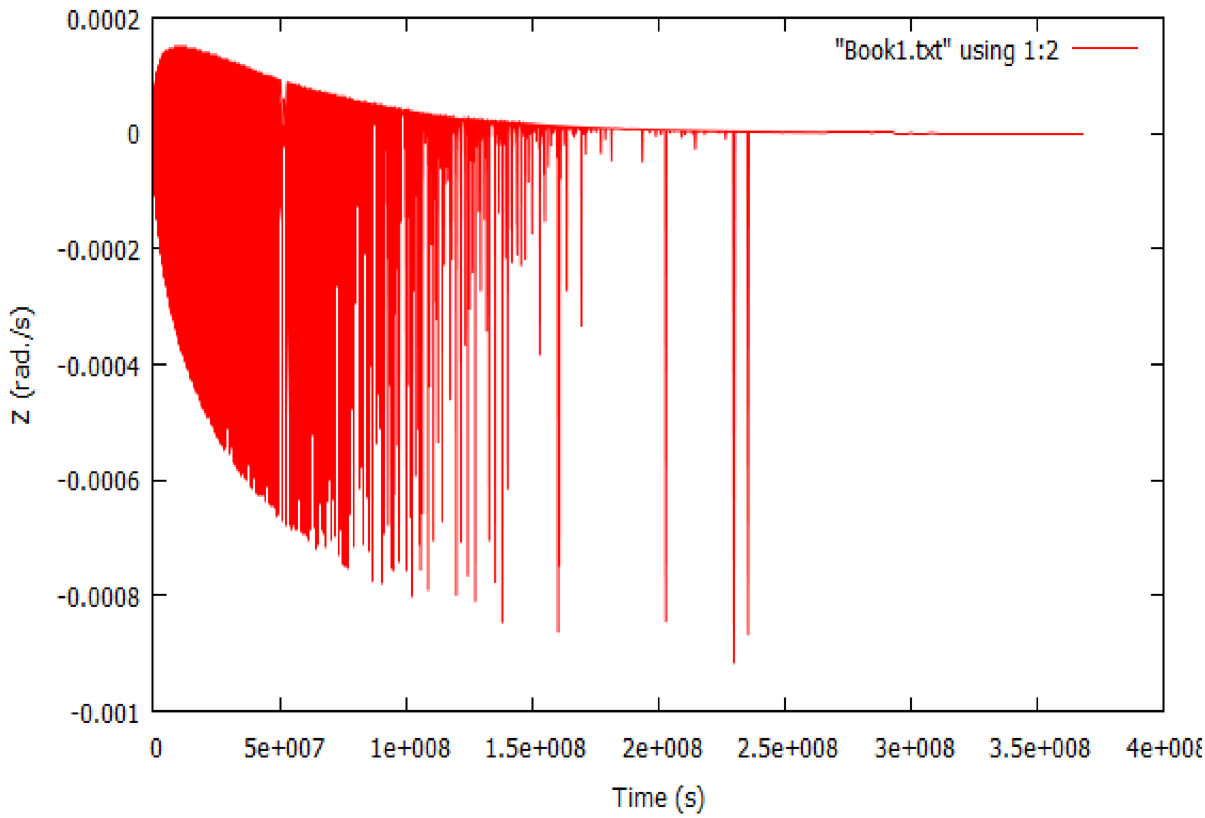

Fig. 3.5. This **r**epresents the graph ofthe characteristic angular frequency Z of the carrier wave against time t .

