Design of Uniform Fiber Bragg grating using Transfer matrix method

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ABSTRACT:
The performance of a uniform fiber Bragg grating is depends on it reflectivity. The reflectivity is again depends on change of effective refractive index and length of the grating. This paper presents the design aspect of an optical fiber Bragg grating for maximum reflectivity. As fiber grating allows considerable amount of energy exchange between different modes of the fiber, couple mode theory which is solved by transfer matrix method is considered as good approximation to calculate the spectral response of fiber Bragg grating.

Key words: *Fiber Bragg grating, couple mode theory, transfer matrix method, reflectivity.*

I. INTRODUCTION:

Fiber Bragg grating is attracting considerable interest for application as sensing element, because of their intrinsic nature and inherent wavelength encoded operation. Fiber optic photosensitivity has indeed opened a new era in the field of fiber optic based devices [1]. The photosensitivity of optical fiber was discovered at the Canadian Communications Research Center in 1978 by Ken Hill et al. [2] during experiments using germanium-doped silica fiber and visible argon ion laser radiation. It was noted that as a function of time, light launched into the fiber was being increasingly reflected. This was recognized to be due to a refractive index grating written into the core of the optical fiber. It has the ability to alter the core index of refraction in a single-mode optical fiber by optical absorption of UV light. The photosensitivity of optical fibers allows the fabrication of phase structures directly into the fiber core, called *fiber Bragg gratings.* Photosensitivity refers to a permanent change in the index of refraction of the fiber core when exposed to light with characteristic wavelength and intensity that depend on the core material.

Fiber Bragg gratings, which operate at wavelengths other than near the writing wavelength , are fabricated by techniques that broadly fall into two categories: those that are holographic [3] and those that are noninterferometric, based on simple exposure to UV radiation periodically along a piece of fiber [4]. The former techniques use a beam splitter to divide a single input UV beam into two, interfering them at the fiber; the latter depend on periodic exposure of a fiber to pulsed sources or through a spatially periodic amplitude mask. There are several laser sources that can be used, depending on the type of fiber used for the grating, the type of grating, or the intended application.

The holographic technique for grating fabrication has two principal advantages. Bragg gratings could be photoimprinted in the fiber core without removing the glass cladding. Furthermore, the period of the photoinduced grating depends on the angle between the two interfering coherent ultraviolet light beams. Thus even though ultraviolet light is used to fabricate the grating, Bragg gratings could be made to function at much longer wavelengths in a spectral region of interest for devices which have applications in fiber optic communications and optical sensors.

The design of fiber Bragg grating depends on various parameters e.g length of the grating, period of gratings, refractive index of core and cladding, mode of excitation conditions and temperature. In this paper, the effect on the reflection spectra of fiber Bragg grating is analyzed at the varied grating length along with the variation of spectral shape with changing refractive index.

II. THEORY:

A fiber Bragg grating consists of a periodic modulation of the refractive index in the core of a single-mode optical fiber. In case of uniform fiber gratings shown in figure1, the phase fronts are perpendicular to the fiber's longitudinal axis with grating planes having constant period.

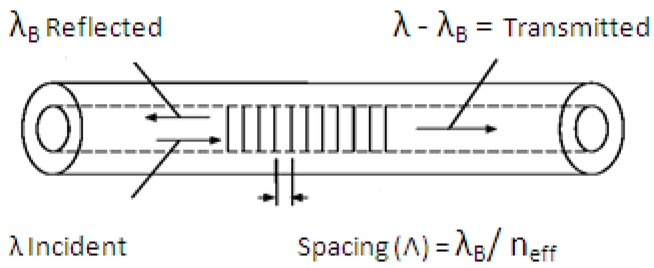

Figure1. Uniform fiber Bragg grating

The periodic perturbation of the refractive index causes the coupling between the different modes of light in the fiber grating. The wavelength for which the coupling between two modes is maximized is given by the following resonance condition:

$$\beta_1 - \beta_2 = m\frac{2\pi}{\Lambda}$$

where β1 and β_2 are the propagation constants of two modes, m is the order of diffraction and Λ is the period of the index modulation. Considering the two modes are identical ($\beta_1 = -\beta_2 = m\frac{2\pi}{\Lambda}$) and $\beta_2 < 0$ (mode propagates in the – z direction), we get the resonant wavelength for Bragg reflection:

$$\lambda = 2n_{eff}\Lambda$$

Couple mode theory:

