Block Mathematical Coding Method of Images Compressed by a SPIHT Algorithm

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ABSTRACT

A method of mathematical block mathematical coding of images compressed by a SPIHT algorithm is developed in the article. Redundancy of information during encoding by a SPIHT algorithm that can further compress the information by entropy coding is identified. An optimization equation which enables us to find an optimal block size for the application in the block mathematical coding is obtained.

Keywords: *compression, image, SPIHT, mathematical coding, block, network*

I. INTRODUCTION

Works [1, 2] show the need for progressive compression of multimedia content, to improve the performance of telecommunication systems and networks. The introduction into telecommunications systems' and networks' infrastructure of intermediate servers of information repacking will significantly reduce the optional content traffic, which will be initially delivered in a lowered quality, and will finish loading to displaying in a full quality if needed only. The existing systems can display only fully accepted piece of multimedia information, which can lead to a complete neglect of an incomplete fragment. An approach proposed in [1, 2] solves a problem of the service denial, which is to display multimedia content without delays, with quality comparable to the amount of the received information. However, the proposed approach may be improved by using a higher multimedia information compression ratio while maintaining the quality of multimedia traffic. The presented article is devoted to solving this problem.

II. COMPARISON OF BIT DENSITIES BY USING DIFFERENT METHODS OF ENCODING INFORMATION

The main methods that claim to be used in a system of intermediate recompression are JPEG2000 and SPIHT. However, the redundancy of information in a packed SPIHT image has a greater degree. This can be shown by calculating the information entropy of the image packing result. Taking into consideration that the coding of both methods is bitwise, we establish a ratio between a unit and a zero bit. The values of ratios may for convenience be taken for each kilobyte of a file, Fig. 1, 2, 3.

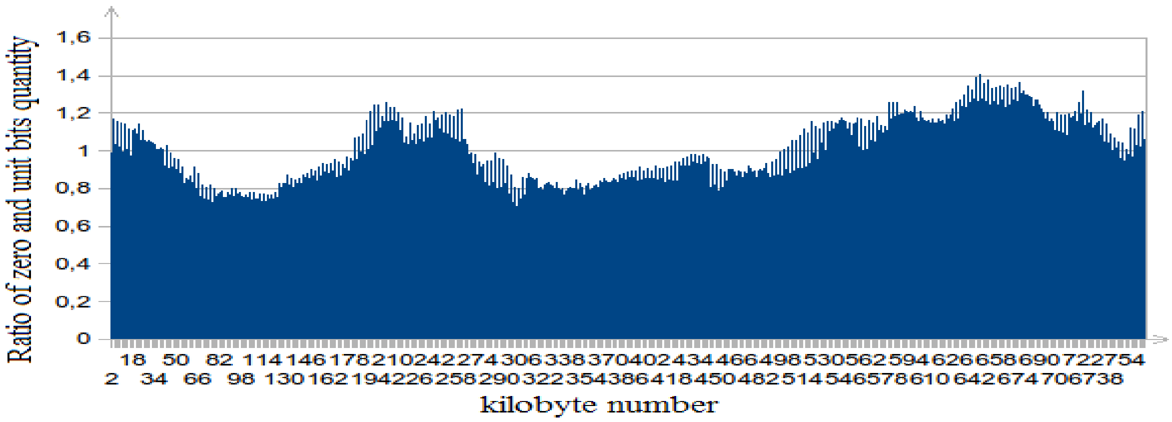

Fig. 1. The ratio between the quantity of a zero and unit bits for the encoded image bmp

The test result shown in Figure 1. graph demonstrates the uneven distribution of zero and unit bits in the source non-coded image. Applying an image coding by method JPEG2000 helps to get rid of redundant information that leads to a uniform probability distribution of reading of a unit and zero bit (Figure 2).

