On A Locally Finite In Ditopological Texture Space

[1,]I.Arockia Rani , [2,]A.A.Nithya

ABSTRACT

The present study deals with the new concept namely α- para compactness in ditopological texture spaces. Also we develop comprehensive theorems using paracompactness and α-open sets. Many effective characterizations and properties of this newly developed concept are obtained.

Keywords : *Texture spaces, Ditopology, Ditopological Texture spaces, α-paracompactness, α-locally finite, α-locally co-finite. 2000 AMS Subject Classification. 54C08, 54A20*

I. INTRODUCTION

In 1998 L.M.Brown introduced on attractive concept namely Textures in ditopological setting for the study of fuzzy sets in 1998. A systematic development of this texture in ditopology has been extensively made by many researchers [3,4,5,7].The present study aims at discussing the effect of α-paracompactness in Ditopological Texture spaces. Let S be a set, a texturing T of S is a subset of P(S). If

(1) (T,⊂) is a complete lattice containing S and φ, and the meet and join operations in (T,⊂) are related with the intersection and union operations in (P(S), ⊂)by the equalities
Λi∈I Ai = ∩i∈I Ai , Ai ∈ T , i ∈ I , for all index sets I, while
Vi∈I Ai = ∪i∈I Ai , Ai ∈ T , i ∈ I , for all finite index sets I.

(2) T is completely distributive.

(3) T separates the points of S. That is, given s1 ≠ s2 in S we have A ∈ T with s1∈ A, s2 ∉ A, or A ∈ T with s2 ∈ A, s1 ∉ A.

If S is textured by T we call (S,T) a texture space or simply a texture.
For a texture (S, T), most properties are conveniently defined in terms of the p-sets Ps =∩{A ∈ T \s ∈ A} and the q-sets, Qs = V{A ∈ T /s ∉ A}: The following are some basic examples of textures.
Examples 1.1. Some examples of texture spaces,
(1) If X is a set and P(X) the powerset of X, then (X; P(X)) is the discrete texture on X. For x ∈ X, Px = {x} and Qx = X \{x}.
(2) Setting I = [0; 1], T= {[0; r); [0; r]/r ∈ I } gives the unit interval texture (I; T). For r ∈ I , Pr = [0; r] and Qr = [0; r).

(3) The texture (L;T) is defined by L = (0; 1], T ={(0; r]/r∈I}For r ∈ L, Pr (0; r] = Qr .

(4) T={φ, {a, b}, {b}, {b, c}, S} is a simple texturing of S = {a, b, c} clearly Pa= {a, b}, Pb ={b}and Pc = {b, c}.
Since a texturing T need not be closed under the operation of taking the set complement, the notion of topology is replaced by that of dichotomous topology or ditopology, namely a pair (τ, κ) of subsets of T, where the set of open sets τ satisfies
1. S, φ ∈ τ
2. G1, G2 ∈ τ then G1 ∩ G2 ∈ τ
3. Gi ∈ τ , i∈ I then Vi Gi ∈ τ ,
and the set of closed sets κ atisfies
1. S, φ ∈ κ
2. K1; K2 ∈ κ then K1 ∪ K2 ∈ κ and
3. Ki ∈ κ, i ∈ I then ∩Ki ∈ κ. Hence a ditopology is essentially a ’topology” for which there is no a priori relation between the open and closed sets.

For $A \in T$ we define the closure $[A]$ or cl(A) and the interior $]A[$ or int(A) under (τ, κ) by the equalities $[A] = \cap\{K \in \kappa / A \subset K\}$ and $]A[= \vee\{G \in \tau / G \subset A\}$:

Definition 1.2. For a ditopological texture space (S; T; τ, κ):

1. $A \in T$ is called pre-open (resp. semi-open, β-open) if $A \subset$ intclA (resp. $A \subset$ clintA; $A \subset$ clintclA). $B \in T$ is called pre-closed (resp. semi-closed, β-closed) if clintB $\subset B$ (resp. intclB $\subset B$; intclintB $\subset$ B)

We denote by PO(S; T; τ, κ) (βO(S; T ; τ, κ)), more simply by PO(S) (βO(S)) , the set of pre-open sets (β-open sets) in S. Likewise, PC(S;T; τ, κ) (βC (S; T ; τ, κ)), PC(S) (βC(S)) will denote the set of pre-closed (β-closed sets) sets.

