

# **Designing Continuous-Time Observers for Linear Hybrid Systems with Application to Three Tank Model**

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## **Abstract**

In this paper, two methods are studied and compared for designing continuous-time observers in linear hybrid systems. The methods are proposed based on Kalman filter and Luenberger observer design methods in linear systems. Three tank system is also considered as a hybrid system with unknown state variables. The linear hybrid model is obtained by linearizing the model of three tank system around its equilibrium point. In addition, it is also assumed that a state feedback controller is utilized for controlling the linearized hybrid system in order to stabilize the system. Simulation results depict the effectiveness and applicability of the proposed methods for state observation in hybrid systems. Comparison of two proposed methods shows that the optimal observer obtained using Kalman filter design method has better performance than Luenberger method.

**Keywords:** Linear hybrid system, Three tank system, Continuous-time observers, Kalman filter, Luenberger observer.

## **Introduction**

In recent years, the study and control of hybrid systems has been considered by many researchers. Such systems are modeled in a special framework provided for describing processes based on continuous dynamics, discrete dynamics and logic rules. Same as many real systems, the states of these systems may be inaccessible and thereby the control of them is not feasible in this condition. The use of observer is therefore required for controlling hybrid system with unknown state variables such that the estimated values are used instead of state variables which are not reachable. There are well-known methods for designing observers in linear systems such as Kalman filter [1], Luenberger observer [2], alpha-beta [3] and Wiener filter [4]. For hybrid systems, the state variables in continuous or discrete parts or both of them may be unavailable and thus continuous-time or discrete-time observers should be designed for each part of hybrid systems. In other words, hybrid systems have continuous and discrete observers. In such systems, discrete dynamic is usually raised from a logic or switching law which is based on design criteria and can be available. Thus, continuous state estimation associated with the continuous dynamics of hybrid systems is more important for control design purpose.

In [5], the authors revealed how to choose observer parameters so that its estimation error is independent of arbitrary plant disturbances. Contrary to previous researches, in [5], the plant disturbances were not assumed to have any mathematical structure. Also, conditions under which an observer can reject plant disturbances were presented. In [6], Gauthier construct an observer for nonlinear and autonomous systems. In [7], an estimation approach was presented for a class of hybrid discrete-time linear systems using Luenberger observers. The proposed Luenberger observer for such a kind of systems relied on the switching among different gains. It was shown that convergence conditions are to ensure the stability of the error dynamics and the related gains may be selected by solving a set of linear matrix inequalities (LMIs). In [8] a methodology for designing dynamical observers for hybrid plants was proposed. In [9], the observability properties of a class of hybrid systems, whose continuous variables are available for measurement, were considered. They depicted that the discrete variable dynamics can be always extended for observable systems to a lattice in such a way that the extended system has the properties that allow the construction of the LU discrete state estimator. In [10], delay approach to continuous-discrete observer design for Lipschitz nonlinear systems was discussed. In [11], a motion sensorless control was proposed for single-phase permanent magnet brushless dc motor based on an I-f starting sequence and a real-time permanent magnet flux estimation.

A well-known case study of hybrid system is three-tank model introduced in [12]. Three tanks system is composed of three tanks of liquid which are connected to each other through pipes and controllable valves. The control objective in this system is usually to preserve the liquid level of tanks in a desirable value and the control objective can be achieved by controlling the liquid flows provided by pumps or adjusting the interconnecting valves. In [13], observability analysis and observer synthesis were studied for a three-tank water process. Observability of the process was considered under various assumptions on measurements. In observer design, the singularity of nonlinear observers was also considered and water level was controlled by using two pumps by a PI controller. In [14], methodologies for designing limited look ahead supervisory controllers were discussed for a class of embedded systems that can be modeled as switching hybrid system (SHS). The authors discussed the controller design and implementation for a three-tank system test-bed with distributed sensor and actuation units. A set of real-time fault adaptive control experiments demonstrated the effectiveness of the

approach .However , disadvantage of their work was considering the pumps as a discrete input. In [15],an identification method was proposed for a class of hybrid systems which were linear and separable in the discrete variables (that were discrete states and discrete inputs). The method took cognizance of the fact that the separable structure of the hybrid system constrains the evolution of system dynamics. In particular, the proposed method identified models corresponding to a certain number of modes, far fewer than the total possible modes of the system. It then generated the models for the remaining modes without any further requirement for input–output data by exploiting the separable structure of the hybrid system. Because of many new parameters to substitute ,the model has a high complexity and high computational effort.