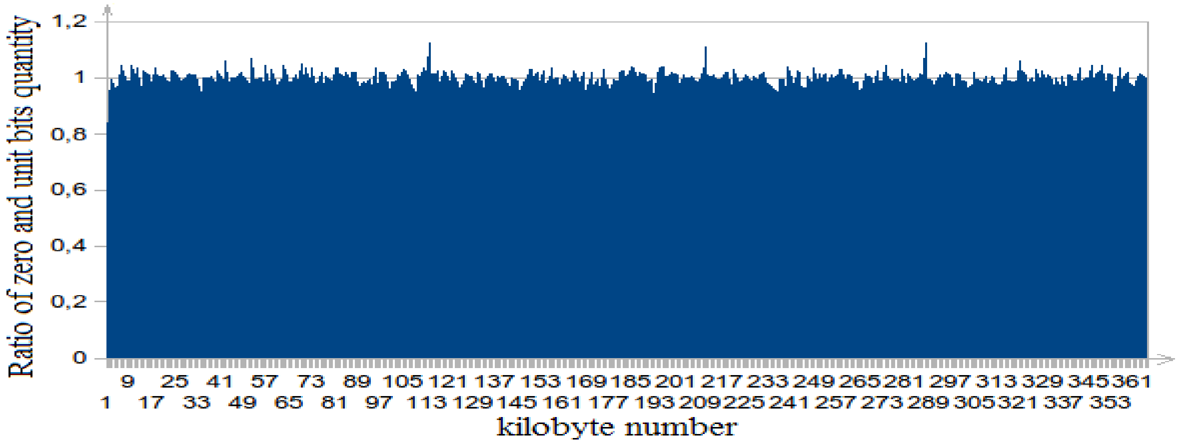

Fig. 2. The ratio between the quantity of a zero and unit bits for JPEG2000 encoded image

This indicates an absence of redundancy in compression by JPEG2000 method. Considering SPIHT coding, it can be noted that the SPIHT algorithm realizes the transmission of bit sets along the appropriate bit segments, resulting in more frequent transmission of a zero bit. Herewith, when the number of significant pixels increases a unit is transmitted more frequently (Fig. 3).

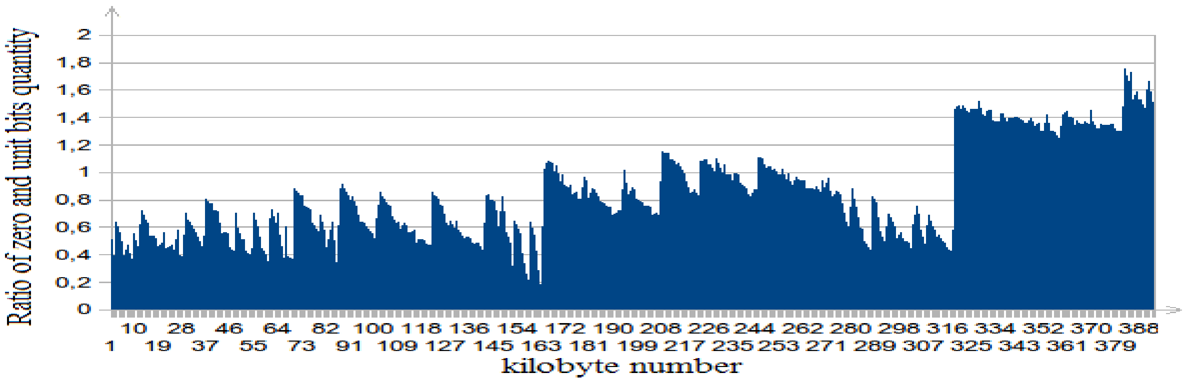

Fig. 3. The ratio between the number of zero and unit bits for the encoded image SPIHT

The situation with the uneven distribution of the ratio of zero and unit bits is the result of peculiarities of SPIHT coding and absence of additional entropy compression, which is present in JPEG2000. If we encounter a situation of uneven bit-encoding when one bit has a fractional value of a bit in the result file, mathematical implementation of compression [9, 10] is suitable, which is in alignment probability of following bit by eliminating of redundant information.

