International Journal Of Computational Engineering Research (ijceronline.com) Vol. 3 Issue. 2

Some Fixed Point Theorems for Expansion Mappings

[1,] A.S.Saluja, [2,] Alkesh Kumar Dhakde [3,] Devkrishna Magarde
[1,]Department of Mathematics J.H.Govt.P.G.College Betul (M.P.)
[2,]IES Group of institutions Bhopal (M.P.)

Abstract
In The Present Paper We Shall Establish Some Fixed Point Theorems For Expansion Mappings In Complete Metric Spaces. Our Results Are Generalization Of Some Well Known Results.

Keywords: Fixed Point, Complete Metric spaces, Expansion mappings.

1. Introduction & Preliminary:

This paper is divided into two parts

Section I: Some fixed point theorems for expansion mappings in complete metric spaces.

Section II: Some fixed point theorems for expansion mappings in complete 2-Metric spaces before starting main result some definitions.

Definition 2.1: (Metric spaces) A metric space is an ordered pair (X, d) where X is a set and d a function on $X \times X$ with the properties of a metric, namely:

1. $d(x, y) \geq 0$. (non-negative) ,
2. $d(x, y) = d(y, x)$ (symmetry),
3. $d(x, y) = 0$ if and only if $x = y$ (identity of indiscernible)
4. The triangle inequality holds:

$$d(x, y) \leq d(x, z) + d(z, y), \text{ for all } x, y, z \text{ in X}$$

Example 2.1: Let $E_n(or\ R^n) = \{x = (x_1, x_2, x_3 .... x_n), x_i \in R, R \text{ the set of real numbers}\}$ and let d be defined as follows:

If $y = (y_1, y_2, y_3.......y_n)$ then $d(x, y) = \left(\sum_1^n | x_i - y_i |^p \right)^{\frac{1}{p}} = d_p(x, y)$ where p is a fixed number in $[0, \infty)$.

The fact that d is metric follows from the well-known Minkowski inequality. Also another metric on S considered above can be defined as follows

$$d(x, y) = \sup_i\{| x_i - y_i |\} = d_\infty(x, y)$$

Example 2.2: Let S be the set of all sequence of real numbers $x = (x_i)_1^\infty$ such that for some fixed $p \in [0, \infty), \sum_1^\infty | x_i |^p \prec \infty$ In this case if $y = y_i$ is another point in S , we define

$d(x, y) = \left(\sum | x_i - y_i |^p\right)^{\frac{1}{p}} = d_q(x, y)$, and from Minkowski inequality it follows that this is a metric space on S .

Example 2.3: For $x, y \in R$, define $d(x, y) = | x - y |$. Then (R, d) is a metric space .In general, for $x = (x_1, x_2, x_3 .... x_n)$ and $y = (y_1, y_2, y_3 .... y_n) \in R^n$, define

$$d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + ...+ (x_n - y_n)^2}$$

Then (R^n, d) is a metric space .As this d is usually used, we called it the usual metric.

Definition 2.2: (Convergent sequence in metric space)

A sequence in metric space (X,d) is convergent to $x \in X$ if $\lim_{n\to\infty} d(x_n, x) \to 0$

Definition 2.3: (Cauchy sequence in Metric space) Let $M = (X,d)$ be a metric space, let $\{x_n\}$ be a sequence if and only if $\forall \varepsilon \in R : \varepsilon > 0 : \exists N : \forall m,n \in N : m,n \geq N : d(x_n, x_m) < \varepsilon$

Definition 2.4: (Complete Metric space) A metric space (X,d) is complete if every Cauchy sequence is convergent.

Definition 2.5: A 2-metric space is a space X in which for each triple of points x,y,z there exists a real function $d(x,y,z)$ such that:

$[M_1]$ To each pair of distinct points x,y,z $d(x,y,z) \neq 0$

$[M_2]$ $d(x,y,z) = 0$ When at least two of x,y,z are equal

$[M_3]$ $d(x,y,z) = d(y,z,x) = d(x,z,y)$

$[M_4]$ $d(x,y,z) \leq d(x,y,v) + d(x,v,z) + d(v,y,z)$ for all x,y,z,v in X.

Definition 2.6: A sequence $\{x_n\}$ in a 2-metric space (X,d) is said to be convergent at x if $\lim_{n\to\infty} d(x_n, x, z) = 0$ for all z in X .

Definition 2.7: A sequence $\{x_n\}$ in a 2-metric space (X,d) is said to be Cauchy sequence if $\lim_{m,n\to\infty} d(x_n, x, z) = 0$ for all z in X .