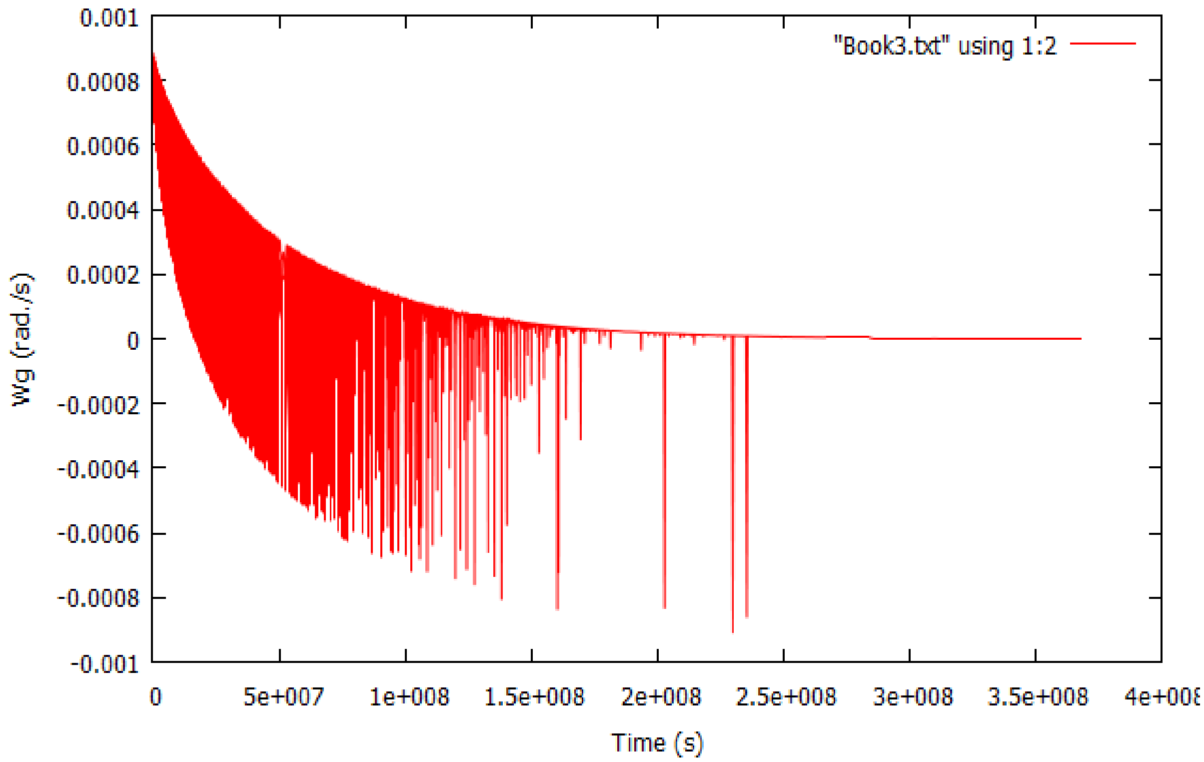

Fig. 3.6. This **r**epresents the graph ofthe group angular velocity Wg of the carrier wave against time t .