Fig. 3 shows the ratio of unit bits to zero ones, which has no constant value, and changes dramatically, not only in the transition from one bit field to another, but also from one image zone to the other. That's why the use of a constant factor for the probabilities of 0 or 1 is not suitable for receiving a minimum size of the output file. Therefore, these probabilities should vary depending on a specific local value. For adaptive arithmetic compression of information, this contains an array of a fixed number of last received data for continuous refinement of probability, for which there is a patent restriction. It is therefore necessary to provide for block coding, for which the probability of values of bits "1" or "0" is separately determined. It is necessary to determine the size of the block and set up the likelihood of a unit bit.

Let's suppose that to record the proportion of unit bits for Ni bits of a file, the recording of fixed-point sizes in b bits is used: p(1)=1-p(0). Then the size of the code within a block will be as follows:

$$N_i^* = -N_i\left[p(0)\log_2 p(0) + p(1)\log_2 p(1)\right] + b\ , \qquad (1)$$

where N^*_i is the size of a packed part; N_i, p(0) is the probability of occurrence of zero bit equal to N_{i0}/N_i; p(1) is the probability of occurrence of a unit bit equal to N_{i1}/N_i. Using the formula of determining the probabilities, let's exclude of (1) the number of unit and zero bits, leaving only the probability of a unit bit and the size of the encoded part of an image:

$$N_i^* = -N_i\left[(1 - p(1))\log_2(1 - p(1)) + p(1)\log_2 p(1)\right] + b \qquad (2)$$

where the first summand depends on the properties of information, and the second is the introduced above constant. A full size of the file will then be the sum of the sizes of packaged blocks:

$$N^* = -\sum_{i-1}^{n} N_i\left[(1 - p(1))\log_2(1 - p(1)) + p(1)\log_2 p(1)\right] + n \cdot b \qquad (3)$$

where n is a number of parts for image packaging. Compression ratio by entropy method depending on p(1) is shown in Fig. 4. Herewith, in the part for encoding, oscillation probabilities of occurrence of "1" can be seen, so that the degree of compression of small data fragments should give a better result. However, with the increase of the number of parts, it is necessary to use a larger amount of information to record probabilities, *nb* in formula (3).

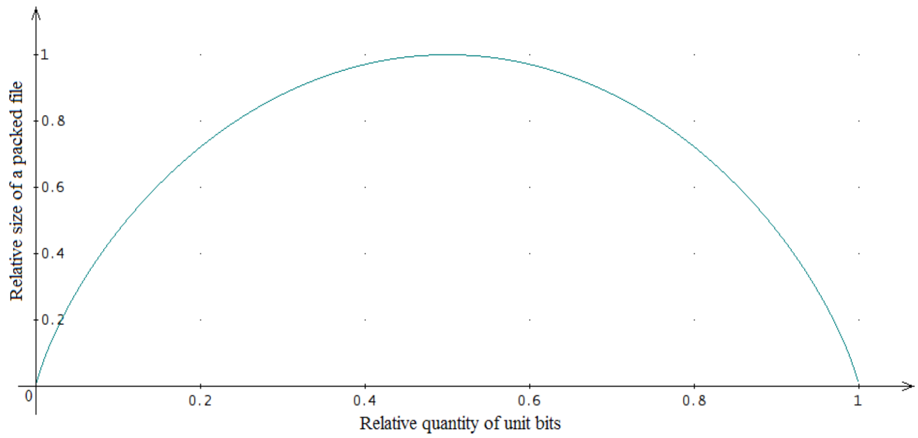

Fig. 4. The dependence of the compression ratio of the share of unit bits

This leads to an optimization problem:

$$N^*(n) = -\frac{N}{n}\sum_{i-1}^{n}\left[(1 - p(1))\log_2(1 - p(1)) + p(1)\log_2 p(1)\right] + n \cdot b \rightarrow \min \qquad (4)$$

The sense of this optimization is a more accurate determination of the probability of occurrence of a unit bit in a code sector, but to record such information it is necessary to allocate a certain amount of information. When the number of periods n is twice increased, the amount of information for recording the coefficients also doubles. This growth must be compensated by improved image compression mode. An example of selection of the intervals is shown in Fig. 5. It is obvious, that the excessive division of the information flow on blocks, the compression ratio may decrease, and as a result a file of a larger size than the original one may be received.