Definition 2.8: A 2-metric space (X,d) is said to be complete if every Cauchy sequence in X is convergent.

2. Basic Theorems

In 1975, Fisher [4], proved the following results:

Theorem (A): Let T be a self mapping of a metric spaces X such that,

$$d(Tx,Ty) \geq \frac{1}{2}[d(x,Tx) + d(y,Ty)] \forall x,y \in X, \ T \text{ is an identity mappings.}$$

Theorem (B): Let X be a compact metric space and $T : X \to X$ satisfies 4. (A) and $x \neq y$ and $x,y \in X$.Then T^r has a fixed point for some positive integer r , and T is invertible.

In 1984 the first known result for expansion mappings was proved by Wang, Li, Gao and Iseki [13].

Theorem (C): "Let T be a self map of complete metric space X into itself and if there is a constant $\alpha > 1$ such that, $d(Tx,Ty) \geq \alpha d(x,y)$ for all $x,y \in X$.

Then T has a unique fixed point in X .

Theorem (D): If there exist non negative real numbers $\alpha + \beta + \gamma > 1$ and $\alpha < 1$ such that

$$d(Tx,Ty) \geq \alpha \min\{d(x,Tx), d(y,Ty), d(x,y)\} \ \forall x,y \in X,$$

T is continuous on X onto itself, and then T has a fixed point.

Theorem (F): If there exist non negative real numbers $\alpha > 1$ such that $d(Tx,T^2x) \geq \alpha d(x,Tx) \forall x \in X$, T is onto and continuous then T has a fixed point.

In 1988, Park and Rhoades [8] shows that the above theorems are consequence of a theorem of Park [7] .In 1991, Rhoades [10] generalized the result of Iseki and others for pair of mappings.

Theorem (G): If there exist non negative real numbers $\alpha > 1$ and T , S be surjective self-map on a complete metric space (X,d) such that;

$d(Tx,Sy) \geq \alpha d(x,y)$ $\forall x \in X$, Then T and S have a unique common fixed point.

In 1989 Taniguchi [12] extended some results of Iseki .Later, the results of expansion mappings were extended to 2-metric spaces, introduced by Sharma, Sharma and Iseki [11] for contractive mappings. Many other Mathematicians worked on this way.

Rhoades [10] summarized contractive mapping of different types and discussed on their fixed point theorems. He considered many types of mappings and analyzed the relationship amongst them, where $d(Tx,Ty)$ is governed by,

$$d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx), d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)$$

Many other mathematicians like Wang,Gao , Isekey [13] , Popa [9] , Jain and Jain [5] , Jain and Yadav [6] worked on expansion mappings Recently , Agrawal and Chouhan [1,2] , Bhardwaj , Rajput and Yadava [1] worked for common fixed point for expansion mapping.

Our object in this paper is, to obtain some result on fixed point theorems of expansion type's maps on complete metric space.

Now In Section I, We will find Some Fixed Point Theorems For Expansion Mappings In Complete Metric Spaces.

3. Main Results

Theorem (3.1): Let X denotes the complete metric space with metric d and f is a mapping of X into itself .If there exist non negative real's $\alpha,\beta,\gamma,\delta > 1$ with $\alpha + 2\beta + 2\gamma + \delta > 1$ such that

$$d(fx, fy) \geq \alpha \frac{[1+d(y, fy)]d(x, fx)}{1+d(x,y)} + \beta[d(x, fx)+d(y, fy)] + \gamma[d(x, fy)+d(y, fx)] + \delta d(x,y)$$

For each $x, y \in X$ with $x \neq y$ and f is onto then f has a fixed point.

Proof: Let $x_0 \in X$ .since f is onto, there is an element x_1 satisfying $x_1 = f^{-1}(x_0)$.similarly we can write $x_n = f^{-1}(x_{n-1}), (n = 1,2,3.........)$