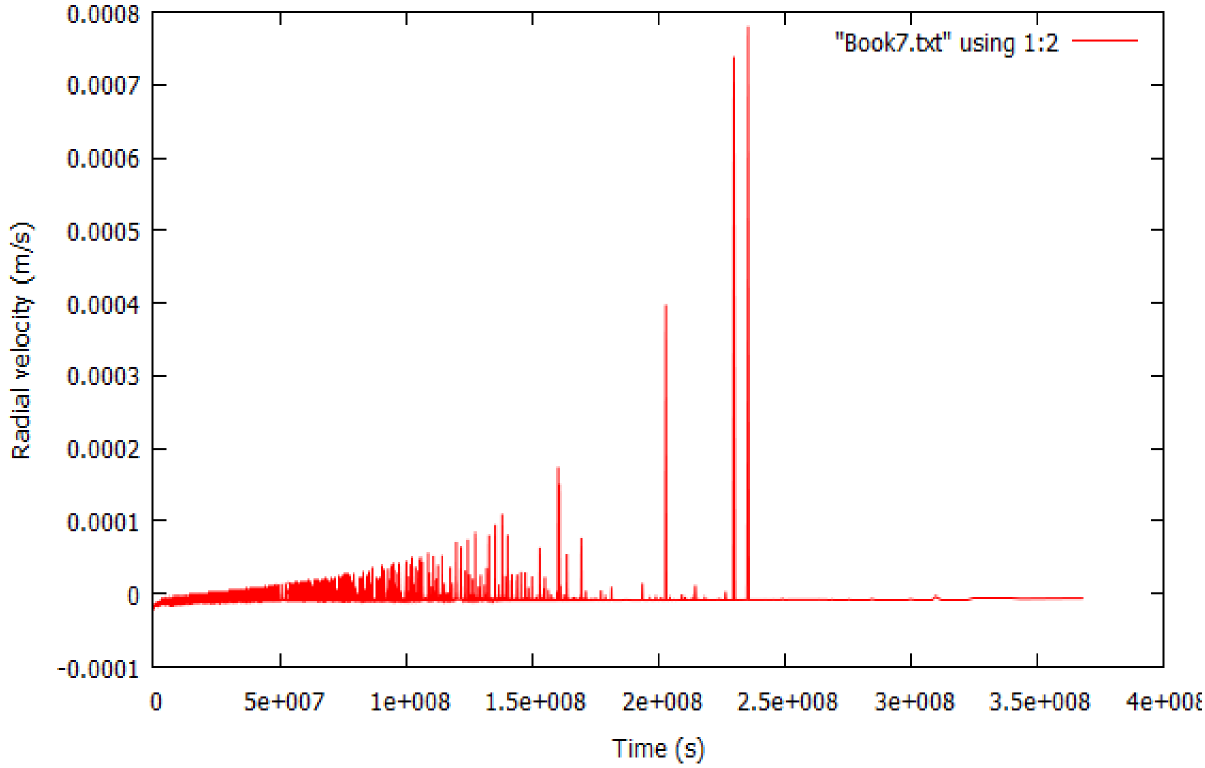

Fig. 3.7. This **r**epresents the graph ofthe radial velocity v_r of the carrier wave against time t .

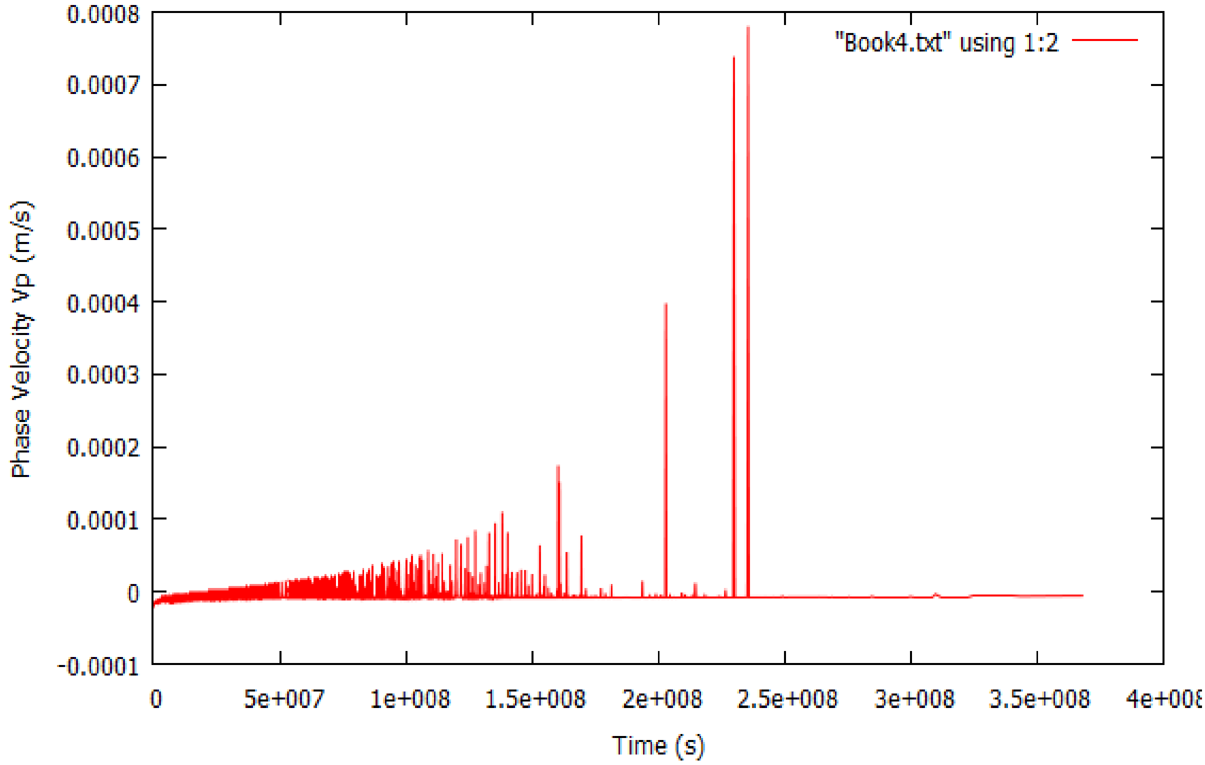

Fig. 3.8. This **r**epresents the graph ofthe phase velocityv_p of the carrier wave against time t .