An important class of theoretical and practical problems in communication and control rises from a statistical nature (Kalman [1]). Such problems are: (i) prediction of random signals,(ii) separation of random signals from random noise and(iii) detection of signals of known form (pulses, sinusoids) in the presence of random noise .In a pioneering work, Wiener [4] revealed that problems (i)and (ii) lead to the so-called Wiener-Hopf integral equation. He also gave a method (spectral factorization) for the solution of this integral equation in the practically important special case of stationary statistics and rational spectra. In [4], the objective is to obtain the specification of a linear dynamic system (Wiener filter) which accomplishes the prediction, separation, or detection of a random signal. In[3](alpha- betha), it was shown that the state vector of a linear system can be reconstructed from observations of the system inputs and outputs. It was shown that the observer, which reconstructs the state vector, is itself a linear system whose complexity decreases as the number of output quantities available increases. The observer may be incorporated in the control of a system which does not have its state vector available for measurement. The observer supplies the state vector ,but at the expense of adding poles to the overall system. In much of modern control theory designs are based on the assumption that the state vector of the system to be controlled is available for measurement. In many practical situations only a few output quantities are available.

Application of theories which assume that the state vector is known is severely limited in these cases. In [2] (Luenberger) shown that the state vector of a linear system can be reconstructed from observations of the system inputs and outputs.

In this paper, the problem of designing a continuous-time states observer to estimate the continuous-time dynamics of hybrid systems is investigated. It is assumed that we know our discrete-time states. In this system, it is assumed that the continuous and discrete dynamics can be linearized. It is also assumed that the system is observable and controllable. Furthermore, a state feedback controller is considered to control the system. The controller operates based on the state estimation obtained from an observer in the continuous-time part of hybrid system. The linear hybrid model is obtained by linearizing the model of three tank system around its equilibrium point. In addition, it is also assumed that a state feedback controller is utilized for controlling the linearized hybrid system in order to stabilize the system. The methods of observer design are based on Kalman filter and Luenberger method .To demonstrate the efficiency of designed observer in estimating the continuous state of model a three tanks system will be considered.

In the next section ,hybrid systems observers will be introduced. In Section 2, three tanks system model is introduced. Section 3presents and compares the proposed methods for designing observers in linearized hybrid systems. Simulation results are presented in Section 4 to show the applicability of proposed methods in three tank model. Finally ,conclusion remarks are drawn in the last section.

## **1- Observers in hybrid systems**

This section is devoted to the introduction of hybrid system formalism and to the formulation of the observability problem for hybrid systems. In this section, we present a general definition of hybrid systems that includes a large class of hybrid systems. The notion of a hybrid system that has been used in the control community is centered around a particular composition of discrete and continuous dynamics. In particular, a hybrid system has continuous evolution and occasional jumps. The jumps correspond to the change of state in an automaton whose transitions are caused by controllable or uncontrollable external events or by the continuous evolution. A continuous evolution is associated to each discrete state, described by differential equations, which may have different structure for each discrete state, and an initial state that is determined whenever a transition into the discrete state of the automaton is taken [16].

Technological innovation pushes towards the consideration of systems of a mixed continuous and discrete nature, which are sometimes called hybrid systems. Hybrid systems arise, for instance, from the combination of an analog continuous-time process and a digital time asynchronous controller. Control theorists and computer scientists (and others) are joining forces to approach the huge challenges in this field. The simplest definition of a hybrid system is a system whose behavior is characterized by several modes of operation. In each mode the evolution of the continuous state of the system is described by its own difference or differential equation. The system switches between the various modes when a particular event occurs. We will typically study hybrid systems in which the continuous input will be designed and the mode transitions are externally induced by state events [12].

Figure1 shows a typical model of hybrid system. In this Figure,  $\sigma$  and  $\Psi$  are discrete input and discrete output and  $u$  and  $y$  are continuous input and continuous output respectively.