From the hypothesis

$$d(x_{n-1}, x_n) = d(fx_n, fx_{n+1})$$

$$\geq \alpha \frac{[1+d(x_{n+1}, fx_{n+1})]d(x_n, fx_n)}{1+d(x_n, x_{n+1})} + \beta[d(x_n, fx_n)+d(x_{n+1}, fx_{n+1})] + \gamma(d(x_n, fx_{n+1})+d(x_{n+1}, fx_n)) + \delta d(x_n, x_{n+1})$$

$$\geq \alpha \frac{[1+d(x_{n+1}, x_n)]d(x_n, x_{n-1})}{1+d(x_n, x_{n+1})} + \beta[d(x_n, x_{n-1})+d(x_{n+1}, x_n)] + \gamma[d(x_n, x_n)+d(x_{n+1}, x_{n-1})] + \delta d(x_n, x_{n+1})$$

$$(1-\alpha-\beta-\gamma)d(x_n, x_{n-1}) \geq (\beta+\gamma+\delta)d(x_n, x_{n+1})$$

$$\Rightarrow d(x_n, x_{n+1}) \leq \left(\frac{1-\alpha-\beta-\gamma}{\beta+\gamma+\delta}\right)d(x_n, x_{n-1})$$

Therefore $\{x_n\}$ converges to x in X.Let $y \in f^{-1}(x)$ for infinitely many n , $x_n \neq x$ for some n.

$$d(x_n, x) = d(fx_{n+1}, fy) = d(fy, fx_{n+1})$$

$$\geq \alpha \frac{[1+d(y, fy)]d(x_{n+1}, fx_{n+1})}{1+d(x_{n+1}, y)} + \beta[d(x_{n+1}, fx_{n+1})+d(y, fy)] + \gamma[d(x_{n+1}, fy)+d(y, fx_{n+1})] + \delta d(x_{n+1}, y)$$

$$\geq \alpha \frac{[1+d(y,x)]d(x_{n+1}, x_n)}{1+d(x_{n+1}, y)} + \beta[d(x_{n+1}, x_n)+d(y,x)] + \gamma[d(x_{n+1}, x)+d(y, x_n)] + \delta d(x_{n+1}, y)$$

Since $d(x_n, x) \to \infty$ as $n \to \infty$ we have $d(x_{n+1}, x_n) = d(x_{n+1}, x) = 0$

Therefore $d(x,y) = 0$ when $x = y$.

$i.e\ y = f(x) = x$

This completes the proof of the theorem.

Theorem (3.2): Let X denotes the complete metric space with metric d and S and T is a mapping of X into itself .If there exists non negative real's $\alpha,\beta,\gamma,\delta>1$ with $\alpha+2\beta+2\gamma+\delta>1$ such that

$$d(Sx,Ty)\geq\alpha\frac{[1+d(y,Ty)]d(x,Sx)}{1+d(x,y)}+\beta[d(x,Sx)+d(y,Ty)]+\gamma[d(x,Ty)+d(y,Sx)]+\delta d(x,y)$$

For each x,y in X with $x\neq y$ and S & T has a fixed point.

Proof: Let x_0 be an arbitrary point in X. Since S & T maps itself there exist points x_1,x_2 in X such that $x_1=S^{-1}(x_0)$, $x_2=T^{-1}(x_1)$

Continuing the process, we get a sequence $\{x_n\}$ in X such that

$$x_{2n+1}=S^{-1}(x_{2n}) \ \& \ T^{-1}(x_{2n+1})=x_{2n+2}$$

We see that if $x_{2n}=x_{2n+1}$ for some n then x_{2n} is a common fixed point of S & T.

Therefore we suppose that $x_{2n}\neq x_{2n+1}$ for all $n\geq 0$

From the hypothesis

$$d(x_{2n},x_{2n+1})=d(Sx_{2n+1},Tx_{2n+2})$$

$$\geq\alpha\frac{[1+d(x_{2n+2},Tx_{2n+2})]d(x_{2n+1},Sx_{2n+1})}{1+d(x_{2n+1},x_{2n+2})}+\beta[d(x_{2n+1},Sx_{2n+1})+d(x_{2n+2},Tx_{2n+2})]$$

$$+\gamma[d(x_{2n+1},Tx_{2n+2})+d(x_{2n+2},Sx_{2n+1})]+\delta d(x_{2n+1},x_{2n+2})$$

$$\geq\alpha d(x_{2n+1},x_{2n})+\beta[d(x_{2n+1},x_{2n})+d(x_{2n+2},x_{2n+1})]+\gamma[d(x_{2n+1},x_{2n+1})+d(x_{2n+2},x_{2n})]+\delta d(x_{2n+1},x_{2n+2})$$

$$\Rightarrow(1-\alpha-\beta-\gamma)d(x_{2n},x_{2n+1})\geq(\beta+\gamma+\delta)d(x_{2n+1},x_{2n+2})$$

$$\Rightarrow d(x_{2n+1},x_{2n+2})\leq\left(\frac{1-\alpha-\beta-\gamma}{\beta+\gamma+\delta}\right)d(x_{2n},x_{2n+1})$$