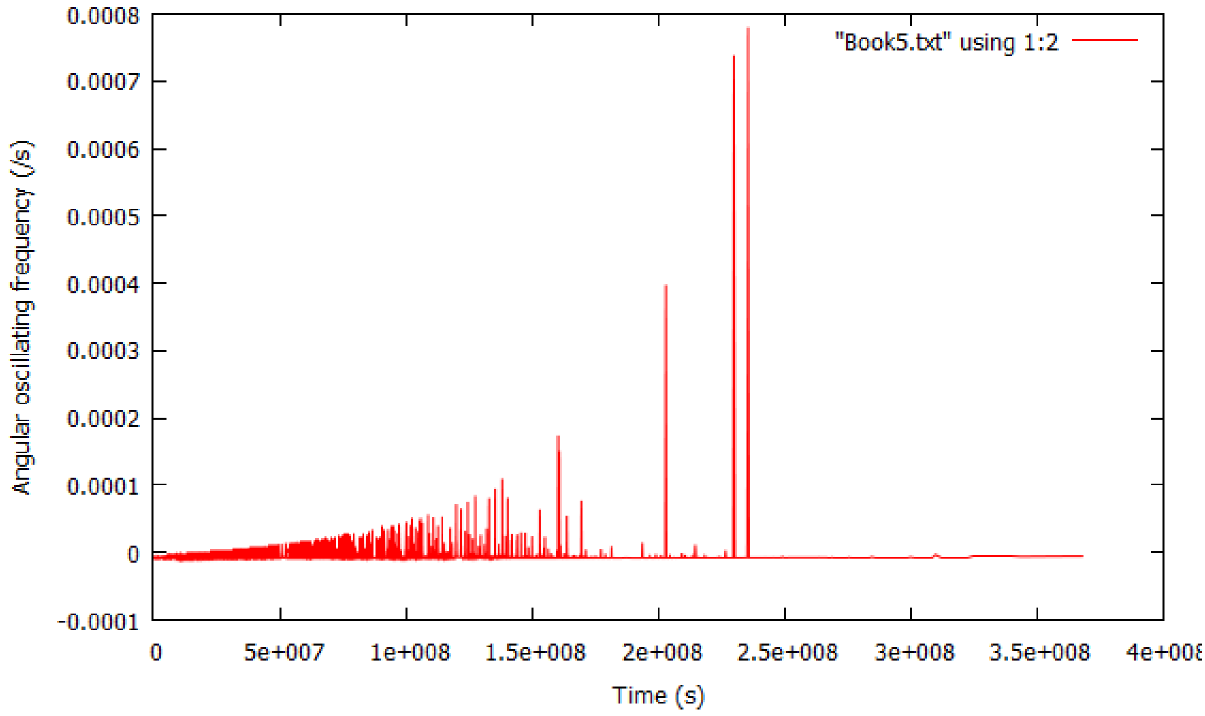

Fig. 3.9. This **r**epresents the graph ofthe angular oscillating frequency ($\dot{\theta}$)of the carrier wave against time t .