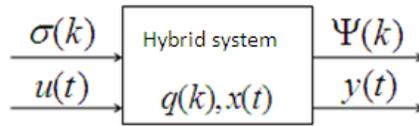


Figure 1. Hybrid system model

State estimation problem has been one of the critical issues in control theory and applications in hybrid systems. A reliable state estimate for a process is indispensable not only for control purpose but also for other applications such as navigation of spacecraft, monitoring, and fault diagnosis in mechanical systems[21]. As is well-known, given the measurements  $y =$

$g(x, u)$  for the process  $\dot{x} = f(x, u)$ , it is possible to reconstruct the full-state by the estimator

$\hat{x} = f(\hat{x}, u) + K(y - \hat{y})$  with innovation process [22]. However, since the mathematical model  $(f, g)$  is only an approximation to the physical process and the actual plant is usually affected by external disturbances, there always exists a discrepancy between the real states and estimates. Like the other feedback systems, the uncertainty effect not considered in the plant model could be significantly reduced by increasing the estimator gain  $K$ . However, those with innovation process alone certainly have a limit to cover all uncertainties with wide ranges in a frequency spectrum. As a simple example, conventional state estimators such as Luenberger observer or Kalman filter produce biased estimates when the plant is under biased external disturbances. Then Hybrid observers that we use in this paper are Kalman filter and Luenberger method.

## 2- Three tank system model

The Schematic of three tank system is shown in Figure 2. Tanks 1 and 3 are filled independently by two pumps.

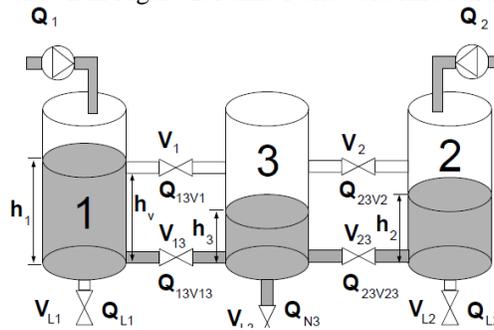


Figure 2. Three tank system model [12]

In this figure,  $Q_1$  and  $Q_2$  are flow of tank 1 and tank 2 respectively changed continuously from 0 to  $Q_{max}$ .  $h_1$ ,  $h_2$  and  $h_3$  are liquid height of each tank, and can be measured by a level sensor. The height of each tank is continuous variable and behavior of each valve is discrete, So, three tank can be considered as a hybrid system. The equations of the three tank can be considered as follows[12]:

$$\dot{h}_1 = \frac{1}{A} (Q_1 - Q_{13V1} - Q_{13V13} - Q_{L1}) \tag{1}$$

$$\dot{h}_2 = \frac{1}{A} (Q_2 - Q_{23V2} - Q_{23V23} - Q_{L2}) \tag{2}$$

$$\dot{h}_3 = \frac{1}{A} (Q_{13V1} + Q_{13V13} + Q_{23V2} + Q_{23V23} - Q_{N3}) \tag{3}$$

Where (definition is same as Figure2)  $Q$  with any indices else{1,2} is the flow of liquid else from pumps,  $H_i, i = \{1, 2, 3\}$  is liquid height of each tank and for simplicity it is assumed that the surface area of each tank have a same surface area  $A$ .

For designing observer, it is assumed that the discrete variables are available, and we consider that valves of  $V_1$  and  $V_2$  and  $V_{L1}$  and  $V_{L2}$  are closed and  $V_{13}$  and  $V_{23}$  are opened. Therefore, the equations of three tank will be as follows:

$$\dot{h}_1 = \frac{1}{A}(Q_1 - Q_{13V13}) \quad (4)$$

$$\dot{h}_2 = \frac{1}{A}(Q_2 - Q_{23V23}) \quad (5)$$

$$\dot{h}_3 = \frac{1}{A}(Q_{13V13} + Q_{23V23} - Q_{N3}) \quad (6)$$

The outlet flows in each tank based on Bernoulli's equation in steady states modeled as a simple orifice can be considered as  $Q_{i3Vi3} = a_i \sqrt{H_i}$  with indices  $i=\{1,2\}$  and  $Q_{N3} = a_3 \sqrt{H_3}$  Where,  $a_1, a_2, a_3$  are proportional constants which depend on the coefficients of discharge, the cross sectional area of each orifice and the gravitational constant and  $H_i$   $i = \{1, 2, 3\}$  is liquid height of each tank .According to Equations (4)- (6), nonlinear state equations described the system dynamics of the three tanks can be explained as

$$\dot{h}_1 = \frac{1}{A}(Q_1 - a_1 \sqrt{H_1}) \quad (7)$$

$$\dot{h}_2 = \frac{1}{A}(Q_2 - a_2 \sqrt{H_2}) \quad (8)$$

$$\dot{h}_3 = \frac{1}{A}(a_1 \sqrt{H_1} + a_2 \sqrt{H_2} - a_3 \sqrt{H_3}) \quad (9)$$

Suppose that for  $Q_1$  and  $Q_2$ , the fluid level in the tanks are at steady state level  $H_1, H_2$  and  $H_3$ .

