

# Geoidal Map and Three Dimension Surface Model Part of Port Harcourt Metropolis from “Satlevel” Collocation Model

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## Abstract:

Geoidal map depicts the geoid configuration of the area under study. Data acquisition for the production of such maps has been very tedious, time consuming and expensive with the use of classical and conventional geoid determination methods. "Satlevel" Collocation is a new method of geoid determination in which the ellipsoidal height from any satellite based system is combined with orthometric height from geodetic levelling to model the geoid. The method enables the geoid to be determined in patches. Geoid so determined can be applied with ellipsoidal height to get orthometric height which height users always prefer. In this work, "Satlevel" Collocation model along with the user's interactive computer program "Orthometric Height on Fly", were used to generate geoidal values in part of Port Harcourt metropolis. The generated data was used to produce the Geoidal map of the study area using SURFER software. The map was overlaid on the Local government map of Rivers State of Nigeria. The production of Geoidal map from "Satlevel" Collocation is easier than the conventional methods, when the initial geoidal coefficients have been determined. This method can be extended to other parts of Nigeria and the world at large.

**Keywords:** Geoid, Geoidal Map, Geoid Modelling, "Satlevel" Collocation, Three Dimension Surface and SURFER

## I. INTRODUCTION

### 1.1 Geoidal Map:

Geoidal map is similar to contour map which is made up of contour while geoidal map is made up of geoidal lines depicting the value of geoid for any points on the earth's surface. Three dimension surface models show the three dimension configuration of the area. The heights of points above the Mean Sea Level (MSL) are used for producing contour map, while geoidal values at different points are used for producing the geoidal map. It is also with reference to the geoid approximated by Mean Sea Level. Therefore, any line on a geoidal surface is an imaginary line drawn on the geoidal map to connect points of the same geoidal height value on, above or below the earth surface. The accuracy required for such maps necessitates the knowledge of the accurate and precise determination of the geoid at the same area. (Amin et al, 2005)

### 1.2. Geoid

The geoid is the surface which coincides with that surface to which the oceans would conform over the entire earth, if free to adjust to the combined effects of the earth's mass attraction (gravitation) and the centrifugal force of the Earth's rotation. Specifically, it is an equipotential surface, meaning that it is a surface on which the gravitational potential energy has the same value everywhere with respect to gravity. The geoid surface is irregular, but considerably smoother than earth's physical surface. Sea level, if undisturbed by tides, currents and weather, would assume a surface equal to the geoid, when the observation is carried out for a numbers of years, usually 18.61years. (Deakin, 1996, Olaleye et al, 2010; Aleem, 2013)

Determination of the geoid has been one of major challenges of geodesists. Gravity data have been used in the past with stokes integration and other approaches. One of such conventional methods was used by Hirvonen (1934) to carry out the first computation on a worldwide scale. He computed the geoidal undulation for 62 points distributed in an East -West band encircling the entire earth surface. For the first time, mean free air anomalies were estimated from the available gravity data covering 5° by 5° blocks. Also, Tanni (1948 and 1949) used large quantity of gravity data to compute the global geoidal heights. He employed the Pratt-Hayford system's gravity reduction method and Airy Heiskanen system. He later computed the global geoidal undulation

in 5° by 5° blocks with a more detailed 1° by 1° blocks geoid for Europe. Furthermore, Heiskanen (1957) computed the gravimetric geoid of Columbus using Free Air anomalies. According to Heiskanen and Moritz (1967), five times gravity data more than Tanni (1934) was available in the work of Heiskanen (1957), who also used electronic computer for numerical integration of Stoke's formula. The geoidal map was produced to depict the geoidal configuration worldwide. Amin et al (2005) predicted the gravimetric geoid and produce the precise geoidal map of the Toshka sector in Southern part of Egypt using least square collocation. Also, Geodesy Group of University of Lagos (GGU, 2006) produced the geoidal map of Nigeria as part of their submissions for the Optimum geoid for Nigeria using classical method (Gravimetric and Earth Geopotential Model 1996 (EGM 96)).