Where $k=\dfrac{1-\alpha-\beta-\gamma}{\beta+\gamma+\delta}<1$ .similarly it can be shown that

$$d(x_{2n+1},x_{2n+2})\leq kd(x_{2n},x_{2n+1})$$

Therefore, for all n ,

$$d(x_{n+1},x_{n+2})\leq kd(x_n,x_{n+1})$$

$$\leq\ldots\ldots\leq(k)^{n+1}d(x_0,x_1)$$

Now, for any $m>n$,

$$d(x_n,x_m)\leq d(x_n,x_{n+1})+d(x_{n+1},x_{n+2})+\ldots\ldots+d(x_{m-1},x_m)$$

$$\leq\left[(k)^n+(k)^{n+1}+\ldots\ldots\ldots+(k)^{m-1}\right]d(x_1,x_0)$$

$$\leq\frac{k^n}{1-k}d(x_1,x_0)$$

Which shows that $\{x_n\}$ is a Cauchy sequence in X and so it has a limit z .Since X is complete metric space, we have $z\in X$, there exist $v,w\in X$ such that $S(v)=z$, $\mathrm{T}(w)=z$

From the hypothesis we have

$$d(x_{2n},z)=d(Sx_{2n+1},Tw)$$

$$\geq \alpha \frac{[1+d(w,Tw)]d(x_{2n+1},Sx_{2n+1})}{1+d(x_{2n+1},w)} + \beta[d(x_{2n+1},Sx_{2n+1})+d(w,Tw)]$$
$$+\gamma[d(x_{2n+1},Tw)+d(w,Sx_{2n+1})]+\delta d(x_{2n+1},w)$$
$$\geq \alpha \frac{[1+d(w,z)]d(x_{2n+1},x_{2n})}{1+d(x_{2n+1},w)} + \beta[d(x_{2n+1},x_{2n})+d(w,Tw)]+\gamma[d(x_{2n+1},Tw)+d(w,x_{2n})]+\delta d(x_{2n+1},w)$$

Since $d(x_n,z)\to\infty$ as $n\to\infty$ we have $d(x_{2n+1},x_{2n})=d(x_{2n+1},z)=0$

Therefore $w=z$ i.e $Sw=Tw=z$

This completes the proof of the theorem.

Section II: Some Fixed Point Theorems In 2-Metric Spaces For Expansion Mappings

Theorem 3.3: Let X denotes the complete 2-metric space with metric d and f is a mapping of X into itself. If there exist non negative real's $\alpha,\beta,\gamma,\delta>1$ with $\alpha+2\beta+2\gamma+\delta>1$ such that

$$d(fx,fy,a)\geq \alpha \frac{[1+d(y,fy,a)]d(x,fx,a)}{1+d(x,y,a)} + \beta[d(x,fx,a)+d(y,fy,a)]+\gamma[d(x,fy,a)+d(y,fx,a)]$$
$$+\delta d(x,y,a)$$

For each x,y in X with $x\neq y$ and f is onto then f has a fixed point.

Proof: Let $x_0\in X$ .since f is onto, there is an element $x_1=f^{-1}(x_0)$.similarly we can write $x_n=f^{-1}(x_{n-1})$ $n=1,2,3.........$

From the hypothesis

$$d(x_{n-1},x_n,a)=d(fx_n,fx_{n+1},a)$$
$$\geq \alpha \frac{[1+d(x_{n+1},fx_{n+1},a)]d(x_n,fx_n,a)}{1+d(x_n,x_{n+1},a)} + \beta[d(x_n,fx_n,a)+d(x_{n+1},fx_{n+1},a)]+\gamma[d(x_n,fx_{n+1},a)+d(x_{n+1},fx_n,a)]$$
$$+\delta d(x_n,x_{n+1},a)$$
$$\geq \alpha \frac{[1+d(x_{n+1},x_n,a)]d(x_n,x_{n-1},a)}{1+d(x_n,x_{n+1},a)} + \beta[d(x_n,x_{n-1},a)+d(x_{n+1},x_n,a)]+\gamma[d(x_n,x_n,a)+d(x_{n+1},x_{n-1},a)]$$
$$+\delta d(x_n,x_{n+1},a)$$
$$\geq \alpha d(x_n,x_{n-1},a)+\beta[d(x_n,x_{n-1},a)+d(x_{n+1},x_n,a)]+\gamma[d(x_{n+1},x_n,a)+d(x_n,x_{n-1},a)]+\delta d(x_n,x_{n+1},a)$$
$$\Rightarrow (1-\alpha-\beta-\gamma)d(x_n,x_{n-1},a)\geq(\beta+\gamma+\delta)d(x_n,x_{n+1},a)$$
$$\Rightarrow d(x_n,x_{n+1},a)\leq \frac{1-\alpha-\beta-\gamma}{\beta+\gamma+\delta}d(x_n,x_{n-1},a)$$