If small variation in  $Q_1$  is  $q_1$  and in  $Q_2$  is  $q_2$ , then the resulting perturbation in levels are  $h_1, h_2$  and  $h_3$  respectively. From Equations (7) up to (9), the equation becomes:

$$(H_1 + h_1) = \frac{1}{A}((Q_1 + q_1) - a_1 \sqrt{H_1 + h_1}) \quad (10)$$

$$(H_2 + h_2) = \frac{1}{A}((Q_2 + q_2) - a_2 \sqrt{H_2 + h_2}) \quad (11)$$

$$(H_3 + h_3) = \frac{1}{A}(a_1 \sqrt{H_1 + h_1} + a_2 \sqrt{H_2 + h_2} - a_3 \sqrt{H_3 + h_3}) \quad (12)$$

From (7)-(12), the equations are

$$\dot{h}_1 = \frac{1}{A}(q_1 - a_1 \sqrt{H_1 + h_1} - \sqrt{H_1}) \quad (13)$$

$$\dot{h}_2 = \frac{1}{A}(q_2 - a_2 \sqrt{H_2 + h_2} - \sqrt{H_2}) \quad (14)$$

$$\dot{h}_3 = \frac{1}{A}(a_1 \sqrt{H_1 + h_1} - \sqrt{H_1} + a_2 \sqrt{H_2 + h_2} - \sqrt{H_2} - a_3 \sqrt{H_3 + h_3} - \sqrt{H_3}) \quad (15)$$

$$\sqrt{H_1 + h_1} = \sqrt{H_1} \left(1 + \frac{H_1}{2H_1}\right) \quad \sqrt{H_1 + h_1} - \sqrt{H_1} \approx \frac{h_1}{2\sqrt{H_1}}$$

Then for small perturbations, we have Therefore,

$$\sqrt{H_2 + h_2} - \sqrt{H_2} \approx \frac{h_2}{2\sqrt{H_2}} \quad \text{and} \quad \sqrt{H_3 + h_3} - \sqrt{H_3} \approx \frac{h_3}{2\sqrt{H_3}}$$

Similarly we have the approximations of Equation is

$$h_1 = \frac{1}{A} \left( q_1 - \frac{a_1 h_1}{2\sqrt{H_1}} \right) \tag{16}$$

$$h_2 = \frac{1}{A} \left( q_2 - \frac{a_2 h_2}{2\sqrt{H_2}} \right) \tag{17}$$

$$h_3 = \frac{1}{A} \left( \frac{a_1 h_1}{2\sqrt{H_1}} + \frac{a_2 h_2}{2\sqrt{H_2}} - \frac{a_3 h_3}{2\sqrt{H_3}} \right) \tag{18}$$

$$h_1 = \frac{1}{A} \left( q_1 - q_{01} - \frac{a_1 h_1}{2\sqrt{H_1}} \right) \quad , \quad h_2 = \frac{1}{A} \left( q_2 - q_{02} - \frac{a_2 h_2}{2\sqrt{H_2}} \right) \quad \text{and}$$

The above Equations can also be written as

$$h_3 = \frac{1}{A} \left( \frac{a_1 h_1}{2\sqrt{H_1}} + \frac{a_2 h_2}{2\sqrt{H_2}} - \frac{a_3 h_3}{2\sqrt{H_3}} - q_{03} \right) \quad \text{where } q_{01}, q_{02} \text{ and } q_{03} \text{ represent perturbations in the outflow at the}$$

drain pipes .we haven't any perturbations in output then if we rewritten this equation with laplace transform we have equations as following

$$(T_1 S + 1)h_1(s) = k_1 q_1(s) \tag{19}$$

$$(T_2 S + 1)h_2(s) = k_2 q_2(s) \tag{20}$$

$$(T_3 S + 1)h_3(s) = k_1 h_1(s) + k_2 h_2(s) \tag{21}$$

Where,

$$T_1 = \frac{A}{a_1} \quad T_2 = \frac{A}{a_2} \quad T_3 = \frac{A}{a_3}$$

$$k_1 = \frac{1}{\frac{a_1}{2\sqrt{H_1}}} \quad k_2 = \frac{1}{\frac{a_2}{2\sqrt{H_2}}} \tag{22}$$

Each value of  $a_1, a_2, a_3, A, H_1, H_2$  can be obtained from plant and those values are  $H_1 = 7, H_2 = 8, H_3 = 10$  and  $a_1 = 10.78, a_2 = 11.03, a_3 = 11.03$  and  $A = 32$ . By solving Equations (22) with those values  $a_1, a_2, a_3, A, H_1, H_2$  the value of  $T_1, T_2, T_3, k_1,$  and  $k_2$  can be determined as following table.