However, it was observed that the classical and conventional methods of geoid determination are time consuming, expensive and laborious. Also, Amin et al, (2005) noted the common defects of all classical techniques for geoid determination to include the inadequate and inhomogeneous distribution of the used of geodetic data. The disadvantages have prevented the widespread use of these methods and hence the advent of simpler and convenient methods such as the use of Global Navigation Satellite System (GNSS) combined with geodetic leveling (Aleem et al, 2011 and Aleem, 2013).

GNSS provide WGS84 ellipsoidal heights and when compared with orthometric heights, from geodetic levelling, it allows for the computation of the geoid, or the geoid-ellipsoid separation in the region of the survey (Aleem, 1996; Olaleye et al, 2010; Aleem et al, 2011). These height differences were used to derive the geoidal models called "satlevel" collocation model by Aleem (2013).

The geoid model will give geoidal undulation at every point of observation. This can be substituted with the ellipsoidal height from GNSS observation to get the orthometric height as given in Equation 1:

$$H = h - N \quad (1)$$

Where:

H = Orthometric height

h = Ellipsoidal height

N = geoidal height

From Equation 1, geoidal height (N) can be computed which can be plotted on the map in form of spot heights as used in contour map. The geoidal height on each point can be interpolated to produce the Geoidal map.

### 1.3 Interpolation of Geoidal Values:

Interpolations of geoidal undulation values follow the same procedures as interpolations of contours which are often done by rough estimate. The procedure is to first plot all the available geoidal heights as point data on the map and estimate the points for the undulation, which are joined to form a continuous geoidal lines. However, a more accurate but time consuming method used to be adopted is to use the formula for drafting contour which can be adopted for geoidal map as given by Aleem (2011):

$$PC = \frac{D}{Z} i \quad (2)$$

Where:

PC is distance from a referenced spot height to the point of geoidal line.

D is the distance between two spot heights

Z is the difference between the two spot heights

i is the difference between the geoidal line and one of the Spot heights.

Distance PC will then be scaled and marked on the map, which are later connected to form the geoidal line.

### 1.4 Significance

Geoidal maps are effective for displaying the geoidal configuration of a place. This is significance in geophysical study as it portrayed the geopotential configuration of the area under study. Therefore, it enhances the analysis of geological structures of the area.

This study is also significant because it reduces the rigour of data acquisition each time the geoidal map of the area is to be produced, when the geoidal coefficients have been determined. The use of software developed by the Author will enhance the productivity.

### 1.5 Software Adopted for Geoidal Map:

There are different types of software in the market for production of contour, digital terrain and three dimensional surface models. Any of these software can be used for production of geoidal map. This study used Surfer software.

#### 1.5.1 Surfer.

Surfer is a Contouring and 3D surface mapping software package from Golden Software Incorporation. It can transform random surveying data, using interpolation, into continuous curved face contour. Surfer's sophisticated interpolation engine transforms your XYZ data into publication-quality maps. It is provided with eighteen different gridding/trend surface algorithms. The methods were grouped into smoothing and exact interpolators. Smoothing interpolators are: Inverse Distance to a Power, Kriging, Polynomial Regression, Radial Basis Function, Modified Shepard's Method, Local Polynomial, Moving Average; while the exact interpolators are: Inverse Distance to a Power, Kriging, Nearest Neighbor, Radial Basis Function, Modified Shepard's Method, Triangulation with Linear Interpolation, and Natural Neighbor. Any of these interpolation methods can be used to approximate the geoid in an area, if adequate data are available. Surfer provides more gridding methods and more control over gridding parameters. Recent edition has added conversion and combination of maps with different coordinate systems.

#### 1.5.2 Other Contour Map Software for Production of Geoidal Map

Other Contour Map Software that can be adopted for production of geoidal map may include: 3DField, Garmin Basemap, mapviewer, Contour storyteller, Function Grapher, Auto plotter, Visual data, Filter test, AutoDEM Li Contour and several other software are available for production of Contour map. Any of these software can also be used for the production of geoidal map of the study area.

## II. THE STUDY AREA

Port Harcourt lies on Latitude:  $4^{\circ}45'N$  and  $5^{\circ}02'N$  and Longitude:  $6^{\circ}52'E$  and  $7^{\circ}09'E$ . It is the seat of Rivers State Government in oil rich Niger delta. Many companies, business organizations and government agencies locate and operate their corporate offices. Many of these organisations have used the services of surveyors for projects that needed height information. The surveyors, unable to get a bench mark to connect, will simply establish a local datum to do the work. This practice has created a situation where many different height values, which are irreconcilable, exist in the area. Therefore, this work reviews the need for simple method of obtaining values for the benchmark, which this study is trying to solve. The points used for the study were plotted on the local government map of Rivers State to show the distribution of points (Figure 1).