Therefore $\{x_n\}$ converges to x in X .Let $y\in f^{-1}(x)$ for infinitely many $n, x_n\neq x$ for such n

$$d(x_n,x,a)=d(fx_{n+1},fy,a)=d(fy,fx_{n+1},a)$$
$$\geq \alpha \frac{[1+d(y,fy,a)]d(x_{n+1},fx_{n+1},a)}{1+d(x_{n+1},y,a)} + \beta[d(x_{n+1},fx_{n+1},a)+d(y,fy,a)]+\gamma[d(x_{n+1},fy,a)+d(y,fx_{n+1},a)]$$
$$+\delta d(x_{n+1},y,a)$$
$$\geq \alpha \frac{[1+d(y,x,a)]d(x_{n+1},x_n,a)}{1+d(x_{n+1},y,a)} + \beta[d(x_{n+1},x_n,a)+d(y,x,a)]+\gamma[d(x_{n+1},x,a)+d(y,x_n,a)]+\delta d(x_{n+1},y,a)$$

Since $d(x_n,x,a)\to\infty$ as $n\to\infty$ we have $d(x_{n+1},x_n,a)=d(x_{n+1},x,a)=0$

Therefore $d(x,y,a)=0\Rightarrow x=y$

i.e $y = f(x) = x$.

Theorem 3.4: Let X denotes the complete metric space with metric d and S & T is a mappings of X into itself .If there exists non negative real's $\alpha, \beta, \gamma, \delta > 1$ with $\alpha + 2\beta + 2\gamma + \delta > 1$ such that

$$d(Sx,Ty,a) \geq \alpha \frac{[1+d(y,Ty,a)]d(x,Sx,a)}{1+d(x,y,a)} + \beta[d(x,Sx,a)+d(y,Ty,a)] + \gamma[d(x,Ty,a)+d(y,Sx,a)]$$
$$+ \delta d(x,y,a)$$

For each x, y in X with $x \neq y$ and S & T has a fixed point.

Proof: Let x_0 be an arbitrary point in X . Since S & T maps into itself there exist points x_1, x_2 in X such that $x_1 = S^{-1}(x_0)$ $x_2 = T^{-1}(x_1)$

Continuing the process, we get a sequence $\{x_n\}$ in C such that

$$x_{2n+1} = S^{-1}(x_{2n}) \quad \& \quad T^{-1}(x_{2n+1}) = x_{2n+2}$$
$$Sx_{2n+1} = x_{2n} \qquad Tx_{2n+2} = x_{2n+1}$$

We see that if $x_{2n} = x_{2n+1}$ for some n then x_{2n} is a common fixed point of S & T.

Therefore we suppose that $x_{2n} \neq x_{2n+1}$ for all $n \geq 0$

From the hypothesis

$$d(x_{2n}, x_{2n+1}, a) = d(Sx_{2n+1}, Tx_{2n+2}, a)$$
$$\geq \alpha \frac{[1+d(x_{2n+2},Tx_{2n+2},a)]d(x_{2n+1},Sx_{2n+1},a)}{1+d(x_{2n+1},x_{2n+2},a)} + \beta[d(x_{2n+1},Sx_{2n+1},a)+d(x_{2n+2},Tx_{2n+2},a)]$$
$$+\gamma[d(x_{2n+1},Tx_{2n+2},a)+d(x_{2n+2},Sx_{2n+1},a)] + \delta d(x_{2n+1},x_{2n+2},a)$$
$$\geq \alpha \frac{[1+d(x_{2n+2},x_{2n+1},a)]d(x_{2n+1},x_{2n},a)}{1+d(x_{2n+1},x_{2n+2},a)} + \beta[d(x_{2n+1},x_{2n},a)+d(x_{2n+2},x_{2n+1},a)]$$
$$+\gamma[d(x_{2n+1},x_{2n+1},a)+d(x_{2n+2},x_{2n},a)] + \delta d(x_{2n+1},x_{2n+2},a)$$
$$\Rightarrow (1-\alpha-\beta-\gamma)d(x_{2n},x_{2n+1},a) \geq (\beta+\gamma+\delta)d(x_{2n+1},x_{2n+2},a)$$
$$\Rightarrow d(x_{2n+1},x_{2n+2},a) \leq \frac{1-\alpha-\beta-\gamma}{\beta+\gamma+\delta} d(x_{2n},x_{2n+1},a)$$