Light guided along the core of an optical fiber will be scattered by each grating plane. If the Bragg condition is not satisfied, the reflected light from each of the subsequent planes becomes progressively out of phase and will eventually cancel out. Additionally, light that is not coincident with the Bragg wavelength resonance will experience very weak reflection at each of the grating planes because of the index mismatch; this reflection accumulates over the length of the grating.

This theory assumes that the mode fields of the unperturbed waveguide remain unchanged in the presence of weak perturbation. This approach provides a set of first-order differential equations for the change in the amplitude of the fields along the fiber, which have analytical solutions for uniform sinusoidal periodic perturbations. A fiber Bragg grating of a constant refractive index modulation and period therefore has an analytical solution. A complex grating may be considered to be a concatenation of several small sections, each of constant period and unique refractive index modulation. Thus, the modeling of the transfer characteristics of fiber Bragg gratings becomes a relatively simple matter, and the application of the transfer matrix method [5] provides a clear and fast technique for analyzing more complex structures.

Suppose two identical modes $Ae^{+i\beta z}$ and $Be^{-i\beta z}$ propagating in opposite directions through a Bragg grating. Due to periodic refractive index perturbation, the coupling coefficient will have a 'dc' and 'ac' component:

$$c(z) = \sigma(z) + 2k(z)\cos\left(\frac{2\pi}{\Lambda}z\right)$$

$$\sigma(z) = \frac{\omega}{c}\frac{\int_0^a\int_0^a \overline{\delta nco}(z)\ \Psi 2\ dx\ dy}{\int_0^\infty\int_0^\infty \Psi 2\ dx\ dy}$$

$$k(z) = \frac{\nu}{2}\sigma(z)$$

Where Ψ is the transverse profile of the identical modes, and a is the core radius of the fiber. The presence of the periodic index perturbation causes the two modes to be coupled such that their amplitudes A and B will vary along the propagation axis (z-axis) as follows:

$$\frac{dA}{dz} = i\sigma A + ikBe^{-}(2\beta - \frac{2\pi}{\Lambda})z$$

$$\frac{dB}{dz} = -i\sigma B - ikAe^{+}(2\beta - \frac{2\pi}{\Lambda})z$$

We can define the detuning δ to be:

$$\delta = \beta - \frac{\pi}{\Lambda} = \beta\left(\frac{1}{\lambda} - \frac{1}{\lambda D}\right)$$

where $\lambda_D = 2n_{eff}\Lambda$ is the design wavelength of the grating. Now introducing another 'dc' self-coupling coefficient:

$$\hat{\sigma} = \delta + \sigma = \beta\,(1 + \frac{\overline{\delta neff}}{neff}) - \pi/\Lambda$$

It is also useful to introduce this substitution:

$$A(z) = R\,(z)\,e^{-i\delta z}$$

$$B(z) = S(z)\,e^{+i\delta z}$$

By introducing this equations we get

$$\frac{dR}{dz} = i\hat{\sigma}R(z) + ikS(z)$$

$$\frac{dS}{dz} = -i\hat{\sigma}S(z) - ikR(z)$$

To solve this system, a new substitution is required:

$$R(z) = r\,(z)e^{+i\delta z}$$

$$S(z) = s\,(z)e^{-i\delta z}$$

This gives us the following system:

$$\frac{dr}{dz} = iks(z)e^{-i2\delta z}$$

$$\frac{ds}{dz} = - iks(z)e^{+i2\delta z}$$

Transfer Matrix Method:

In the T-matrix method, the coupled mode equations are used to calculate the output fields of a short section dl_1 of grating for which the three parameters are assumed to be constant. Each may possess a unique and independent functional dependence on the spatial parameter z. For such a grating with an integral number of periods, the analytical solution results in the amplitude reflectivity, transmission, and phase. These quantities are then used as the input parameters for the adjacent section of grating of length δl_2. The different transfer matrix elements are shown as below:

$$F = \begin{bmatrix} \cosh\left(\frac{\Omega L}{N}\right) + i\sinh\left(\frac{\Omega L}{N}\right) & -\frac{k}{\Omega}\sinh\left(\frac{\Omega L}{N}\right) \\ -\frac{k}{\Omega}\sinh\left(\frac{\Omega L}{N}\right) & \cosh\left(\frac{\Omega L}{N}\right) - i\sinh\left(\frac{\Omega L}{N}\right) \end{bmatrix}$$

We can connect the fields at the two ends of the grating through

$$\begin{bmatrix} u(L) \\ v(L) \end{bmatrix} = T \begin{bmatrix} u(0) \\ v(0) \end{bmatrix}$$

Where $T = T_N * T_{N-1} * T_{N-2} * \ldots\ldots\ldots T_i \ldots\ldots\ldots T_1$

For entire grating $T=\begin{bmatrix} T11 & T12 \\ T21 & T22 \end{bmatrix}$

The reflection coefficient is calculated by the relation

$$R = \frac{T21}{T11}$$

Solution of the Couple Mode Equations:

If the grating is uniform along *z*, then the coupled mode equations are coupled first-order ordinary differential equations with constant coefficients, for which closed-form solutions can be found when appropriate boundary conditions are specified. The reflectivity of a uniform fiber grating of length L can be found by assuming a forward-going wave incident from $z = -\infty$, passing through the grating and requiring that no backward-going wave exists for $z \geq L$, i.e., $u(L) = 1$, and $v(L) = 0$. The amplitude and power reflection coefficients $\rho = v(0)/u(0)$ and $r = |\rho|^2$, respectively, can then be shown to be

$$\rho = \frac{v(0)}{u(0)} = -\frac{k\sinh(\Omega L)}{\hat{\sigma}\sinh(\Omega L) + i\Omega\cosh(\Omega L)}$$

$$R = |\rho|^2 = \sinh^2(\Omega L)/(\cosh^2(\Omega L) - \hat{\sigma}^2/k^2)$$

Where $\Omega = \sqrt{k^2 - \hat{\sigma}^2}$

III. RESULTS AND DISCUSSION:

The parameters used for the simulation of the fiber Bragg grating are as given in the table.1

Table1: simulation parameters

core refractive index	1.64
λ_B (center wavelength)	1550 nm
λ_1 (lower limit of wavelength)	1546.9 nm
λ_2 (upper limit of wavelength)	1553.1 nm
Δ_{neff}	0.5e-4 to 3.5e-4
ν (fringe visibility)	1
Length (L)	2 mm to 35 mm
N (number of grating)	100
Λ (grating period)	L/N

For different values of grating length (table2), Reflected spectra was obtained and analyzed. From the spectra, it was confirmed that the spectral properties of uniform gratings comes out to be similar to sinc function. The reflection spectra for different grating lengths are shown in the figure 2,3,4.

Table2: Reflectivity for different grating length and for different deln

Grating Length in mm	Reflectivity (%) For deln(δ_n)=1e-4	Reflectivity (%) For deln(δ_n)=2e-4	Reflectivity (%) For deln(δ_n)=2.5e-4
2	14.77	44.83	58.85
3	29.43	70.21	82.57
4	44.83	85.52	93.22
5	58.77	93.2	97.51
8	85.44	99.39	**99.99**
10	93.22	**99.99**	99.99
12	96.92	99.99	99.99
14	98.61	99.99	99.99
15	99.09	99.99	99.99
17	**99.99**	99.99	99.99
20	99.99	99.99	99.99
22	99.99	99.99	99.99
25	99.99	99.99	99.99
27	99.99	99.99	99.99
30	99.99	99.99	99.99
32	99.99	99.99	99.99
35	99.99	99.99	99.99