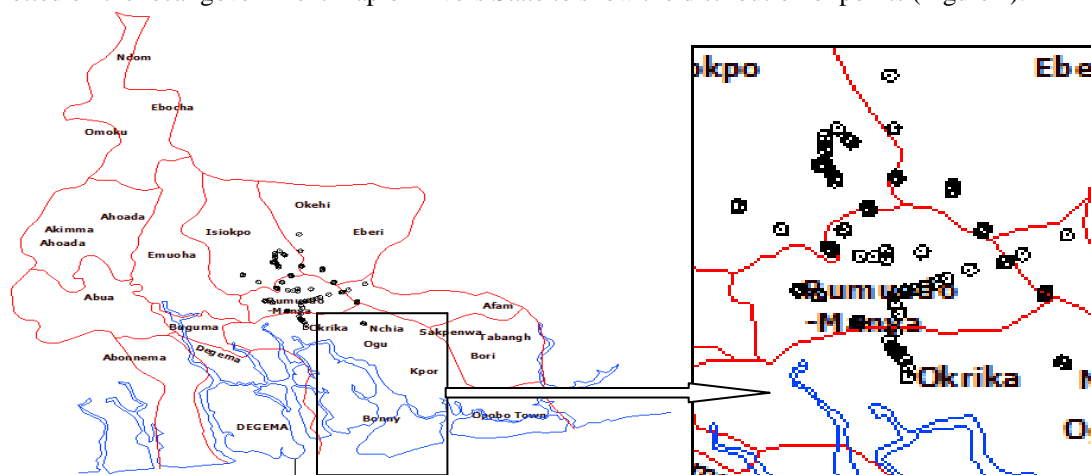


Figure 1: The Distribution of Points Used for the Study in Port Harcourt

### III. METHODOLOGY

#### 3.1 Materials and Data

The equipment needed for the exercise are:

- GNSS receiver and its accessories to acquire data for ellipsoidal height
- Level and its accessories to acquire data for orthometric height
- Computer and its accessories for computation and analysis
- Surfer software for plotting the geoidal values.
- Any software or program that can implement least square adjustment.

In this work Microsoft Office excel was used for all the computations, However Aleem, (2013) has developed a computer program for all the procedures called “Orthometric Height on Fly”. This was used to validate the results.

#### 3.2 Data Acquisition

Levelling operation was carried out to obtain data for orthometric height.

GNSS observation was carried out to acquire geodetic coordinates which includes the geodetic latitude ( $\phi$ ), geodetic longitude ( $\lambda$ ) and ellipsoidal height (h). 86 points with good spatial distribution within the study area were covered. Other relevant data such constant for semi major axis, flattening and other parameter for the World Geodetic System WGS84 referenced ellipsoid were collected from various literatures and INTERNET websites for data analysis and processing.

#### 3.3 Data Processing

Levelling reduction was carried; to obtain height of each point. These height data were assumed to be the orthometric height.

GNSS observation was processed to get the three dimensional coordinates, the geodetic latitude ( $\phi$ ), geodetic longitude ( $\lambda$ ) and ellipsoidal height (h), which were used in “satlevel” collocation model.

#### 3.4 “Satlevel” Collocation Model:

“Satlevel” Collocation model was developed by Aleem, (2013). The model is of the form:

$$N_i = N_L + A_1 (\cos^3 \phi_i \cos \lambda_i + \sin^2 \phi_i \cos \phi_i \cos \lambda_i + \cos^3 \lambda_i \cos \phi_i + \sin^2 \lambda_i \cos \phi_i \cos \lambda_i) + A_2 (\cos^3 \phi_i \sin \lambda_i + \sin^2 \phi_i \cos \phi_i \sin \lambda_i + \cos^2 \lambda_i \cos \phi_i \sin \lambda_i + \sin^3 \lambda_i \cos \phi_i) + A_3 (\cos^2 \phi_i \sin \phi_i + \sin^3 \phi_i + \cos^2 \lambda_i \sin \phi_i + \sin^2 \lambda_i \sin \phi_i) + r_i \tag{3}$$

Where:

$N_L$  is the long wavelength part of the geoid undulation in the area.