where $k = \dfrac{1-\alpha-\beta-\gamma}{\alpha+\beta+\gamma} < 1$

$$d(x_{2n+1},x_{2n+2},a) \leq kd(x_{2n},x_{2n+1},a)$$

Similarly $d(x_{2n+2},x_{2n+3},a) \leq kd(x_{2n+1},x_{2n+2},a)$

Therefore we obtain $d(x_{n+1},x_{n+2},a) \leq kd(x_n,x_{n+1},a)$

Which shows that $\{x_n\}$ is a Cauchy sequence in X and so it has a limit μ is complete metric space, we have $\mu \in X$, there exists $v, w \in X$ such that $S(v) = \mu$, $T(w) = \mu$

From the hypothesis we have

$$d(x_{2n}, \mu, a) = d(Sx_{2n+1}, Tw, a)$$

$$\geq \alpha \frac{[1+d(w,Tw,a)]d(w,Sx_{2n+1},a)}{1+d(x_{2n+1},w,a)} + \beta[d(x_{2n+1},Sx_{2n+1},a)+d(w,Tw,a)]$$

$$+\gamma[d(x_{2n+1},Tw,a)+d(w,Sx_{2n+1},a)]+\delta d(x_{2n+1},w,a)$$

$$\geq \alpha \frac{[1+d(w,\mu,a)]d(w,x_{2n},a)}{1+d(x_{2n+1},w,a)} + \beta[d(x_{2n+1},x_{2n},a)+d(w,Tw,a)]$$

$$+\gamma[d(x_{2n+1},Tw,a)+d(w,x_{2n},a)]+\delta d(x_{2n+1},w,a)$$

Since $d(x_{2n},\mu,a)\to\infty$ as $n\to\infty$ we have $d(x_{2n+1},x_{2n},a)=d(x_{2n+1},w,a)=0$. Therefore $d(w,\mu,a)=0$ implies $w=\mu$ i.e $Sw=Tw=\mu$

This completes the proof of the theorem.

References

[1]. Agrawal, A.K. and Chouhan, P. “Some fixed point theorems for expansion mappings” Jnanabha 35 (2005) 197-199.

[2]. Agrawal, A.K. and Chouhan, P. “Some fixed point theorems for expansion mappings” Jnanabha 36 (2006) 197-199.

[3]. Bhardwaj, R.K. Rajput, S.S. and Yadava, R.N. “Some fixed point theorems in complete Metric spaces” International J.of Math.Sci. & Engg.Appls.2 (2007) 193-198.

[4]. Fisher, B. “Mapping on a metric space” Bull. V.M.I. (4), 12(1975) 147-151.

[5]. Jain, R.K. and Jain, R. “Some fixed point theorems on expansion mappings” Acta Ciencia Indica 20(1994) 217-220.

[6]. Jain, R. and Yadav, V. “A common fixed point theorem for compatible mappings in metric spaces” The Mathematics Education (1994) 183-188.

[7]. Park, S. “On extensions of the Caristi-Kirk fixed point theorem” J.Korean Math.Soc.19(1983) 223-228.

[8]. Park, S. and Rhoades, B.E. “Some fixed point theorems for expansion mappings” Math.Japonica 33(1) (1988) 129-133.

[9]. Popa, V. “Fixed point theorem for expansion mappings” Babes Bolyal University, Faculty of Mathematics and physics Research Seminor 3 (1987) 25-30.

[10]. Rhoades B.E. “A comparison of various definitions of contractive mappings” Trans. Amer Math.Soc.226 (1976) 257-290.

[11]. Sharma.P.L., Sharma.B.K. and Iseki, K.“Contractive type mappings on 2-metric spaces” Math.Japonica 21 (1976) 67-70.

[12]. Taniguchi, T. “Common fixed point theorems on expansion type mappings on complete metric spaces” Math.Japonica 34(1989) 139-142.

[13]. Wang, S.Z. Gao, Z.M. and Iseki, K. “Fixed point theorems on expansion mappings” Math.Japonica .29 (1984) 631-636.