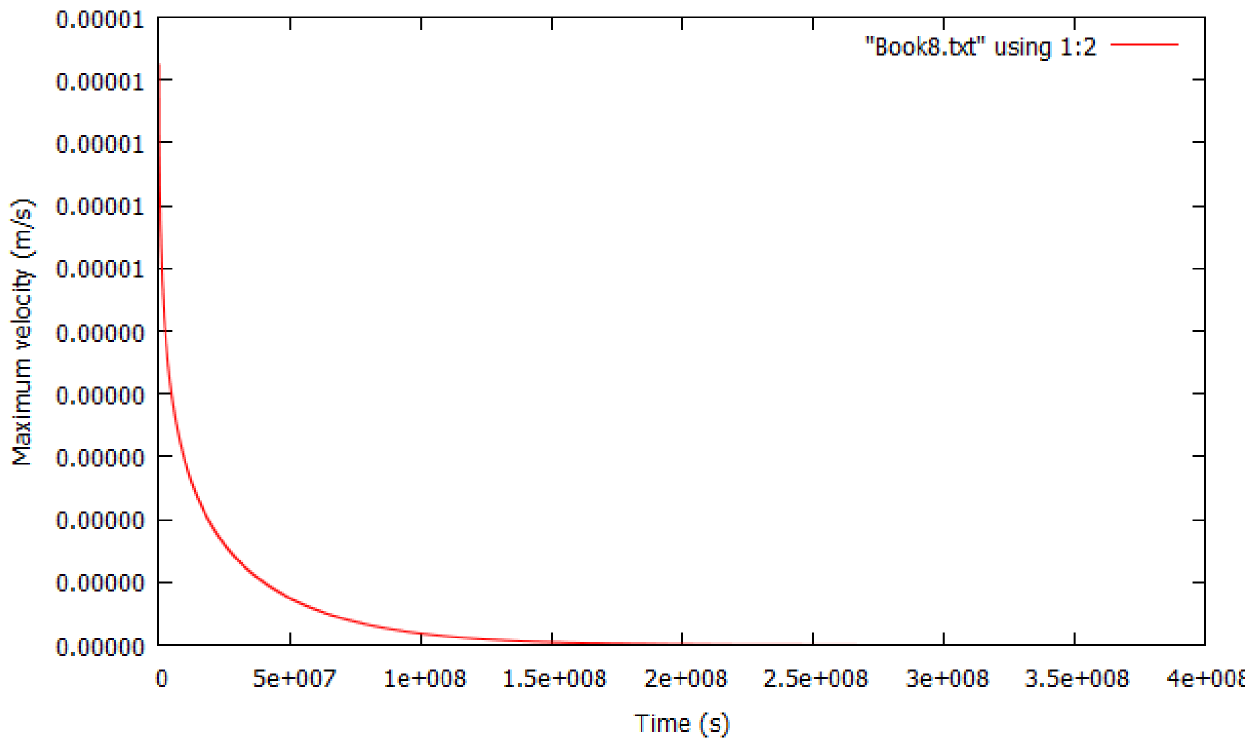

Fig. 3.10. This **r**epresents the graph ofthe maximum velocity v_{max} of the carrier wave against time t .

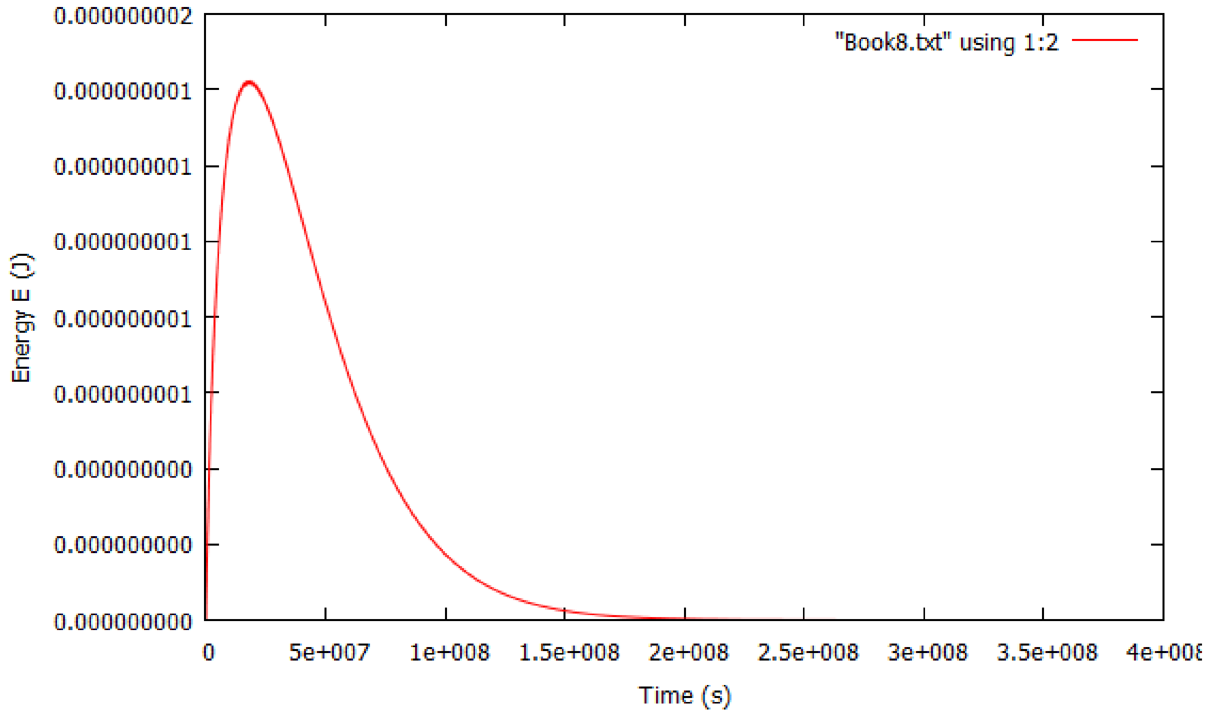

Fig. 3.11. This **r**epresents the graph ofthe attenuated Energy Eof the carrier wave against time t .