$A_1, A_2$  and  $A_3$  are the geoidal coefficients which are unknown coefficients to be determined.

$\phi$  and  $\lambda$  are the WGS ‘84 geodetic coordinates (Latitudes and Longitudes)

$r_i$  is residue at an observation point.

The “satlevel” collocation model geoidal coefficients of the area were computed using least squares adjustment observation equation method. The “satlevel” collocation model geoidal coefficients were used to compute the geoidal undulation (Table 1) of each point in the area. The geoidal undulations were plotted into a chart (Figure 3) to form the geoidal surface. The geoidal undulation generated from “satlevel” collocation and the geodetic coordinates of the area were plotted into geoidal map using Surfer software (Figure 4).

Three dimensional geoid surface model (Figure 5) was plotted using the geoidal undulation and the Geodetic coordinates (Table 1)

### IV. Results and Discussions

#### 4.1 Results

The results of the orthometric heights acquired from the geodetic levelling and ellipsoidal heights from GNSS observation were substituted into Equation 1 to obtain the values of the geoidal undulation as shown in Tables 1.

Stations	Latitude [°]	Longitude [°]	Ellipsoidal Heights (h) [m]	Orthometric Heights (H) [m]	Geoid (N) [m]
GPS 02	4.988341858	7.005441514	42.542	23.638	18.904
GPS 03	4.981133603	6.949840522	40.065	21.24	18.825
GPS 04	4.972244803	6.951180808	38.771	19.938	18.833
GPS 05	4.988165797	6.959676808	41.357	22.523	18.834
GPS 06	4.976870211	6.950525386	39.485	20.657	18.828
GPS 07	4.968417417	6.950765697	38.351	19.516	18.835
GPS 08	4.956065461	6.949389547	36.427	17.585	18.842
GPS 60	4.91610835	6.881154569	20.982	02.189	18.793
XSV 662	4.873506919	6.99841315	27.603	08.648	18.955
ZVS 3003	4.847971022	7.047811589	32.308	13.282	19.026

Table 1: Sample of Orthometric heights and Geoidal Undulations of some of the Stations

Geoidal undulation for the “Satlevel” Collocation (Table 2) model was computed using Equation 3 for Port Harcourt metropolis.

**Table 2: Sample of Local and “Satlevel” Collocation Geoidal Undulations**

STATIONS	Geoidal Undulation computed from Equation (1) [m]	“Satlevel” Geoidal Undulation Computed from Equation (3) [m]	Residuals
GPS 02	18.9040	18.8910	0.013
GPS 03	18.8250	18.8288	-0.0038
GPS 04	18.8330	18.8368	-0.0038
GPS 05	18.8340	18.8360	-0.002
GPS 06	18.8280	18.8327	-0.0047
GPS 07	18.8350	18.8390	-0.004
GPS 08	18.8420	18.8457	-0.0037
GPS 60	18.7930	18.7817	0.0113
XSV 662	18.9550	18.9548	0.0002
ZVS 3003	19.0260	19.0229	0.0031

The data acquired Table (1) were used to plot chart (Figure 2):

The values of the Geoidal coefficients computed using least square adjustment observation equation as:

$N_L = -5477.15802$   
 $A_1 = 2714.485748$   
 $A_2 = 375.21354$   
 $A_3 = 214.34789$

The results of the “satlevel” collocation geoidal undulations were plotted into chart (Figure 3):