It is observed that the reflectivity increases with the increase in length of the grating up to a particular length and beyond that it becomes saturated. After that, if the length is incremented further, reflectivity maintains the same value of 99.99%. The reflectivity reaches the maximum

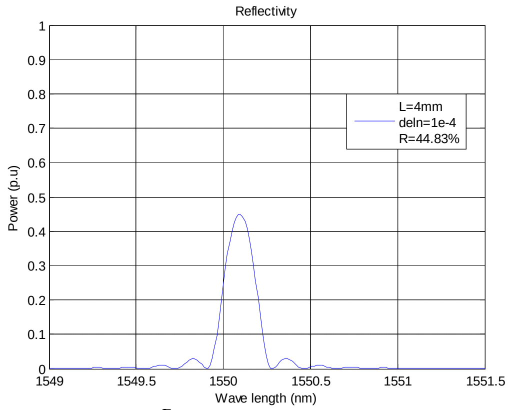

Fig.2 Reflective spectrum at L= 4mm

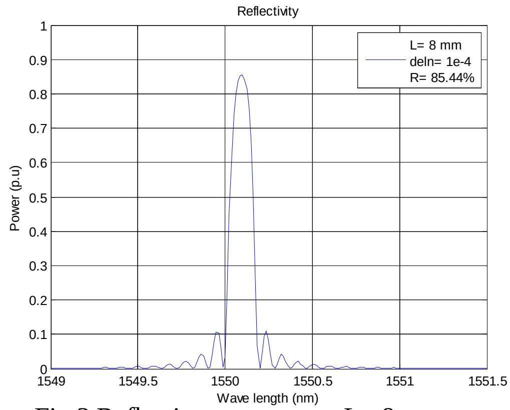

Fig.3 Reflective spectrum at L= 8mm

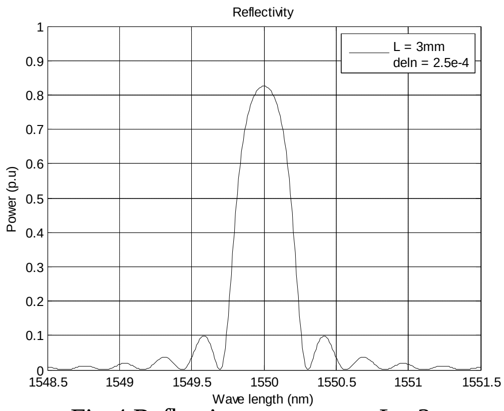

Fig.4 Reflective spectrum at L= 3mm

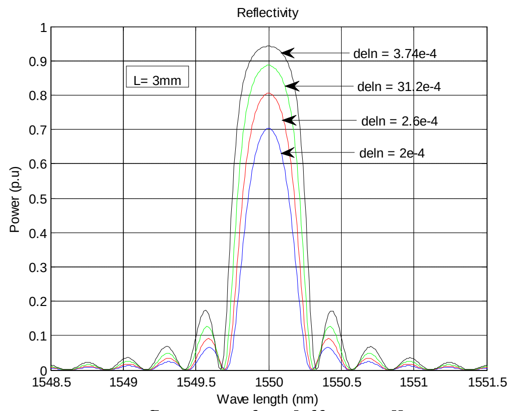

Fig.5 Reflectivity for different dlen

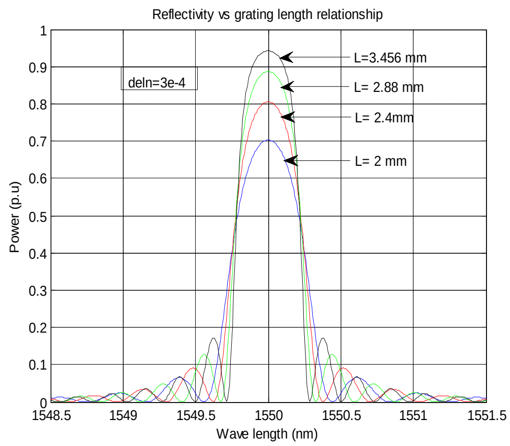

Fig.6 Reflectivity for different L

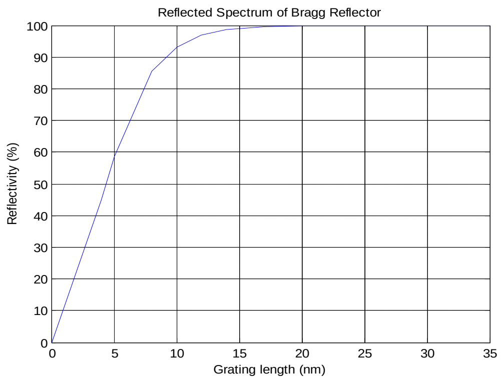

Fig.7 Reflectivity vs grating length

value but it is accompanied with a significant increase in the reflectivity of side lobes. It is observed from the fig.6 that the simulated fiber Bragg grating showed better performance as the grating length increased and achieved 99.99% at the grating length 17mm with effective refractive index of 1e-4.

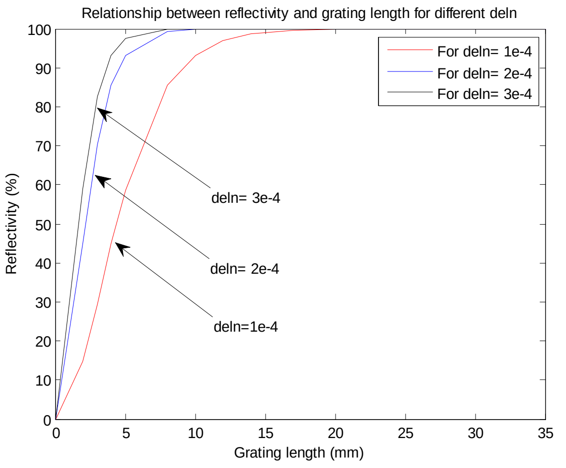