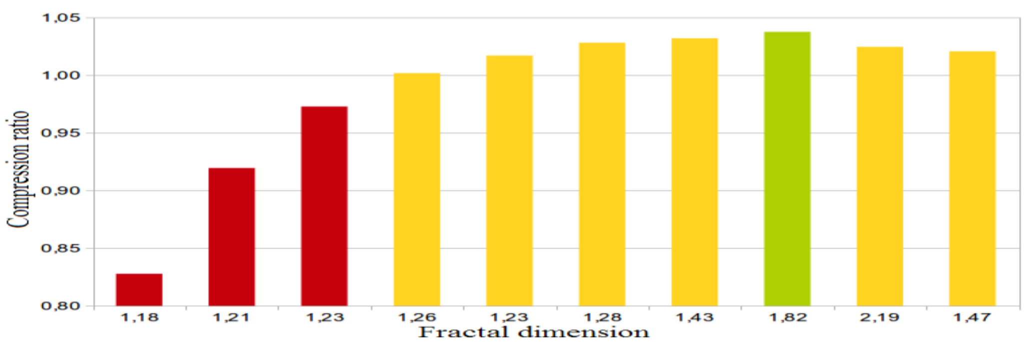

Fig. 5. An experiment of compression by blocks (vertically compression ratio, horizontally fractal dimension of a change in the estimate of probability sampling of a unite bit, intervals changed twice)

We may decide on further reduction of the blocks with generalized information about the proportion of unit bits from the analysis of formula (2). Let's suppose that the block size is halved with the notation:

$$p_{2i,2n}(1) = p_a, p_{2i+1,2n}(1) = p_b, p_{i,n} = p, \qquad p = \frac{p_a + p_b}{2}.$$

One can understand how the degree of compression of one block is changed from the following correlation:

$$K = \frac{2\left(p\log_2(p) + (1-p)\log_2(1-p)\right)}{p_a\log_2(p_a) + (1-p_a)\log_2(1-p_a) + p_b\log_2(p_b) + (1-p_b)\log_2(1-p_b)}, \qquad (5)$$

where $K(p_a,p_b)$ is symmetric relatively to the arguments function which is equal to 1, with $p_a=p_b$, which means there is no change in the size of compressed information. In this case, there will be no benefit from partitioning the data stream into smaller parts. When $p_a \neq p_b$, then $K>1$, and this (Fig. 6) means a decrease of the initial amount of multimedia data, which will be sent to the telecommunication network.

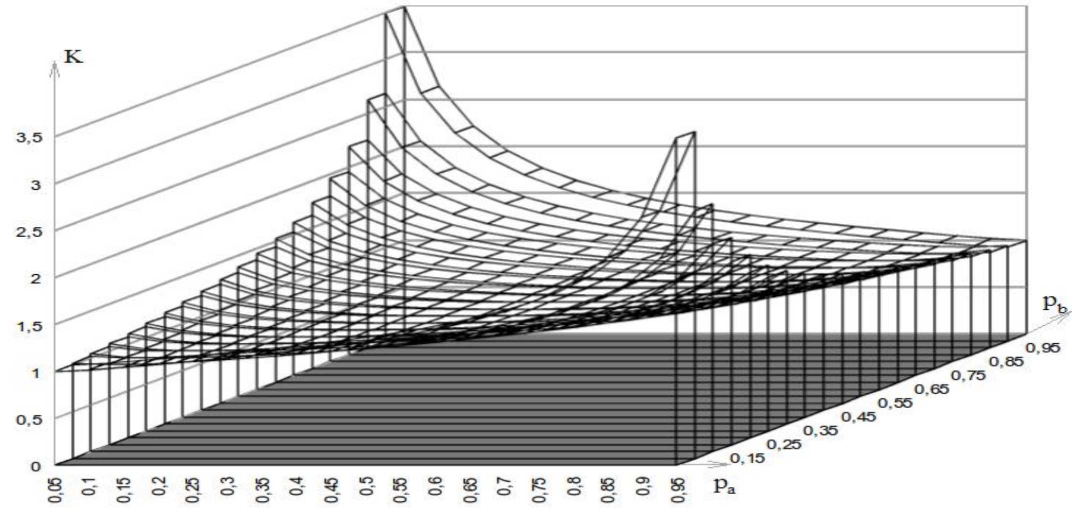

Fig. 6. The dependence of the coefficient of the information K amount reduction while splitting the block of coding $p=p_a+p_b$

