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Design and construction of a didactic robotic arm with four degrees of freedom

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Abstract
The design and construction of a didactic robotic arm of four degrees of freedom (4GL) for an institutional interest, constituted the starting point of the research presented that focused on the determination of the substantive elements provided by the process of mathematical modeling in the training of professional skills in future engineers. It was developed at the National Technological Institute of Mexico / Tantoyuca campus, with students from the 6th semester of Electronic Engineering in order to investigate the role played by students in an active learning environment based on projects and mathematical modeling. The results obtained report findings, moments where the student takes an active and leading role in their learning process, developing activities of professional interest that demand challenges.
The mathematical model obtained allows us to know the dynamics and the direct kinematics of the robot arm with four degrees of freedom, as well as determining the position and orientation of the robot's end, with respect to a coordinate system that is taken as a reference, known the values of the joints and the geometric parameters of the elements of the robot arm.
Key words: Electronic Engineering, Mathematical modeling process, Professional skills, Problem resolution,Project Based Learning, robotic arm

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I. INTRODUCTION

Globalization processes currently demand professionals with the capacity to solve problems, who make good decisions, creative and innovative in solutions appropriate to the context. Currently in higher education in Mexico, the aim is to develop new pedagogical strategies that allow the training of engineering professionals with the competencies required in increasingly dynamic and globalized social and work environments. These new methodologies seek to enhance the development of competencies: learning to learn, organize and plan, analyze and synthesize, apply knowledge to practice, express yourself clearly orally and in writing, critical and self-critical capacity, work collaboratively, ability of initiative and leadership and knowing a second language (Schmal, 2012).

In 21st century higher education, learning focuses on competencies and attributes of graduation. Faced with this need, CACEI (the Accreditation Council for Engineering Education) proposes that higher education institutions offer quality education and points out essential attributes in the graduate (solving engineering problems, carrying out adequate engineering design processes, conducting experimentation). adequate, communicate effectively, acknowledge your ethical and professional responsibilities, constantly update yourself, work as a team).

Given these demands, it is necessary to focus the design of new pedagogical approaches and a paradigm shift in the teaching-learning process in the training of future engineers. The design of a project-based learning model and the process of mathematical modeling (ABP-mathematical modeling) are proposed. Methodologies that require the student to include their learning process and request mental-physical effort to find the optimal solution to the phenomenon under investigation.

Project Based Learning engages students in projects that provide solutions to a problem situated in reality and make them participate in the development of specific solutions. An active-collaborative learning model in which a related problem is addressed in a physical or social environment. In an environment that demands challenges, analysis, reflections and debates. Where strategies are proposed to tackle the phenomenon, brainstorming, solution tactics, search for pertinent information, solution approach and communication of results before a scientific community or society.

The mathematical modeling process involves processes of simplification, idealization and structuring of a phenomenon, mathematization of the phenomenon, formulation of the mathematical model, study, interpretation and validation of data in order to justify the coherence of the mathematical model. Contreras and Martínez-Cruz (2009) consider that mathematization is inscribed in the actions involved in the construction of a representation that accounts for the aspects described in a daily or scientific situation.

Modeling is a very important process in learning mathematics, it allows students to observe, reflect, discuss, explain, predict, revise and in this way construct concepts in a meaningful way; an environment where experimentation and simulation are used to study the variables in real time that include the mathematical model of the phenomenon under study. Biembemgut and Hein (2004) suggest some strategies to approach the mathematical modeling process: exposition of the topic, delimitation of the problem, formulation of the problem, development of programmatic content, presentation of analogous examples, formulation of a model and resolution of the same from the model, interpretation of the solution and validation of the model. This process demands challenges from the student that require physical and intellectual effort, in which the ability to explore, investigate and learn about the world around them is put into play. Actions that place him in a leading role in his own learning process.

It is necessary to underline, the research proposes a learning model focused on the professional interest of the student, in an engineering context where optimal solutions are requested that demand challenges.

A didactic-pedagogical model framed in project-based learning and the process of mathematical modeling, an environment where mathematics is used as a tool to awaken the critical and creative sense in the development of tasks located in the study of a phenomenon.

The nature of this type of active learning is built facing challenges, cooperation between student-teacher. Fidalgo, Sein, and García (2017) Challenge-based learning can be applied in the context of the student's academic environment, an environment where students are involved in solving the challenge posed, this method includes cooperative learning, debate and communication; strategy that encompasses an applied vision of various academic subjects.