Figure.8Reflectivity vs grating length for different deln values

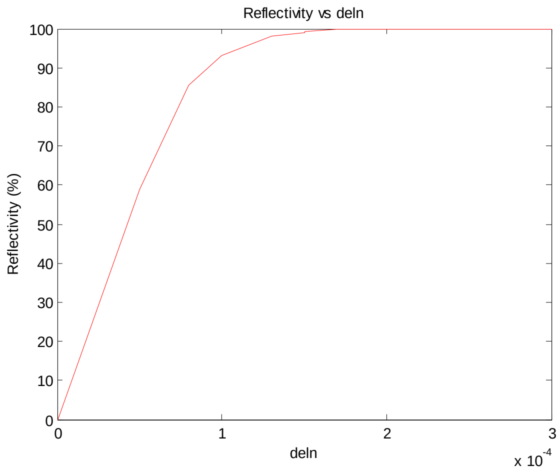

Figure.9 Relationship between reflectivity vs refractivity

From the figure.8 and figure.9 it is observed that the reflectivity changes due to the change of length of the grating as well as the change of the refractive index of the grating. Figure.8 shows that the length required to achieved 99.99% reflectivity for the grating having δ_n = 3e-4 is more than the other grating having different δ_n values.

IV. CONCLUSION:

This work draws the following conclusions:

1. The reflectivity of fiber grating increases with the increase in grating length. For strong grating, it has to be long and in the same time it has to have a large index change.
2. The grating with longer length with small index change has the narrow band width.
3. Grating may become saturated if it already met 99.99% reflectivity, beyond which increasing the length will only affect the bandwidth of the grating.
4. Bandwidth will increase as coupling coefficient (k) increases and will get smaller as L increases.

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