In Figure 6, with the schedule $K(p_a,p_b)$ it can be seen that the degree of information compression is greatly increased at the maximum difference between p_a and p_b. For making a decision to reduce encoding blocks, it is necessary to have an estimate of $|p_a-p_b|$. The evaluation may be performed with the help of the concept of fractal dimension of a numerical sequence [11] (Fig. 7, 8).

1. Fractal dimension F is close to 2 (Fig. 7). As the number of reports of the curve increases, the curve gradually fills the plane – new fractures commensurate with the scale of the previous ones. As a consequence, we get the high compression ratio from (4).

2. Fractal dimension of F is close to 1 (Fig. 8). In this case, the specified curve will almost repeat the previous one, and specifying the probability of a sample unit to increase the information compression rate will not succeed.

For a sequence of probabilities $\{p_0,p_1,...,p_{2n-1}\}$ on larger blocks we'll get a set of probabilities:

$$\{(p_0+p_1)/2,(p_2+p_3)/2,...,(p_{2n-2}+p_{2n-1})/2\}$$

at a conditionally unit interval. The lengths of the curves for the first and second sequences are as follows:

$$L_1 = \sum_{i=0}^{n-1}\frac{p_{2i}+p_{2i+1}}{2}\cdot\frac{1}{n}, \qquad L_2 = \sum_{i=0}^{2n-1} p_i \frac{1}{2n}.$$

A local fractal dimension of the curve will be:

$$F(n) = \log_2(L_2/L_1).$$

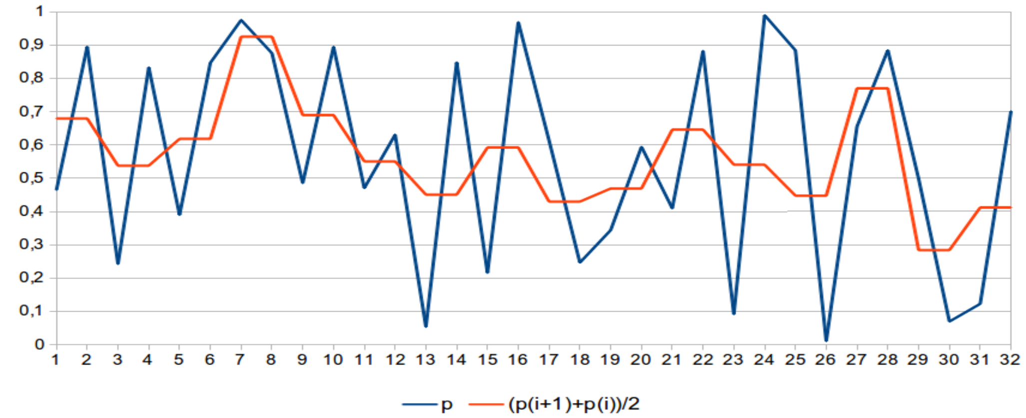

Fig. 7. Clarification of a line with the fractal dimension 1.8

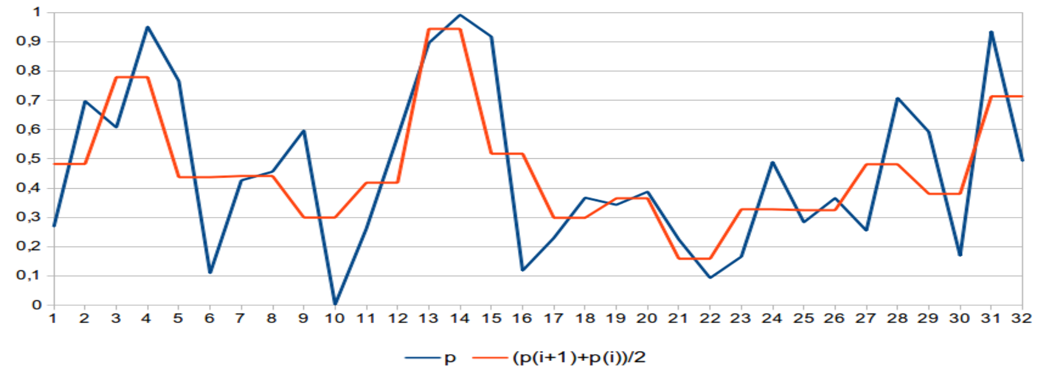

Fig. 8. Clarification of a line with the fractal dimension 1.12

In order to determine the fractal dimension to evaluate the improvement of data compression, we consider two extreme cases:

The dimension is understood here as the number of F, which shows to what extent a should be raised to get a coefficient of the curve elongation, having reduced the "line" at a times. With a length of L, the line has n segments of $\{(p_i,i/n);(p_{i+1},(i+1)/n)\}$. The average length of one segment will be $\langle L\rangle=L/n$. When the number of blocks increases from n_1 to n_2 $(n_1<n_2)$, the line lengthening will be approximately equal to $L_2\approx L_1(n_2/n_1)^F$ and $\langle L2\rangle=L_1(n_2/n_1)^F/n_2$. Here and below, the notation $\langle L\rangle$ is used for an expected value. Finally, the average difference between the successive values of probabilities of unit bits emergency will be:

$$\left\langle \left|\Delta p_2\right|\right\rangle = \left\langle \left|\Delta p_2\right|\right\rangle \left(\frac{n_2}{n_1}\right)^{F-1}. \tag{6}$$

For the initial breakdown of bits sequence which consists of n_1 segments of the fractal dimension F switched to a breakdown of $n_2=zn_1$ segments. It follows from (6) that:

$$\left\langle \left|\Delta p_2\right|\right\rangle = \left\langle \left|\Delta p_2\right|\right\rangle z^{F-1}. \tag{7}$$

The meaning of this expression is in a constant increase in the density difference of unit bits to tackle increasingly shorter subsequences from the main sequence. An increase of the compression ratio K to some limit follows from the correlations (5) and (7). The most pessimistic estimate of the compression ratio change will be if we take p=0,5, $p_a=p-\Delta p/2$, $p_b=p+\Delta p/2$ (Fig. 6):

$$K(F,z) = \frac{n \log_2 0{,}5}{\sum_{i=1}^{n}\left[\left(0{,}5+z^{F-1}/2\right)\log_2\left(0{,}5+z^{F-1}/2\right)+\left(0{,}5-z^{F-1}/2\right)\log_2\left(0{,}5-z^{F-1}/2\right)\right]} \tag{8}$$

However, adding each block with the information about unit bits share, more information of size zb is needed. The optimization task (4) is reduced to finding a minimum of an equation with one kind of variable z:

$$N \cdot K(F,z) + zb \to \min . \tag{9}$$

The objective function (9) is not linear, so its minimum is sought by numerical methods. The problem is quickly solvable because z can take only integer values and (9) has only one minimum because of monotony (8) and a summand zb.

III. CONCLUSIONS

As a result of the research, a method of mathematical block coding of images compressed by an algorithm SPIHT has been developed. In developing this method, the following results were obtained:

1. The results of image compression based on wavelet transformation of JPEG2000 and SPIHT are examined.
2. The presence of redundant information while using SPIHT coding is shown.
3. The degree of compression of SPIHT code with the help of a block arithmetic compression is investigated.
4. A weighting function which allows receiving the size of the block on which the probability of getting a single bit is taken by a constant is obtained.
5. An algorithm allows correct restoring of non-fully received information that retains the properties of the progressive image transmission.

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