**Table 1: value of parameters of Equation (19)-(21)**

$T_1 = 2.2439$
$T_2 = 2.0514$
$T_3 = 1.8349$
$k_1 = 0.0701$
$k_2 = 0.0641$

### 3- Designing Observers in Hybrid Systems

Schematic of a hybrid system with continuous-time and discrete-time observers for such systems are shown in Figure 3. In

this Figure,  $\tilde{q}$  and  $\tilde{x}$  are estimation of location observer and continuous observer respectively. It can be seen that both observers can be used for state observation in this system. However, in this paper it is assumed that only continuous-time state variables are unknown.

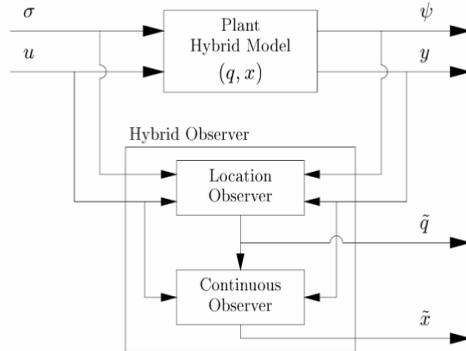


Figure 3. General structure of observers in a hybrid system

Suppose that the discrete states of the three tanks are available. So, only the continuous variable of the three tank system which is the height of liquid in the third tank in the schematic (as shown in Figure 4) should be estimated.

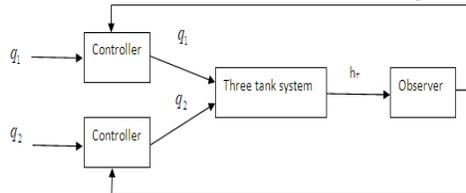


Figure 4. Observer of three tank system

For designing observer, the three tank system equation is considered as defined in Section 2. The structure of Luenberger observer is shown in Figure 5.

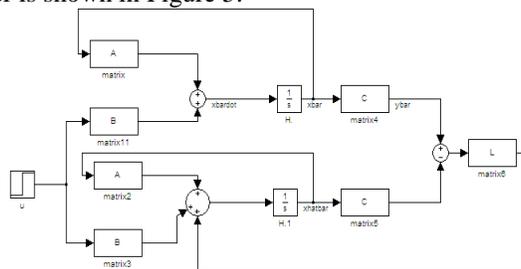


Figure 5. Luenberger Observer schematic

A symmetric positive definite matrix  $P$  in Luenberger method as theorem in [7], is the solution of the algebraic Lyapunov inequalities

$$(A_i - L_i C_i)^T P (A_i - L_i C_i) - P < 0 \quad i = 1, 2, \dots, k \quad (23)$$

The observer involves an estimation error which asymptotically convergent to zero.  $L$  is the observer gain.

Kalman filter is another method which is used for designing observers. Structure of Kalman observer is shown in Figure 6.

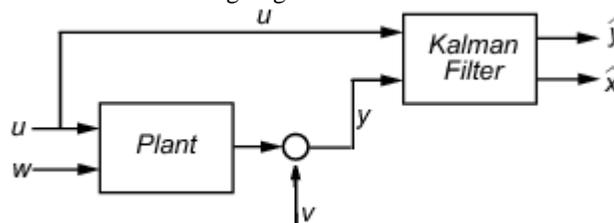


Figure 6. Schematic of Kalman Observer

Where  $u$ ,  $w$ ,  $v$  and  $y$  are input, white process noise, and white measurement noise and measured output, respectively. The state estimates are  $\hat{y}$  and  $\hat{x}$ . Note that  $\hat{y}$  estimates the third tank output.

#### 4- Simulation Results

In this section, the observers are designed using Kalman filter and Luenberger observer design methods and in presence of a state feedback controller. It is supposed that the discrete states of the three tanks are available. So, only the continuous variable of the three tank system i.e. the liquid level of third tank will be estimated, as shown in Figure 4.

It is assumed that both continuous and discrete dynamics of hybrid system are linear. Discrete dynamics of system are known and the system does not have any perturbation.