IV. DISSCUSION OF RESULTS

The fig.3.1 represents the variation of the spatial oscillating part of the carrier wave with time. It oscillates between the values of ± 1 and converges to zero when the multiplier attainsa maximum value $\lambda = 19332$. Generally, the first significant feature of the spectra given by the figs 3.1, 3.2, 3.3 and 3.4 is the definite singularity at time t = 5 x 10^7s (19 months) or about 2 years. This definite singularity is an indication of the attenuating constituent parameters of the physical system of the resident 'host wave' due to the presence of the interfering 'parasitic wave'. However, the host system further renormalizes to cancel the systems defect and hence the continuous bold spectra.

The second obvious significant feature common to the figures is the depletion in the spectra at t = 1.7 x 10^8 s (65 months) or about 5 years. The interpretation of this depletion is that the interfering 'parasitic wave' is now taking dominant control of the resident host system. Thus the constituent parameters of the 'parasitic wave' are gradually becoming equal to those of the 'host wave'. This results to the decay in the active constituents of the host system. The destructive effect of the interfering wave in the host system is now becoming very intense and difficult to control.Finally, the graphs show that the spectra of the physical system represented by the carrier wave develops failure and hence breakdown if uncontrolled around $t \geq 2.5 \times 10^8\, s$ (96 months) or about 8 years. This is marked by the faint and sharp separations in the spectra lines of figs 3.1 - 3.4. The physical system described by the constituted carrier wave goes to zero when $t = 367894196s$ which is about 12 years.

The Fig. 3.2 shows that within the first 2 years (t = 5 x 10^7s) when the multiplicative factor $0 \leq \lambda \leq 14633$, the total phase angle E experiences both positive and negative increase in values. Also the figure reveals a bold spectrum in the total phase angle of the constituted carrier wave in the interval of the multiplicative factor $0 \leq \lambda \leq 18874$and time $0 \leq t \leq 1.7$x10^8s ($0 \leq t \leq 5$years). In this regime the phase angle of the 'host wave' is fluctuating between both positive and negative values thereby undergoing constructive and destructive interactions with that of the 'parasitic wave'. Beyond this interval, that is, when the time $t \geq 1.7 \times 10^8\, s$ and the multiplier $18875 \leq \lambda \geq 19332$, the spectrum lines of the total phase angle becomes faint and sharp showing a steady depletion in the total phase angle of the carrier wave.The total phase angle of the constitutive carrier wave has maximum and minimum value of ± 1.5468 rad. However, the spectrum does not finally go to zero rather it diverges even when the multiplier has attained its maximum value $\lambda = 19332$.

In Fig. 3.3, the amplitude of the carrier wave is initially imaginary in the interval $0 \le \lambda \le 1866$ and $0 \le t \le 122s$ while the amplitude of thecarrier wave is $i\,0.00418631m \le A \le i\;0.00405311m$. However, it is the absolute values of the amplitude that we used in the graphical presentation. Subsequently, the amplitude is made up of the imaginary and real part, $A = A_1 + iA_2$ in the interval of the multiplier $0 \le \lambda \le 6432$, and the time $0 \le t \le 6287092s$ and the amplitude of thecarrier wave is $i\,0.00418631m \le A \le i\;0.00018918m$. This shows that the motion is actually two-dimensional (2D). Thus A_1 and A_2 are the components of the amplitude in x and y - directions, and A is tangential to the path of the moving amplitude in the carrier wave. This region of real and imaginary values of the amplitude is an indication of the physical system to guide and renormalizes the system against the effect of the interfering'parasitic wave'.