This active learning model seeks to be the managerial process of the paradigm shift in higher education, an action that will provide students with the opportunity to build their learning process from their own experiences that request challenges and stimulate curiosity and develop professional skills .Offering the possibility of exploring, manipulating, suggesting hypotheses, making and recognizing mistakes, justifying, exposing and debating.

In an environment where communication is the main vehicle of learning and evaluation is considered as a process of learning construction and continuous improvement. On the other hand, the mathematical modeling process is considered as a scientific activity in mathematics that is involved in obtaining models of the other sciences. This scientific process is a tool for the representation of phenomena, with the purpose of obtaining observations, data, reflections, proposals, conclusions.

The intention of this research is to attend to and solve an institutional problem located in the electronic engineering area of the InstitutoTecnológico Superior de Tantoyuca. The problem: the electronic engineering laboratory does not have a didactic instrument for modeling, simulation and control; to carry out real-time practices in the subjects (vector calculus, linear algebra, mechanism analysis, programming and robotics). For the solution, it is required to design and build a robot arm based on programming and analysis of mechanisms, based on mathematical models in order to predict and reproduce the movements using joints with different arm lengths. With this, achieve real-time practices with complex movements and simulation of the joints of four degrees of freedom: rotation, position, orientation, center of mass and speed of the joints.

General purpose

Design and build a robot arm taking mathematical models as the point of joint design to obtain direct kinematics with a homogeneous transformation matrix using the Denavit-Hartenberg algorithm and obtain robot dynamics, using the Matlab programming tool to real-time simulation for decision-making in the design and construction of the 4GDL robotic arm (four degrees of freedom).

Specific objectives

- Apply the mathematical modeling process to obtain the mathematical model that represents and reproduces the joints with different dimensions of rotation, position and orientation of the robotic arm.
- Apply homogeneous transformation matrices.

• Calculate the direct kinematics from the homogeneous transformation matrix with the denavit-hartenberg algorithm.
• Use Matlab programming software to simulate the 4GL robot arm.

II. DEVELOPMENT

The importance of granting an active role to students in their learning process is fundamental and inescapable, the potential that these collaborative-active learning methodologies generate are of considerable benefit to students. Rué Domingo (2016) asserts that active and collaborative learning is one of the aspects that define effective educational practice; Students learn more when they are intensively involved in their education and are asked to think and implement what they are learning in different settings. For Rué Domingo (2016) the elements that are combined in active learning are: purpose, meaning, complexity, autonomy, reflection and metacognition. Learning must be active, not passive. In learning-focused classes, students need to be actively involved (McCombs, 2001). They must have active learning opportunities and, to a large extent, act in diverse contexts and build their own knowledge (Stroh and Sink, 2002). They must learn by doing, not passively sitting by listening.

Students learn when they are involved in their learning construction process, understanding that physical and mental energy are required to build their own learning. This can be achieved when the student manages to reflect and asks himself how much I have learned, what I need, the questioning of his own beliefs and values, what are the necessary things to approach the subject. That is why it is important to plan didactic strategies that are going to be used to develop: critical thinking, decision making, self-directed learning, cooperation between equals, conflict resolution, lifelong learning, social skills, etc. It seems, then, that this learning approach can give an effective response to the new training needs of students in their process as future engineers.

Using project-based learning (ABP) as a learning teaching strategy brings tremendous benefits to students. For Cobo Gonzales and Valdivia Canotte (2017) project-based learning is a methodology that is developed in a collaborative way that confronts students with situations that lead them to make proposals in the face of certain problems.

This type of project-based learning is built on an active-collaborative learning framework, an environment that encourages students to investigate in a real engineering context, where they are asked to propose various solutions and choose the optimal one. Rojas (2005) exposes some benefits of ABP; prepares students for jobs. Students are exposed to a wide variety of skills and competencies such as collaboration, project planning, decision making, and time management. In addition, it ventures into decision-making, where essential issues must be identified, the participation of other people with the appropriate profiles must be sought and their points of view analyzed; with the intention of building solid evidence to justify decision making.