The matrices A,B,C are known from state-space model of three tank system where

$$A = \begin{bmatrix} -0.9906 & -0.4858 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0 & -1.032 & -0.5313 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.0625 & 0 \\ 0 & 0 \\ 0 & 0.0625 \\ 0 & 0 \end{bmatrix}, C = [0 \quad 0.03819 \quad 0 \quad 0.03493]$$

and matrices P, L (gain observer of Luenberger method) are selected as follows:

$$P = 10e-003 * \begin{bmatrix} 0.1331 & 0.1175 & -0.0873 \\ 0.1175 & 1.4445 & -0.0371 \\ -0.0873 & -0.0371 & 0.0960 \end{bmatrix},$$

$$L = \begin{bmatrix} 552.4211 \\ -21.2599 \\ 650.3919 \end{bmatrix}$$

In addition, matrix K as in [1] for Kalman filter is designed to be:

$$K = \begin{bmatrix} 0.0009 & 0.0019 & 0.0008 & 0.0017 \\ 0.0008 & 0.0016 & 0.0007 & 0.0014 \end{bmatrix}$$

System output in absence of observer and controller is shown in Figure 7.

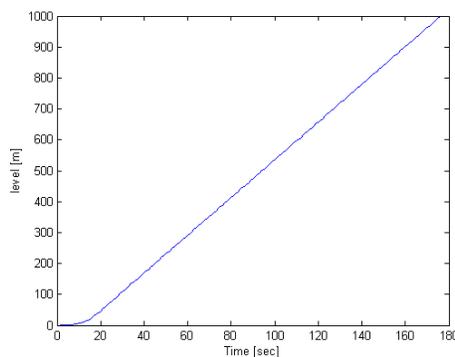
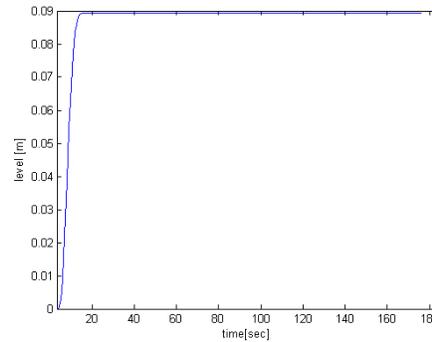


Figure 7. System output without any control and observe

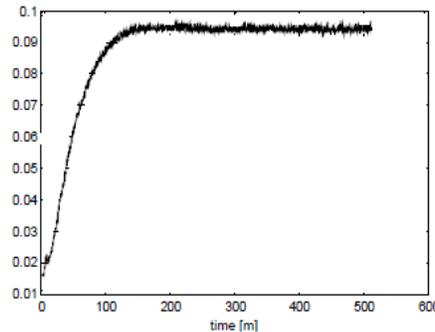
It can be seen that the level of third tank with the swinging form goes to infinity.

When the state feedback control is added to the system, system output (the level of the third tank) is in the form of Figure 8.



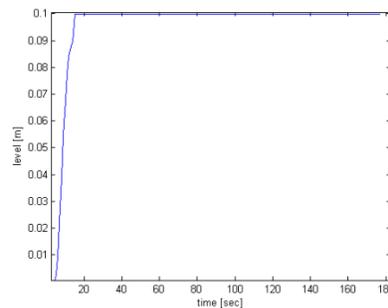
**Figure 8. System output when we have state feedback control**

When the state feedback control and Luenberger observer are used ,the result could be in the form of Figure 9 and the output could be estimated as 9.97082



**Figure 9. System output when we have state feedback control and Luenberger observer**

When the model with the proposed Kalman filter method is used, the output is as the form shown in Figure 10.



**Figure 10. System output with state feedback control and Kalman filter**

where the third tank height becomes and keep to 9.99999 cm.

It is obvious that without observer and controller in the system, the output goes to infinity because of the nature of the system. The controller could keep the output to steady state, but it is far from the ideal point. When the Luenberger method is used, the result will come near to ideal but Kalman filter is the better way to force it to ideal point.

## 5- Conclusion

In this paper, a three tanks system has been considered as a hybrid system and by considering a closed loop control system ,the problem of observers design for this system has been studied .Luenberger and Kalman filter observers have been designed by considering the state feedback controller as the system controller. The system controller uses an estimation of the output ,i.e. the pump inlet ,from observer in every time in order to achieve the desired level of the third tank. This filter can be applied easily to any hybrid systems. Comparing two observers show that Kalman filter can perform better response for hybrid systems specially when there exist noise, disturbance or uncertainty.

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