As in [3] we consider the sets $Q_s \in T$, $s \in S$, defined by $Q_s = \vee\{P_t \mid s \notin P_t\}$

By [1.1] examples, we have $Q_x = X/\{x\}$, $Q_r = (0, r] = P_r$ and $Q_t = [0, t)$ respectively. The second example shows clearly that we can have $s \in Q_s$, and indeed even $Q_s = S$.

Also, in general, the sets Q_s do not have any clear relation with either the set theoretic complement or the complementation on T. They are, however, closely connected with the notion of the core of the sets in S.

Definition.1.3 For $A \in T$ the core of A is the set core(A) $= \{ s \in S \mid A \not\subset Q_s \}$.

Clearly core(A) $\subset$ A, and in general we can have core(A) $\not\subset$ T . We will generally denote core(A) by A^b

2. Dicovers and α-locally finite

Definition 2.1 A subset C of $T \times T$ is called a difamily on (S,T). Let $C=\{(G_\alpha , F_\alpha)/\alpha \in A\}$ be a family on (S,T). Then T is called a dicover of (S,T) if for all partitions A_1, A_2 of A, we have

$\cap_{\alpha \in A_1} F_\alpha \subset \vee_{\alpha \in A_2} G_\alpha$

Definition 2.2 Let (τ, κ) be a ditopology on (S,T). Then a difamily C on (S, T, τ, κ). is called α-open(co-α-open) if dom(C) $\subset$ αO(S) (ran(C) $\subset$ αO(S)).

Definition 2.3 Let (τ, κ) be a ditopology on (S,T). Then a difamily C on (S, T, τ, κ). is called α-closed(co-α closed) if dom(C) $\subset$ αC (S). (ran(C) $\subset$ αC (S)).

Lemma 2.4 [3] Let (S, T) be a texture. Then $P=\{(P_s , Q_s) \mid s \in S^b \}$ is a dicover of S.

Corollary 2.5 [3] Given $A \in T$, $A = \emptyset$, there exists $s \in S^b$ with $P_s \subset A$.

Definition 2.6 Let (S, T) be a texture, C and C' difamilies in (S, T). Then C is said to be a refinement of C0 , written C < C0. If given A C B we have

A0 C0 B0 with $A \subset A0$ and $B0 \subset B$. If C is a dicover and C < C0 , then clearly C0 is a dicover.

Remark 2.7 Given dicovers C and D then $C \wedge D = \{(A \cap C, B \cup D) \mid A\ C\ B, C\ D\ D\}$ is also a dicover. It is the meet of C and D with respect to the refinement relation.

Definition 2.8 Let $C = \{G_i , F_i) \mid i \in I\}$ be a difamily indexed over I. Then C is said to be (i)Finite (co-finite) if dom(C) (resp., ran(C)) is finite.

(ii) α-Locally finite if for all $s \in S$ there exists $K_s \in$ αC (S) with $P_s \not\subset K_s$ so that the set $\{i \mid G_i \not\subset K_s\}$is finite.

(iii) α-Locally co-finite if for all $s \in S$ with $Q_s \neq S$ there exists $H_s \in$ αO(S) with $H_s \not\subset Q_s$ so that the set $\{I \mid H_s \not\subset F_i\}$ is finite.

(iv) Point finite if for each $s \in S$ the set $\{i \mid P_s \subset G_i\}$ is finite.

(v) Point co-finite if for each $s \in S$ with $Q_s \neq S$ the set $\{i \mid F_i \subset Q_s\}$ is finite.

Lemma 2.9 Let (S, T, τ, κ) be a ditopological texture space and C be a difamily then, the following are equivalent:

1. $C=\{(G_i , F_i) \mid i \in I\}$ is α locally finite.

2. There exists a family $B = \{Bj \mid j \in J\} \subset T/\{\emptyset\}$ with the property that for $A \in T$ with $A = \emptyset$, we have $j \in J$ with $Bj \subset A$, and for each $j \in J$ there is $Kj \in \alpha C(S)$ so that $Bj \not\subset Kj$ and the set $\{i \mid Gi \not\subset Kj\}$ is finite.
Proof. Straightforward.

Lemma 2.10 Let (S, T, τ, κ) be a ditopological texture space and C be a difamily then, the following are equivalent:
(a) $C=\{(Gi , Fi) \mid i \in I\}$ is α-locally co-finite.