Various authors have proposed active pedagogical strategies, as an alternative to traditional teaching, in which the student becomes the axis of the training process. Some of those strategies are: problem-based learning (ABP); project-oriented learning (AOP); competency-based learning (ABC) (Vega, Portillo, Cano &Navarrete, 2014).

Driver and Bell (1986) state, if we want the objective of science learning to be fulfilled, we need to take into account the latest knowledge provided by research on the scientific knowledge of students, about the acquisition and development of concepts , especially the data that hold that learning is an active process in which students build and rebuild their own understanding in light of their experiences.

The mathematical modeling process is considered as a scientific activity in mathematics that is involved in obtaining models of the other sciences, it does not happen automatically or immediately, it requires a certain period of time in which the researcher uses knowledge mathematicians, the knowledge of the context and the phenomenon to be investigated and their abilities to describe, establish and represent the relationships between variables in such a way that a model can be built that represents the phenomenon under investigation. Berrío (2011) observes how students, when committing themselves to the study of phenomena, not only interpret and (re) construct mathematical models, but also (re) construct their considerations regarding the phenomenon itself, becoming a prominent factor towards dynamization. . In engineering, the mathematical modeling process constitutes an indispensable professional competence of practical utility, where mathematics is considered as a tool to investigate, model and solve real case problems that arise in everyday life.

For Biembengut and Hein (2004), the modeling process involves a series of procedures, namely, choosing the theme; recognition of the situation / problem; delimitation of the problem; familiarization with the subject to be modeled; theoretical reference; problem formulation; hypothesis; formulation of a mathematical model, development; resolution of the problem from the model, application; interpretation of the solution and validation of the model. The modeling process is considered as a tool in engineering to study real world phenomena; with the purpose of obtaining conclusions from the phenomenon under study.

It is important to highlight that the mathematical modeling process aims to find a mathematical model that allows us to better explain a real contextual situation in terms of the phenomenon under study. Considering that it is important to distinguish when talking about mathematical modeling between model and modeling; modeling refers to the process while model refers to the result or product of that process. Modeling is described as a process that starts from a real situation or problem and that ends, through a succession of steps or phases, in a mathematical model that responds to the real problem or situation initially raised. In the process, previous steps are taken up, with which the process adopts a cyclical behavior.

Mathematical modeling is a process involved in obtaining a mathematical model. A mathematical model of a phenomenon or problem situation is a set of mathematical symbols and relationships that represents, in some way, the phenomenon in question. The model allows not only to obtain a particular solution, but also to serve as a support for other applications or theories. The model allows not only to obtain a particular solution, but also to serve as a support for other applications or theories. In practice, this set of symbols and relationships can be linked to any branch of mathematics, in particular, to the fundamental instruments of mathematical applications (Biembengut, 1999, p.20).

In the present investigation, an active-collaborative learning model is proposed that contributes to the process of building professional skills in future engineers. The didactic strategy is framed in the processes: Project-based learning and mathematical modeling. This didactic-pedagogical approach involves the student in a project located of interest and in connection with the environment. An approach that calls for mental-physical effort to find the optimal solution of the phenomenon under investigation.

Project Based Learning (ABP)

In project-based learning, students face phenomena that demand challenges, usually throughout an academic period, applying the fundamental concepts and principles learned previously. The ABP provides an active learning experience that engages the student in a challenging and meaningful project. According to Camacho (2014) project-based learning is located within the broad framework of collaborative and experiential university education: it promotes active, critical and creative learning, where students commit to their learning process and that of their peers . In addition, it favors the development of other transversal competences and is a fundamental element for the motivation of the students. Learning process, which approaches a concrete reality in an academic environment, through the completion of a work project. The contradictions that arise and the pathways to their solution contribute to making this object of pedagogical influences an active subject. Likewise, project-based learning offers the opportunity to achieve significant knowledge by solving situations in the professional field. According to DíazBarrigaArceo (2015), it is required to work on real projects and tasks, from daily life or from a professional competence field, facing practical, concrete and realistic experiences. Project-based learning is a method that attaches great importance to the process of researching around a topic to solve complex problems from open solutions, or to the process of tackling difficult topics that allow the generation of new knowledge (De Miguel , 2005).