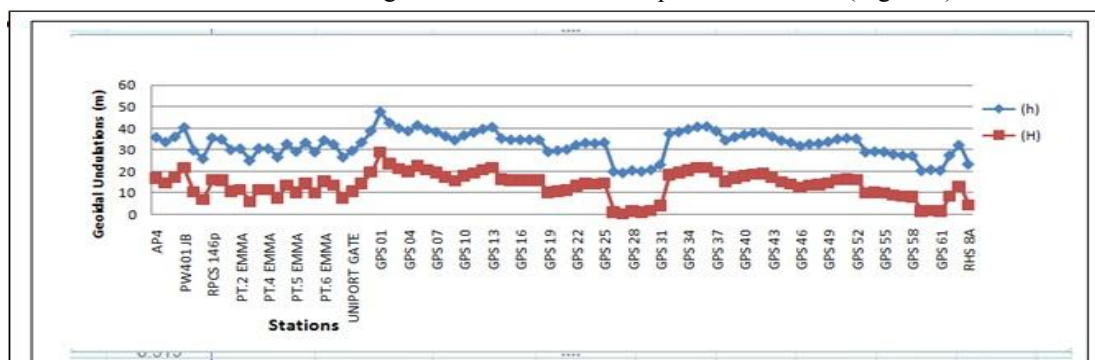


Figure 4.1a: Chart showing the relationship between Ellipsoidal and Orthometric Heights in Port Harcourt

Figure 2: Chart showing the relationship between Ellipsoidal and Orthometric Heights in Port Harcourt

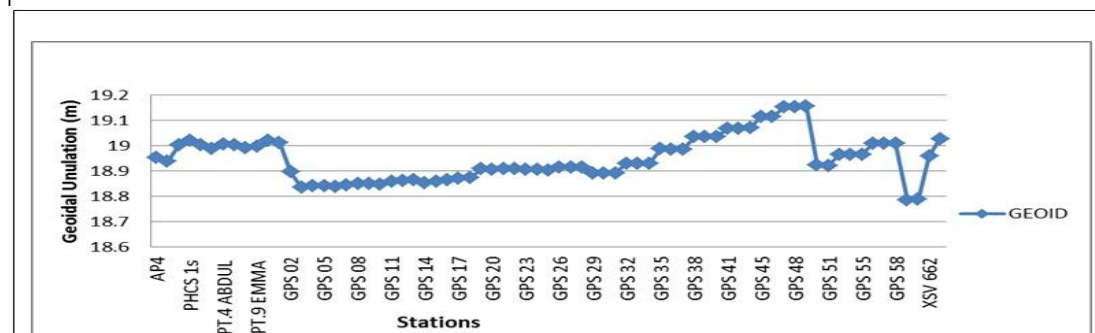


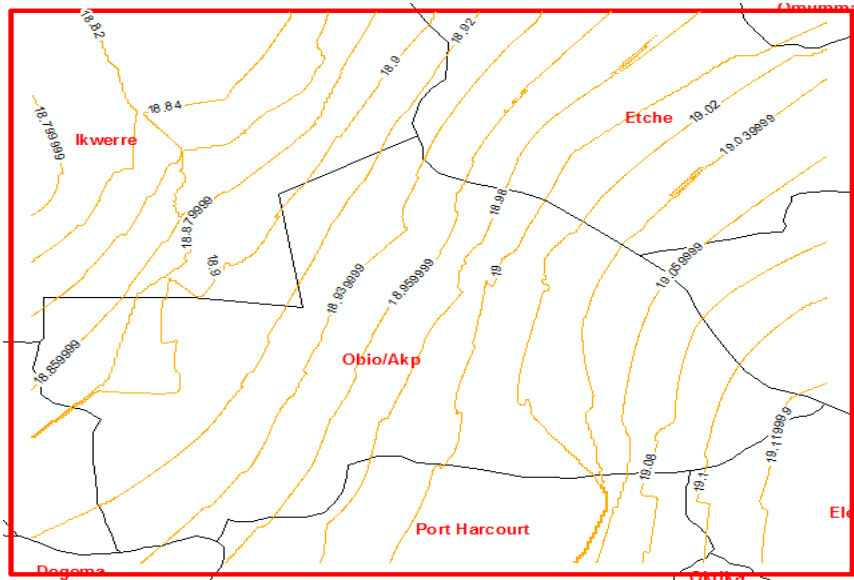
Figure 3: Chart showing the Geoidal surface from the “Satlevel” Collocation Models for Port Harcourt

The “Satlevel” collocation Geoidal map (Figure 4):

The geoidal map from geoidal undulation in Table 1 of Port Harcourt was plotted and overlaid on the Local Government map of Rivers State

**Figure 4: Geoidal Map Plotted From “Satlevel” Collocation Model for Port Harcourt**

The 3-D Geoid surface Model (Figure 5) was also plotted using Surfer software.