(b) There exists a family $B = \{Bj \mid j \in J\} \subset T/\{S\}$ with the property that for $A \in T$ with $A \neq S$, we have $j \in J$ with $A \subset Bj$, and for each $j \in J$ there is $Hj \in \alpha O(S)$ so that $Hj \not\subset Bj$ and the set $\{i \mid Hi \not\subset Fj\}$ is finite.
Theorem 2.11 The difamily $C=\{(Gi , Fi) \mid i \in I\}$ is α locally finite if for each $s \in S$ with $Qs \neq S$ we have $Ks \in \alpha C(S)$ with $Ps \not\subset Ks$, so that the set $\{i \mid Gi \not\subset Ks\}$ is finite.
Proof. Given $C=\{(Gi , Fi) \mid i \in I\}$ is α locally finite ,then by Lemma 2.9 there exists a family $B = \{Bj \mid j \in J\} \subset T/\{\emptyset\}$ with the property that for $A \in T$ with $A \neq \emptyset$, we have $j \in J$ with $Bj \subset A$, and for each $j \in J$ there is $Kj \in \alpha C(S)$ so that $Bj \not\subset Kj$ and the set $\{i \mid Gi \not\subset Kj\}$ is finite. Now take $B = \{Ps \mid Qs \neq S\} = \{Ps \mid s \in Sb\}$, and for $A \in T$ and A is nonempty, then by corollary 2.5 there exists $s \in Sb$ with $Ps \subset A$.
Therefore for every $s \in S$, there exists $Ks \in \alpha C(S)$ with $Ps \not\subset Ks$ such that $\{i \mid Gi \not\subset Ks\}$ is finite.
Theorem 2.12 Let (S, T, τ, κ) be a ditopological texture space and C be a α-locally finite dicover and $s \in S$. Then there exists A C B with $s \in A$ and $s \notin B$.
Proof. Given $C=\{(Ai , Bi) \mid i \in I\}$ be α-locally finite. Take $K \in \alpha C(S)$ with $s \notin K$ and $\{i \in I \mid Ai \not\subset K\}$ is finite. Now partition the set I into two sets such that $I1=\{i \in I \mid s \in Ai\}$ and $I2 = \{i \in I \mid s \notin Ai$, since C is a dicover it should satisfy
$\cap_{i \in I1} Bi \subset \vee_{i \in I2} Ai$
now $\vee_{i \in I2} Ai$ does not have s according to our partition, which implies $s \notin \cap_{i\in I1} Bi$. Thus we arrived at for all $i\in I1$, $s \in Ai$ and $s \notin Bi$.(i.e) $s \in A$ and $s \notin B$.

Theorem 2.13 Let (S, T, τ, κ) be a ditopological texture space and $C = \{(Ai, Bi) \mid i \in I\}$ be a difamily.
(1) If C is α-locally finite, then dom(C) is α closure preserving.
(2) If C is α-locally co-finite, then ran(C) is α interior preserving.
Proof. (1) Let I 0 subset of I. We have to prove $\alpha cl(\vee_{i \in I0} (Ai)) = \vee_{i\in I0} \alpha cl(Ai)$. To prove $\alpha cl(\vee_{i\in I0} (Ai)) \subset \vee_{i\in I0} \alpha cl(Ai)$ suppose this is not true, we get there exists $s \in S$ with $s \in \alpha cl(\vee_{i\in I0} (Ai))$ and $s \notin \vee_{i\in I0} \alpha cl(Ai)$

$$\alpha cl(\vee_{i\in I0} Ai) \not\subset Qs \text{ and } Ps \not\subset \vee_{i\in I0} \alpha cl(Ai) -------- (*)$$

Since C is α-locally finite, we have $\{i \in I \mid Ai \not\subset \vee_{i\in I0} \alpha cl(Ai)\}$ is finite. Now partition I 0 into two sets such that
$I1 = \{i \in I0 \mid Ai \not\subset K\}$ and $I2 = I0/I1$

Now $\vee_{i\in I0} Ai =(\vee_{i\in I2} \cup \vee_{i\in I1})Ai$
$=(\vee_{i\in I2} \cup \cup_{i\in I1})Ai$
$\alpha cl(\vee_{i\in I0} Ai) \subset K \cup \cup_{i\in I1} Ai$
using * we can say $\vee_{i\in I0} \alpha cl(Ai) \subset Qs$, which is a contradiction. Therefore $\alpha cl(\vee_{i\in I0} (Ai)) = \vee_{i\in I0} \alpha cl(Ai)$.Hence αclosure is preserving.
(2) It is the dual of (1).

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