The mathematical modeling process

The process of mathematical modeling occupies an increasingly important place in the training of engineers, either in terms of skills or content. Modeling offers the learning process a role that is appropriate to the current era in education; in which the results and the decision making assisted by the engineers are in real time. The objective is to provide the student with mathematical tools that support him to model real situations situated in the context of engineering. In engineering, the mathematical modeling process constitutes an indispensable competence; models are used extensively in engineering, industry and manufacturing. For example, engineers use skyscraper computer models to predict their resistance and how they would behave in an earthquake. Aircraft manufacturers use elaborate mathematical models to predict the aerodynamic properties of a new design before actually building the aircraft, etc.

The mathematical modeling process involves a series of actions or phases that make the construction or interpretation of a model not develop instantly, the cycle begins with the choice or determination of a phenomenon or problem in the real world until evaluation, interpretation and validation of the mathematical model. It is essential to identify in this process all the factors involved in the study of the phenomenon and make the necessary simplifications to build the mathematical model that represents the phenomenon.

Biembengut and Hein (2006) the modeling process involves a well-articulated series of phases: choice of theme; recognition of the problem phenomenon / situation & delimitation of the research phenomenon; familiarization with the subject to be modeled & theoretical referential; formulation of the problem & hypothesis; formulation of a mathematical model & development; problem resolution from the & application; model interpretation of the solution and model validation & evaluation. The elaboration of a mathematical model requires, on the part of the modeler, knowledge of both mathematics and other sciences, in addition, a good dose of intuition and creativity to interpret the context and discern what are the variables involved (Biembengurt and Hein, 1999, pp . 12-13).

III. MATHEMATICAL MODEL

The concept of mathematical model has been present in many fields of engineering, some definitions of mathematical model are raised that refer largely to the vision of mathematics in relation to reality. A Mathematical Model is defined as a mathematical construction aimed at studying a particular real-world system or phenomenon. This model can include graphs, symbols, simulations, and experimental constructions. (Giordano Weir and For, 1997). A mathematical model is a mathematical description of a real world phenomenon. As defined by Biembengut and Hein (2004), a mathematical model of a phenomenon or problem situation is a set of symbols and mathematical relationships that represents, in some way, the phenomenon in question.

The purpose of this model is to understand the phenomenon, experiment and make predictions regarding future behavior. The generation of a mathematical model is not done immediately, on the contrary, it requires a certain period of time in which the researcher is challenged to test his knowledge of the subject in question, knowledge of the context, skills to describe, establish and represent the existing relationships, identify variables with the intention of building a mathematical model that represents the phenomenon under investigation, in order to obtain conclusions of the phenomenon under study and subject it to a procedure of observation and experimentation of its behavior with the intention of identifying the behavior of the variables and predict at a certain moment the present and future behavior of the situation for decision-making.

Cruz (2010) considers that the design of didactic experiences in the training of engineering students should be content-oriented, the development of mathematical thinking, the usual work of an engineering professional and the development of strategic thinking oriented towards use. of design, via mathematical modeling. Engineering design is not a finished product, but a methodology that relies on knowledge, inventiveness, creativity and awareness of the concept of urgency, to visualize a real problem, formulate it in technical terms, explore possible solutions, evaluate alternatives, propose one or more forms or solution routes, evaluate the possible processes that need to be used and their corresponding results, select one of the best solutions based on a set of criteria, execute the necessary actions to carry out a particular proposal and evaluate the process and the results of each and every one of the actions, permanently making adjustments and corrections and issuing judgments and recommendations that are supported by facts, preferably quantifiable. For Krick (1996) he considers engineering design to be a five-phase process with inputs and outputs: problem formulation; problem analysis; investigation; decision making; specifications (details of the proposed solution).

IV. METHODOLOGY

The methodology used in this research is a strategic articulation of the processes: project-based learning (ABP) and the process of mathematical modeling.

Phases of the mathematical modeling process in the development of the Project

Phase I (Formulation of the problem): This phase specifies the real problem situated in a context of interest (delimitation of the phenomenon or problem). Defining a real model by simplifying, structuring and idealizing the phenomenon under investigation, the first task is to formulate the phenomenon using the language of mathematics (the representation of reality is carried out through the process of mathematization). The theoretical and physical considerations of the phenomenon are studied up to an interpretation of the associated data (obtaining data involves mathematization or the use of mathematical skills).