## V. CONCLUSION AND RECOMMENDATION

### 5.1 Conclusion

In this study, levelled heights were established along with GPS observation in some parts of Port Harcourt to model the geoid in the study area. We have coordinated some of the points collocated with both GNSS and geodetic levelling in the area. 'Satlevel' Collocation Model was used to get the geoidal coefficients, which were used to get the geoidal undulation for each of the points. Ellipsoidal heights from GNSS observation were used to get the orthometric heights, and used to plot the geoidal map with surfer software.

### 5.2 Recommendations

The area of coverage needs to be extended and more data should be acquired to improve the results. Contracts for the production of maps should be awarded to the Institutions of Higher learning in the Nigeria. This will enable the participating staff to acquire more experience and impart same to the students, so as to improve the qualities of graduates for future challenges.

## REFERENCES

- [1] K. F. Aleem. GPS and Remote Sensing of Environment". *The Graduate Surveyors*. Journal of NISS University of Nigeria, Enugu Campus. 12(1) 24 – 29. 1996
- [2] K. F. Aleem. Drafting and Mapping (GET 102). *Lecture Notes*. Department of Geomatics Engineering Technologies, Yanbu Industrial College, Kingdom of Saudi Arabia. UNPUBLISHED. 2011
- [3] K. F. Aleem. Adaptation of a Global Orthometric to a Local Height Datum Using "Satlevel" Collocation Model". A Ph.D Thesis. Department of Surveying and Geoinformatics, University of Lagos Nigeria. UNPUBLISHED. 2013.
- [4] K. F. Aleem, J. B. Olaleye, O. T. Badejo and J. O. Olusina. "A Combination of Ellipsoidal Height from Satellite Method and Orthometric Height for Geoid Modelling". *A publication of the International Global Navigation Satellites Society IGNS* 2011. <http://ignss.org/Conferences/PastPapers/2011ConferencePastPapers/2011PeerReviewedPapers/tabid/108/Default.aspx>
- [5] M. M. Amin, S. M. El-Fatairy and R. M. Hassouna "Precise Geoidal Map of the Southern Part of Egypt by Collocation Toshka Geoid". *From Pharaohs to Geoinformatics. FIG Working Week 2005* Cairo, Egypt April 16-21, 2005
- [6] Geodesy Group University of Lagos (GGU). Determination of an Optimum Geoid for Nigeria. *A Project Report* Sponsored by Centre for Geodesy and Geodynamics (CCG) and the National Space Research and Development Agency (NASRDA) Abuja, Nigeria. 2006.
- [7] Golden Software, Inc (2012) "Surfer 11: Powerful Contouring, Gridding, and 3D Surface Mapping Software for Scientists and Engineers". Online Article accessed on 12<sup>th</sup> November 2012 <http://www.goldensoftware.com/products/surfer/surfer.shtml>
- [8] L. S. Heliani, Y. Fukuda, and S. Takemoto. "Simulation of the Indonesian Land Gravity Data using a Digital Terrain Model Data. *Earth Planets Space*", 56(1): 15–24. 2004
- [9] W. A. Heiskanen. The Columbus Geoid. *Trans. Am. Geophys. Union*, 38:841-848. 1957
- [10] Heiskanen, W.A., and H. Moritz. *Physical Geodesy*. San Francisco, California: W.H. Freeman and Co. 1967
- [11] J. B. Olaleye,.; K. F. Aleem, J. O. Olusina and O. E. Abiodun . Establishment of an empirical Geoid Model for a Small Geographic Area: A Case Study of Port Harcourt, Nigeria. *Surveying and Land Information Science*. 70(1): 39-48(10). 2010 <http://www.ingentaconnect.com/content/nsps/salis/2010/00000070/00000001/art00006>
- [12] Tanni, L. On the Continental Undulation of the Geoid as Determined from Present Gravity Materials:. Helsinki Publishers Isostatic institute. International Association of Geodesy. No. 18. 1948
- [13] Tanni, L. (1949). *The Regional Rise of Geoid in Central Europe Materials*. Helsinki Publishers Isostatic institute. International Association of Geodesy. No. 22.