There is constant agitation by the intrinsic parameters of the 'host wave' to suppress the destructive influence of the interfering 'parasitic wave' in this region. The intrinsic parameters of the 'host wave' are posing a serious resistance to the destructive tendency of the 'parasitic wave'.The effect of the imaginary decay in the amplitude is unnoticeable or inadequately felt by the physicalsystem in this interval. Although, unnoticeable as it may, but so much imaginary destructive harm would have been done to the intrinsic constituent parameters of the 'host wave'.

Beyond this interval the amplitude of the carrier wave begins to fluctuate with only real values in the interval of the multiplier $6433 \le \lambda \le 19233$and the time 6289274 s $\le t \le$241907463 sor(73 days$\le t \le$8 years)and the amplitude of thecarrier wave is $0.00337988m \le A \le 0.00054097m$. In this region the interfering wave is now taking absolute effect on the dynamic mechanical system of the 'host wave'. In other words, counting from the moment the 'parasitic wave' interferes with the 'host wave', there would be absolute indication and manifestation of the 'parasitic wave' after 73 days. The non-consistent attenuating behavior in this interval is a consequence of the fact that the amplitude of the carrier wave do not steadily go to zero, rather it fluctuates. The fluctuation is due to the constructive and destructive interference of both the 'host wave' and the 'parasitic wave'. In the regions where the amplitude of the carrier wave is greater than either of the amplitude of the individual wave, we have constructive interference, otherwise, it is destructive.

Our calculation shows that there is a steady exponential decrease in the values of the amplitude in the interval $242359220s \le t \le 367894196s$ (8 years$\le t \le$12 years) when the raising multiplier $19234 \le \lambda \le 19332$. This consistent decrease leads to a gradual reduction and weakening in the initial strength of theconstituents of the system of the host. Consequently, the amplitude of the constituted carrier wave consistently attenuates to zero when the raising multiplier $\lambda \ge 19332$and the time$t \ge 367894196$ s or about 12 years. The amplitude of thecarrier wave thus varies asymptotically between $0.00053612m \le A \le 0.00010378m$.

The graph of the carrier wave against time is shown in fig. 3.4. Of course we know that the carrier wave is the product of the amplitude and the spatial oscillating phase. Consequently, the calculated values of the amplitude which is the maximum displacement from some origin are usually greater those of the carrier wave. As we have said before now, the spectrum of the carrier wave is similar to those of figs, 3.1 and 3.2. The carrier wave experiences a steady damping process and it is consistently attenuated to zero. The spectrum of the carrier wave was initially bold with several regular discontinuities most noticeably in the time interval 1.5 x 10^8 s$\le t \le$2 x 10^8 s (about 5 years). The attenuation of the carrier wave to zero is rapid in the interval when the raising multiplier $19234 \le \lambda \le 19332$ and the time 2.5 x 10^8 s$\le t \le$4 x 10^8 s. This of course represents the interval of the predominance of the dynamic constituents of only the interfering 'parasitic wave',while those of the resident 'host wave' are critically undergoing damping.

Thus the phenomenon of AIDS actually occurs in the interval when the raising multiplier $19234 \le \lambda \le 19332$and the time 242359220 s$\le t \le$367894196 s or (8 years$\le t \le$12 years). Consequently, within this interval the physical system under study can no longer annul the destructive effect of the 'parasitic wave'. The carrier wave which describes the coexistence of the 'host wave' and the 'parasitic wave' ceases to exist around 12 years after interference. This is as a result of the fact that all the active constituents of the 'host wave' would have been completely attenuated by the influence of the interfering 'parasitic wave'.

The spectrum of the characteristic angular velocity Z and that of the group angular velocity w_g is represented in figs. 3.5 and 3.6. They are both negatively attractive showing a significant affinity between the resident 'host wave' and the 'parasitic wave'. It is ctear from the fiures that both physical quantities of the carrier wave attenuate to zero after a sufficiently long time. However, the spectrum of the characteristic angular velocity has a wider spectrum than that of the group angular velocity.The spectrum of figs. 3.7, 3.8 and 3.9 are also very similar. The radial velocity, the phase velocity and the oscillating angular frequency of the carrier wave increases positively with increase in time. They all attenuate to zero at $t \geq 2.4 \times 10^{8}$ s (about 8 years). This exemplary behavior is synonymous with most physical system to produce enhanced efficiency before the system malfunction or complete failure sets in.