Phase II Solve (definition of elements, data collection and analysis): Transform the real phenomenon into a mathematical model; formulate the mathematical model that represents the problem by identifying the variables (independent and dependent). The importance of the analysis of the initial conditions of the phenomenon under investigation is highlighted, in order to obtain the model that relates and articulates the variables. The model is treated mathematically in different representations (graphical, numerical and algebraic) transiting between representations with the intention of constructing solid arguments that justify the model.

Phase III Interpret (formulation and simulation of the mathematical model): Experiment with the mathematical model in order to deduce mathematical conclusions and interpret the data obtained in light of the study phenomenon. The intention is to reflect and offer correct explanations and / or to build the necessary predictions. Once the mathematical model is built, the appropriate mathematical techniques will be used to carry out the relevant experiments or simulations to observe and analyze the behavior of the variables with the intention of building predictions and justifications for the phenomenon under study.

Phase IV evaluate (validation and documentation of the mathematical model): Validate the model built in the light of solutions obtained, analyzing the veracity of the data obtained (experiment, verify and test the predictions). Comparing them against new data and relative to the studied phenomenon. In this phase it is necessary to check the precision of the model, see how well it describes the original real situation and how well it predicts past and / or future behavior. If the predictions (established hypotheses) do not relate appropriately to reality, then either the model needs to be fine-tuned or a new one needs to be formulated and the cycle started (return to phase 1).

It is important to note that the process of mathematical modeling in a flexible process of improvement continues with approximations to the ideal model requested by the phenomenon under study. In the design of experiments, observations are made, data are adjusted and the model that represents the phenomenon in real time continues to be improved.

Evidence of the methodology to build the mathematical model of the "4GL Didactic Robot Arm"
The objective of the methodology was to find the mathematical model to obtain the rotation, position and orientation of the joints of the robot arm through direct kinematics using homogeneous transformation matrices, using the Denavit-Hartenberg algorithm. Also, the use of Matlab numerical programming software to check and simulate the mathematical model.
Previous knowledge:Research fundamentals, vector calculus, linear algebra, mechanism kinematics analysis, programming, robotics, code management in Matlab.
Specific topics:Matrix operations, Cartesian reference system, rotation matrices, translation matrices, homogeneous transformation matrices to obtain the equation of the position and orientation of the joints, Matlab code to support mathematical modeling, use of the Peter I toolbox. Corke, Denavit-Hartenberg algorithm for the assignment of the didactic robot arm parameters.
Phase 1The type of robot to be built is defined, in this project the construction of a vertical angular, revolution or anthropomorphic Robot was chosen, which simulates the parts of the body, using rotational joints and to activate their movements stepper electric motors are used.
Phase 2.In order to carry out its analysis, it was necessary to know the measurements of the four links of the robot, these are carried out from the axis of the joint N-1 to the axis of the joint N. where N-1 is the initial position of the measurement and N is the final position of each link. Figure 1 shows the model of the 4GL robot arm.

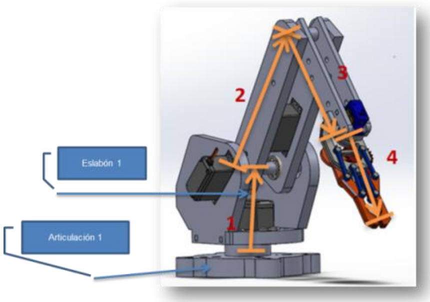

Figure 1.Model of the 4GL didactic Robot arm.Source: own design

Phase 3.The "homogeneous transformation matrix T translation" was designed to obtain the equation of the position and orientation of the joints. Vector and matrix algebra were used to represent and describe the location of an object in three-dimensional space with respect to a fixed reference system. Since a robot can be considered as a kinematic chain formed by rigid objects or links linked together by joints. In this way, the direct kinematic problem is reduced to finding a homogeneous transformation matrix of translation T that relates the position and orientation of the end of the robot with respect to the fixed reference system located at its base. This matrix T is the function of the joint coordinates and is represented by:

$$T = \begin{bmatrix} nx & ox & ax & px \\ ny & oy & ay & py \\ nz & oz & az & pz \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{n} & \boldsymbol{o} & \boldsymbol{a} & \boldsymbol{p} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The resolution of the direct kinematic problem consists in finding the relationships that make it possible to know the spatial location of the end of the robot from the values of its joint coordinates.
In the case of analysis, the four degrees of freedom robot is made up of four links joined by four joints, so that each joint-link pair constitutes a degree of freedom. A solidarity reference system can be associated to each link and, using the homogeneous transformations, it is possible to represent the relative rotations and translations between the different links that make up the robot. Normally, the homogeneous transformation matrix representing the position and relative orientation between the systems associated with these four consecutive robot links is usually called the $\mathbf{{}^{0}A_{4}}$ **matrix**:

$$\mathbf{{}^{0}A_{4} = {}^{0}A_{1}\,{}^{1}A_{2}\,{}^{2}A_{3}\,{}^{3}A_{4}}$$

Thus, given the robot with four degrees of freedom, we have that the position and orientation of the final link will be given by the matrix T:

$$T = {}^{0}\mathbf{A}_4 = {}^{0}\mathbf{A}_1\,{}^{1}\mathbf{A}_2\,{}^{2}\mathbf{A}_3\,{}^{3}\mathbf{A}_4$$

Although any reference system linked to each element can be used to describe the relationship between two contiguous elements, the usual form that is usually used in robotics is the Denavit-Hartenberg (D-H) representation. Denavit and Hartenberg proposed in 1955 a matrix method that allows to establish in a systematic way a coordinate system {S1} linked to each link i of an articulated chain, being able to determine the kinematic equations of the complete chain.

According to the representation of D-H, choosing appropriately the coordinate systems associated with each link, it will be possible to go from one to the next through four basic transformations that depend exclusively on the geometric characteristics of the link.

These basic transformations consist of a succession of rotations and translations that allow the reference system of element i to be related to the system of element i-1. The transformations in question are as follows (it is important to remember that the passage from the $\{S_{i-1}\}$ to $\{S_i\}$ system by means of these four transformations is guaranteed only if the $\{S_{i-1}\}$ and $\{S_i\}$ systems have been defined according to certain rules that will be explained later):

1. Rotation around the zi-1 axis an angle θ_i.
2. Translation along zi-1 a distance d_i; vector d_i (0,0, d_i).
3. Translation along xi a distance a_i; vector a_i (0,0, a_i).
4. Rotation around the xi axis by an angle α_i.

Since the product of matrices is not commutative, the transformations must be carried out in the indicated order. In this way you have to:

$$^{i-1}\mathbf{A}_i = \mathbf{T}(z,\theta_i)\ \mathbf{T}(0,0,d_i)\ \mathbf{T}(a_i,0,0)\ \mathbf{T}(x,\alpha_i)$$

Making the product between matrices, we obtain the representative matrix of each of the links:

$$^{i-1}A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{i-1}A_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i\sin\theta_i & \sin\alpha_i\sin\theta_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\alpha_i\cos\theta_i & -\sin\alpha_i\cos\theta_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Phase 4.Obtaining the mathematical model of the four degrees of freedom robot.Direct kinematics with homogeneous matrices

Table 1.D-H parameters for the robot (the values of the links given in meters).

Articulation	θ	d	a	α
1	θ_1	$d_1 = 0.09948$	0	90
2	θ_2	0	$a_2 = 0.15$	0
3	θ_3	0	$a_3 = 0.11062$	0
4	0	0	$a_4 = 0.10555$	0

We make the product of the matrices for each link based on the Denavit-Hartenberg (D-H) algorithm process, the following matrices are obtained.

$$A_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{1}A_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}^{2}A_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & a_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{3}A_4 = \begin{bmatrix} 1 & 0 & 0 & a_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

We calculate the position and orientation of the first and second link of the robot with respect to the base coordinate system.

$$ {}^{0}A_2 = \ {}^{0}A_1 \ {}^{1}A_2 $$

$$ {}^{0}A_2 = \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\cos\theta_1\sin\theta_2 & \sin\theta_1 & a_2\cos\theta_1\cos\theta_2 \\ \cos\theta_2\sin\theta_1 & -\sin\theta_1\sin\theta_2 & -\cos\theta_1 & a_2\cos\theta_2\sin\theta_1 \\ \sin\theta_2 & \cos\theta_2 & 0 & d_1 + a_2\sin\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

We calculate the position and orientation of the first, second and third link of the robot with respect to the base coordinate system.

$$ A_3 = \ {}^{0}A_1 \ {}^{1}A_2 \ {}^{2}A_3 = \ {}^{0}A_2 \ {}^{2}A_3 $$

$$ {}^{0}A_3 = \begin{bmatrix} \cos(\theta_2+\theta_3)\cos\theta_1 & -\sin(\theta_2+\theta_3)\cos\theta_1 & \sin\theta_1 & \cos\theta_1(a_3\cos(\theta_2+\theta_3) + a_2\cos\theta_2) \\ \cos(\theta_2+\theta_3)\sin\theta_1 & -\sin(\theta_2+\theta_3)\sin\theta_1 & -\cos\theta_1 & \sin\theta_1(a_3\cos(\theta_2+\theta_3) + a_2\cos\theta_2) \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 & d_1 + a_3\sin(\theta_2+\theta_3) + a_2\sin\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

We calculate the position and orientation of the four links of the robot with respect to the base coordinate system, in this way we obtain the transformation matrix T that indicates the location of the final system.

$$ T = {}^{0}A_4 = \ {}^{0}A_1 \ {}^{1}A_2 \ {}^{2}A_3 \ {}^{3}A_4 = \ {}^{0}A_3 \ {}^{3}A_4 $$

$$ T = \ {}^{0}A_4 $$

$$ = \begin{bmatrix} \cos(\theta_2+\theta_3)\cos\theta_1 & -\sin(\theta_2+\theta_3)\cos\theta_1 & \sin\theta_1 & \cos\theta_1(a_3\cos(\theta_2+\theta_3) + a_4\cos(\theta_2+\theta_3) + a_2\cos\theta_2) \\ \cos(\theta_2+\theta_3)\sin\theta_1 & -\sin(\theta_2+\theta_3)\sin\theta_1 & -\cos\theta_1 & \sin\theta_1(a_3\cos(\theta_2+\theta_3) + a_4\cos(\theta_2+\theta_3) + a_2\cos\theta_2) \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 & d_1 + a_3\sin(\theta_2+\theta_3) + a_4\sin(\theta_2+\theta_3) + a_2\sin\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Based on the expression of the Transformation Matrix T we have the final position of the robot

$$ Px = \cos\theta_1(a_3\cos(\theta_2+\theta_3) + a_4\cos(\theta_2+\theta_3) + a_2\cos\theta_2) $$
$$ Py = \sin\theta_1(a_3\cos(\theta_2+\theta_3) + a_4\cos(\theta_2+\theta_3) + a_2\cos\theta_2) $$
$$ Pz = d_1 + a_3\sin(\theta_2+\theta_3) + a_4\sin(\theta_2+\theta_3) + a_2\sin\theta_2 $$

Phase 6. Using the MATLAB tool, the mathematical model of the Robot's position of four degrees of freedom is checked. For direct kinematics calculations with homogeneous matrices in MATLAB, a script was made. Getting the following

```
>> robot_4GDL
robot =
brazorobotico 4GDL:: 4 axis, RRRR, stdDH, slowRNE
+---+-----------+-----------+-----------+-----------+-----------+
| j |     theta |         d |         a |     alpha |    offset |
+---+-----------+-----------+-----------+-----------+-----------+
|  1|         q1|    0.09948|          0|     1.5708|          0|
|  2|         q2|          0|       0.15|          0|          0|
|  3|         q3|          0|    0.11062|          0|          0|
|  4|         q4|          0|    0.10555|          0|          0|
+---+-----------+-----------+-----------+-----------+-----------+
```

Transformation Matrix =

1	0	0	0.3662
0	0	-1	0
0	1	0	0.09948
0	0	0	1

In figure 2 Simulation of the 4GL didactic robot arm obtained from the Matlabsoftware and Figure 3 shows the front and side views of the design of the 4GL didactic robot arm.

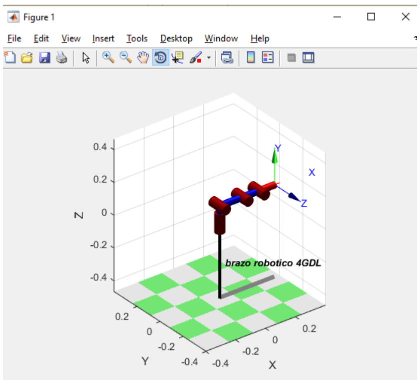

Figure 2.4GDL robot in MATLAB. Source: own design

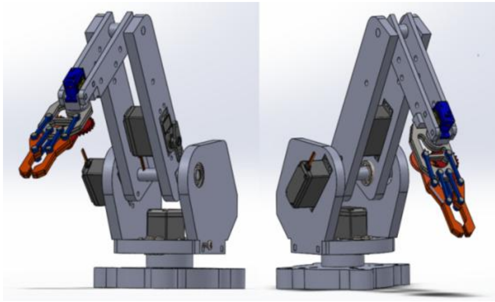
Figure 3. Views of the 4GL robot arm. Source: owndesign