The maximum velocity attainby the carrier wave is shown in fig. 3.10. The velocity decreases to zero when the time $t \geq 2 \times 10^{8}$ (about 6 years). Although, the velocity is zero but there is still a residual velocity that keeps the carrier wave going.

Finally, the energy of the constituted carrier wave as shown in fig. 3.11, first increases beforeit decreases exponentially to zero when the time is $t \geq 2 \times 10^{8}$ s. The explanation is that the energy attenuation process is not instantaneous and consistent. The lack of consistency is as a result of the constituents of the resident 'host wave' in the carrier wave putting a serious resistance to the destructive influence of the interfering 'parasitic wave'.

V. CONCLUSION

The existence and life span of any physically active system described by the constituted carrier wave is thus determined by the functional index γ. If the index factor issay zero $\gamma = 0$, the carrier wave would have lasted for 2943609286 s (93 years)before it would have attenuated to zero. This study shows that the process of attenuation in most physically active system does not obviously begin immediately. The wave function that defines the activity and performance of most system is guided by some internal factor which enables it to resist any external interfering influence which is destructive in nature. The anomalous behaviour exhibited by the carrier wave at some point during the damping, is due to the resistance pose by the carrier wave in an attempt to annul the destructive effects of the interfering wave. It is evident from this work that when a carrier wave is undergoing attenuation, it does not steadily or consistently come to rest; rather it shows some resistance at some point in time during the damping process, before it finally comes to rest. Consequently, the existence or the life span of any physically active system is determined by the resistance ofits basic intrinsic parameters to the destructive influence of any external factor.

5.1 Suggestions for further work

This study in theory and practice can be extended to investigate wave interference and propagation in three- dimensional (3D) system. The constituted carrier wave we developed in this work can be utilized in the deductive and predictive study of wave attenuation in exploration geophysics and telecommunication engineering. This work can also be extended to investigate energy attenuation in a HIV/AIDS patient.

APPENDIX: Vector representation of the superposition of the 'host wave' and the 'parasiticwave'.

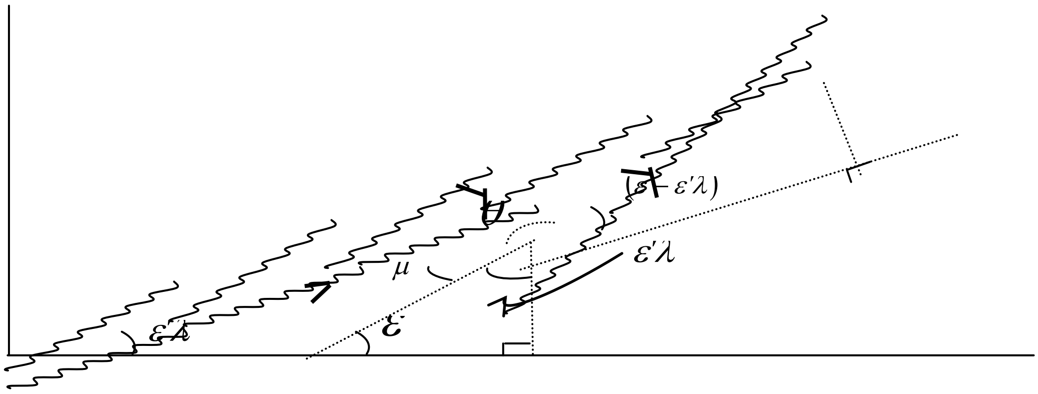

Fig.A1. This represents the resultant wave or the constitutive carrier wave y after the superposition of the 'parasitic wave' y_2 on the 'host wave' y_1.
$\mu + \varepsilon'\lambda + 180^0 - \varepsilon = 180^0$; $\mu = \varepsilon - \varepsilon'\lambda$; $\theta = 180^0 - (\varepsilon - \varepsilon'\lambda)$; $\theta = \pi - (\varepsilon - \varepsilon'\lambda)$

REFERENCES

[1] **David Halliday, Robert Resnick and Jearl Walker**
Fundamentals of Physics, 6th Edition, John Weley and Sons, Inc. New york 2001 : p378.

[2]. **Lain G. Main (1995).** Vibrations and waves in physics
Cambridge University Press, third edition.

[3] **Lipson S.G., Lipson H. and Tannhauser (1996).** Optical physics
Cambridge University Press third edition.

[4] **Brillouin L. (1953).** Wave propagation in periodic structure
Dover, fourth edition New York.