V. RESULTS AND DISCUSSION

The evidences report findings on the role that didactic activities play in a context of professional interest and the sense of reality in the implementation of the mathematical modeling process within the process of building professional skills in the training of future engineers. An alternative look at the meaning and application of mathematics at the engineering level is also exposed.The findings found during the development of the mathematical modeling process at different times are presented below.At first, the problem was set in a context of professional interest, the students accept the challenge demanded by the project "design and construction of a 4GL didactic robot arm". The first task was to identify the initial conditions, prior knowledge, and the variables involved. So also structure the first approach to the mathematical model that represents the phenomenon under investigation. To carry out these tasks, the students actively participated in the presentation of hypothesis-ideas, collaborative work and debates. The mathematization process was used to study the theoretical and physical considerations of the phenomenon until interpreting the associated data. Using matrix operations to obtain homogeneous transformation matrices that represent the position and orientation of the joints of the robot arm. Activities where mathematical concepts give meaning and application to mathematics, allowing to approach and study a phenomenon under investigation.

In a second moment, the project is delimited and directed, they rethink hypotheses, debate and discuss the solution methods; activities that allow students to use cognition and metacognition. A second approach to the mathematical model is formulated that represents the problem identifying and articulating variables. The model is represented mathematically in different representations (graphical, numerical and algebraic) transiting between representations with the intention of building solid arguments that justify the model. In order to build the mathematical analysis model, it was necessary to know the requirements: technical and physical specifications of the robot arm. And through the process of mathematization, the mathematical model that represents the robot arm is obtained through a homogeneous transformation matrix of translation T, in order to obtain the equation of the position and orientation of the joins.

In a third moment, the group debate is opened, a process of validation and refinement of the proposals made in the previous phases, moments where the collaborative group work among the students is observed. The teacher takes a promoter role in the exposition of the ideas, stimulating the participation of the students and promoting the establishment of justifications. To justify the arguments presented, experiments are carried out with the mathematical model of the robot with four degrees of freedom, which allows predicting the position and orientation of the four links of the robot with respect to the base coordinate system. Using the homogeneous transformation matrix "T", the location of the final system of the robot arm is indicated. To perform this task, the MATLAB mathematical software is used, the relevant experiments or simulations are performed to observe and analyze the behavior of the variables with the intention of building predictions and justifications of the mathematical model with respect to the position of the Robot with four degrees of Freedom. Experimenting with the mathematical model manages to deduce mathematical conclusions and interpret the data obtained in light of the study phenomenon. The intention was to reflect and offer correct explanations and / or to build the necessary predictions of the model.

VI. CONCLUSIONS

Experimentation is intended to capture the attention and motivate the student, in addition to generating a space where the student has the opportunity to interact with variables under study in real time, discover the explanation of the phenomenon under investigation using metacognitive processes. The evidence reports findings from a project-based learning environment and the mathematical modeling process, situated in the student's professional interest with an active-collaborative learning dynamic and a leading role for the student. The intention of this didactic methodology was to generate the inclusion and development of competences in real environments, facing authentic situated projects that demand challenges. Methodology that considers the process of mathematical modeling with a focus on the use of mathematics applied to the other sciences, linked to the study of situations and solution of real-world problems.

Thus, mathematical modeling is also considered as the complete process of moving from a real situated problem to a representative mathematical model, where students take an active role and acquire the challenge to find the solution to the engineering context phenomenon, as well as the acquisition technological and collaboration skills. In this sense, the Organization for Economic Cooperation and Development (OECD, 2010) exposes the importance of the development of mathematical skills and specifically of mathematical modeling skills.The nature of this type of project allows the student to focus on the training process as a future engineer. Likewise, it is important to consider the orientation of the contents, the development of mathematical thinking, the usual work of an engineering professional and the development of strategic thinking oriented towards the engineering design process.

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